A Regularized Nonlinear Diffusion Approach for Texture Image Denoising

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Abstract

In this paper a new partial differential equation based method is presented with a view to denoising images having textures. The proposed model combines a nonlinear anisotropic diffusion filter with recent harmonic analysis techniques. A wave atom shrinkage allied to detection by gradient technique is used to guide the diffusion process so as to smooth and maintain essential image characteristics. Two forcing terms are used to maintain and improve edges, boundaries and oscillatory features of an image having irregular details and texture. Experimental results show the performance of our model for texture preserving denoising when compared to recent methods in literature.

1. Introduction

A quite relevant subject in the image processing area is image denoising. Applications in this area include image transmission, image recording, and image information processing, among others. The basic idea of denoising is to recover a damaged image, which was originally obtained by means of some deterioration model, that is, given an image with noise, the objective is to obtain a noiseless image by means of an answer to the corresponding inverse problem. For this, many denoising methods assume that the desired image is sectionally smoothed while noise is characterized by a high frequency oscillation. To deal with this problem, various techniques based on nonlinear methods have been proposed, these include adaptive smoothing, anisotropic diffusion, variational methods, wavelet techniques, iterative regularization and nonlinear inverse scale methods [1], [2], [3], [4], [7], [16], [17], [18], [19]. Nevertheless, these models are not appropriate for treating natural highly detailed images because they have a marked presence of oscillatory characteristics such as textures and intrinsic structures, in addition to noise.

To try to bypass these problems, there is a growing tendency of researches that consider a combination of before mentioned techniques allied to recent ones of harmonic analysis such as curvelets and wave atoms, in addition to independent applications of these two transformations as shown in [8], [9] and [22]. Over the last two years, new hybrid methods have been proposed with a view to integrate partial differential equations, especially anisotropic diffusion and variational methods, with curvelets and wave atoms, which were presented in [14], [15], [16], [20]. However, as these models are directly based on those transformations, practical applications tend to substantially smooth the image, besides reproducing the Gibbs phenomenon which is the problem of oscillations close to points of discontinuous.

To solve the problem, in this paper we propose a Partial Differential Equation (PDE) based on nonlinear regularized diffusion capable of restoring a noisy image having a high concentration of textures and fine details without the necessity of excessively smoothing important image characteristics such as edges, boundaries and textures. The proposed method combines the ideas described in [1], [2] and [17], that is, a balanced anisotropic diffusion PDE to recent harmonic analysis works [8], [9] and [10] about wave atoms so as to produce a hybrid filter balanced by a certain diffusivity term.

This paper is organized as follows. In section 2 we present some existing models for the noise removal from images having textures. In section 3 we describe the proposed model. Numeric implementation is discussed in section 4, while in section 5 we show the experimental results and an analysis of its performance in comparison to other methods. We end with section 6 with our conclusions about the proposed filter.

2. PDE-based methods for denoising

Many PDE-based techniques have been proposed aiming to smooth an image having noise. In most cases, those techniques are directly obtained by introducing a diffusive character evolutionary equation or variations of the classical total variational model presented by Rudin et al.(ROF) [21]. In both of these categories, a constant preoccupation has always been the reconstruction of the image without losing edges and boundaries.

Aiming to maintain these characteristics and simultaneously remove noise, Barcelos et al. [2] proposed the following nonlinear anisotropic PDE-diffusion,

$$\frac{\partial u}{\partial t} = g|\nabla u| div\left(\frac{\nabla u}{|\nabla u|}\right) + \alpha(1-g)(I-u), \quad (1)$$

where $I(x) = u(x,0)$ represents the observed image (with noise), $u(x,t)$ is its smoothed version on a scale $t \in \mathbb{R}_+$, Ω is the domain of both images and α is a tuning parameter. The boundary conditions for the above problem are of the Neumann type. Here, $g = g(s) \in [0,1]$ is given by

$$g(s) = \frac{1}{1+s^2}, \quad (2)$$

with $s = \sqrt{\kappa}\,|\nabla(G_\sigma * u)|$, where G_σ is the Gaussian function, σ denotes the standard deviation of the image I and κ is a constant. Function g is frequently, in literature, called a diffusivity term or regularizer term and is used to detect edges and locally control the degree of diffusion incidence, that is, if ∇u has a small average value near neighborhood of a point x (average calculated by the convolution product $G_\sigma * u$), then x is considered an interior point and the applied diffusion by equation (1) will be intense. On the other hand, if the average value in the neighborhood of x is large, then x is considered as being a point belonging to a boundary area and the diffusion would be small, smoothing homogeneous regions and preserving edges. Thus, the anisotropic filter (1) would smooth the homogeneous regions and preserve edges and boundaries.

Indeed the model (1) does not produce satisfactory results from images with textures, because the diffusivity term g acts locally on the point x, based on the calculation of the "average value" (convolution) of its neighboring points, however, points contained in texture regions possess sudden value oscillations, even in small neighborhoods, which can possibly interfere in obtaining that average.

In [16], Plonka et al. proposed a PDE-based method directed to the problem of images with textures. The proposed model was

$$\frac{\partial u}{\partial t} = dig(g\,\nabla(Pu)), \quad (3)$$

with Neumann boundary condition, $g = g(|\nabla(Pu)|)$ given by the diffusivity term presented in (2), P represents the projection operator which is determined by a curvelet shrinkage [5], [6].

The equation (3) can be seen as TV diffusion [21] managed by the regularization term Pu, which identifies the texture in u. Here, the edge detection obtained by g is more sensitive than that presented in the previous model, since it is based on a diffusivity term that best captures texture, as is the case with curvelet transforms. The idea of the authors was to use the diffusive process to minimize the pseudo-Gibbs artifacts produced by the curvelets.

A year later, a variant of equation (3) was proposed by the same authors in [20], based on the same diffusion process presented above with the addition of a reaction term $(SI - u)$, intending to preserve and improve data marked as texture. In that paper, I denotes the observed image and S is an operator which can be defined as from a wavelet, curvelet or wave atom shrinkage.

The idea of anisotropic diffusion guided by harmonic analysis techniques to preserve and improve the characteristics of an image can also be found in [14] and [15].

Another important method in literature is the model based on the behavior of local adaptability considered by Gilboa et al. [11] which is defined in relation to the classic model ROF of total variation proposed in [21], given as follows

$$\inf_{(u,I)\in BV(\Omega)\times L^2(\Omega)} \left\{ \int_\Omega |\nabla u| dx + \frac{\lambda}{2}\int_\Omega |I-u|^2 dx \right\} \quad (4)$$

where $BV(\Omega)$ denotes the space of functions of bounded variation and λ is a parameter to be chosen. The first term in (4) denotes the total variation of u while the second is a data-fidelity term. The authors of [11] investigated the model (4) and proposed a new version having the capacity to preserve textures during the diffusion process. Instead of taking λ constant as was adopted in (4), they defined $\lambda = \lambda_I(x)$ variable in all Ω, attributing to λ the capacity to represent local properties of the image, that is, in the identification of regions characterized by textures. The PDE obtained in [11] based on a modified version of (4) is given by

$$\frac{\partial u}{\partial t} = div\left(\Phi'(\nabla u)\frac{\nabla u}{|\nabla u|}\right) + \lambda_I(x)(I-u+C) \quad (5)$$

where C is a constant. Here, $div\left(\Phi'(\cdot)\frac{\nabla u}{|\nabla u|}\right)$ represents the diffusion adaptive process while $\lambda_I(x)(I-u+C)$ is the reaction term, which is locally balanced by λ, thus contributing to the maintenance of the texture in u.

Other recent models of texture-preserving denoising coming from the ideas presented in [11] can be found in [12] and [13].

3. The proposed model

Let u be an image on a gray scale and I its version containing noise, represented by the functions $u : \Omega \rightarrow \mathbb{R}$ and $I : \Omega \rightarrow \mathbb{R}$, respectively.

We assumed that the domain Ω is a rectangular region of $\mathbb{R}^2$ and that the noise present in I is additive, that is,

$$I(\mathbf{x}) = u(\mathbf{x}) + \eta(\mathbf{x}),\ \mathbf{x} = (x, y) \in \Omega, \tag{6}$$

where η represents the noise (in this paper we adopted the Gaussian noise) with a mean 0 and variance σ_η^2

$$\int_\Omega \eta(\mathbf{x}) d\mathbf{x} = 0 \quad \text{and} \quad \int_\Omega \eta^2(\mathbf{x}) d\mathbf{x} = \sigma_\eta^2\ . \tag{7}$$

Moreover, we supposed that the original image u contained oscillatory structures such as texture and irregular details. The objective here is to reduce the noise level of input image I, that is, the impact of the noise $\eta(\mathbf{x})$ must be minimum in the output smoothed image, making it visually closer to u without the loss of textures and other oscillatory details.

Encouraged by [2], [8], [11], [16] and [20], in this paper we propose a nonlinear balanced PDE,

$$\frac{\partial u}{\partial t} = h|\nabla u| div \left(\frac{\nabla u}{|\nabla u|} \right) + \alpha(1-h)(I-u) + \\ + \beta(1-h)(SI-u) \tag{8}$$

where h is the diffusivity term, S is the operator described in [20] and α, β are weighting parameters.

The term $|\nabla u| div \left(\frac{\nabla u}{|\nabla u|} \right)$ is called the mean curvature flow (MCF) which is responsible for the diffusion process in the model. It diffuses u in an orthogonal direction of its gradient vector ∇u and does not diffuse it in any other direction.

In our model, we propose a regularizer term $h = h_\mu(\sqrt{\delta}\,|\nabla Su|)$ as follows

$$h = \begin{cases} g(\sqrt{\delta}\,|\nabla Su|), & \text{if} \quad g(\sqrt{\delta}\,|\nabla Su|) \leq \mu \\ g(\sqrt{\kappa}\,|\nabla u|), & \text{if} \quad g(\sqrt{\delta}\,|\nabla Su|) > \mu \end{cases} \tag{9}$$

where g is taken as being (2) and δ, κ are tuning parameters. Here, μ is a threshold value that controls the capturing of gray shades in relation to $g(\sqrt{\delta}\,|\nabla Su|)$.

We can rewrite the equation (9) for $\mathbf{x} \in \Omega$ as follows:

$$h_\mu(\mathbf{x}) = g(O_1(\mathbf{x}))\chi_{D_\mu} + g(O_2(\mathbf{x}))\chi_{D_\mu^c}, \tag{10}$$

where $O_1(\mathbf{x}) = \sqrt{\delta}\,|\nabla Su|$ and $O_2(\mathbf{x}) = \sqrt{\kappa}\,|\nabla u|$ are operators, $D_\mu = \{\mathbf{x} \in \Omega : g(O_1(\mathbf{x})) \leq \mu\}$, $\Omega = D_\mu \cup D_\mu^c$ and μ is a suitable fixed threshold value.

Taking a hybrid diffusivity term is justified by the following facts: while $g(O_1(\mathbf{x}))$ adequately captures texture regions, that is, $g(O_1(\mathbf{x})) \sim 0$ at those points, the homogeneous regions of the image, where diffusion must be more intense, they are also defined by shades of gray close to black. On the other hand, the term g set $g(O_2(\mathbf{x})) \sim 1$ in homogeneous regions, but in the regions characterized by texture and other intrinsic details, g does not efficiently capture the information. In this case, we would like $g(O_1(\mathbf{x}))$ as well as $g(O_2(\mathbf{x}))$ to get near the extremities, but they should not reach such values as this would completely annul the contribution of the diffusion or of the two reaction terms considered in (8), creating abrupt image discontinuities. Fig.1 below clearly illustrates this fact:

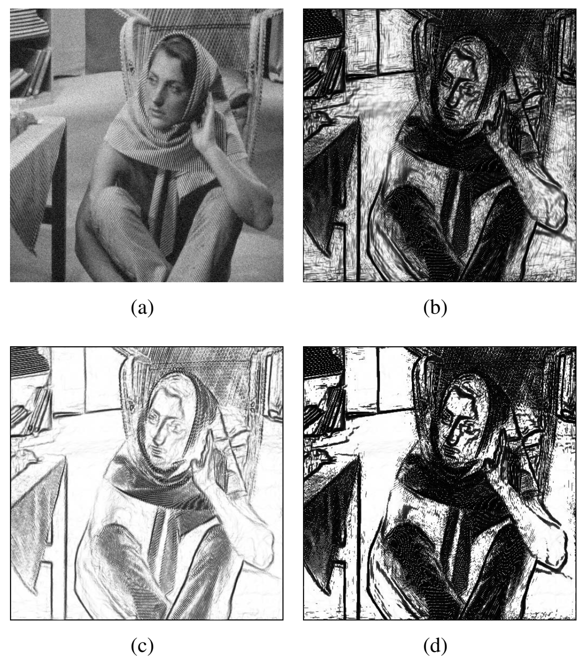


Figure 1. The proposed diffusivity term h. (a) Observed image; (b) term $g(O_1(\mathbf{x}))$; (c) $g(O_2(\mathbf{x}))$ and (d) h.

The equation (8) can be understood as a balancing between smoothing and "keeping close" the I and SI components. This balancing is governed by the term h, which is used as edge/ texture detector and also to control the velocity of the diffusion. As described before, it can be observed that the homogeneous regions of the image has a big h. Then, $(1-h) \sim 0$ and thus the reaction terms $(I-u)$ and $(SI-u)$ act in a practically insignificant way in the composition of (8). Consequently, the MCF equation presented in the first part of (8) will considerably smooth the image. On the other hand, for the regions, h is small, which implies in $(1-h) \sim 1$. This makes the forcing term $(I-u)$ retain edges and boundaries of the noise image I simultaneously with the reaction term $(SI-u)$ to preserve and reconstruct oscillatory characteristics of the image such as texture and other details. Thus, both reaction terms have the function of reducing the degenerative effects of the diffusion process and of restoring important image features.

Here we use S for the (8) reaction term and also to identify regions defined by textures. This operator is defined based on the recent and useful harmonic analysis tool called wave atoms [8], [9] and [10].

Wave atoms are a variant of 2D wavelet packets obeying the important relationship of the wavelength $wavelenght \sim (diameter)^2$ parabolic scale, which improves the sparse representation of certain oscillatory patterns. This means that distorted oscillatory functions (e.g. oriented textures) have a significant sparser expansion in wave atoms than in other representations such as wavelets, Gabor atoms or curvelets. Wave atoms composition elements have high directional sensitivity and are anisotropic, which makes them ideal for applications where the intention is to identify regions characterized by oscillatory patterns such as textures or subtle details.

Since the objective is to characterize surfaces having textures (fingerprint, seismic profile, engineering surfaces, photographs, etc), there is a great advantage in applying wave atoms transformations to the proposed problem.

For further information as to wave atoms systems, we suggest [8], [9] and [10].

The extension of proposed method to the treatment of color images can be done by considering the representative system of color RGB, applying the filter in each of the three color channels independently, since each of these channels consists of a gray scale image. In this case, the equation of degradation (6) should also be extended by the classical vector addition.

4. Numerical approximations

We assume that all the images are numerically represented by matrices where each element u_{ij} denotes the intensity value of the image $u(x,y)$ at the point $(x,y)=(x_i,y_j)$, with $x_i = i\Delta x$ and $y_j = j\Delta y$. To simplify, we adopt $\Delta x = \Delta y = 1$. We denoted $u(x_i,y_j,t_n)$ by u_{ij}^n, where $t_n = n\Delta t$. Here, t represents the scalar "temporal" parameter and Δt its step.

The derivatives in (8) are numerically approximates using finite difference techniques, which are quite precise when implementing computational algorithms.

The temporal derivative $\dfrac{\partial u}{\partial t}$ is calculated in (x_i,y_j,t_n) by the advanced difference $\dfrac{\partial u}{\partial t} \sim \dfrac{u_{ij}^{n+1}-u_{ij}^n}{\Delta t}$. The degenerated diffusion

$$|\nabla u| div\left(\frac{\nabla u}{|\nabla u|}\right) = \frac{u_{xx}u_y^2 + -2\,u_x u_y u_{xy} + u_{yy}u_x^2}{u_x^2+u_y^2}$$

is numerically approximated by the use of central difference schemes.

In (9), $|\nabla u|$ is obtained by the direct calculation of the Euclidian norm with quantization similar to that shown before, while with to compute the signals obtained from S we proposed wave atom shrinkage as follows:

$$Sf = (WA)^{-1}\Theta_\theta(WA)(f), \qquad (11)$$

where f can be taken as u or I, WA denotes the wave atom transform [8], $(WA)^{-1}$ the inverse transformation and Θ_θ the hard threshold function, which is given by

$$\Theta_\theta = \Theta_\theta(z) = \begin{cases} z, & |z| \geq \theta \\ 0, & |z| < \theta, \end{cases} \qquad (12)$$

with θ being the threshold value. For the numerical implementation of wave atoms transforms, we have adopted the discretization suggested in [8], where the authors proposed a model based on the fast Fourier transform and a wrapping trick. For further details about discrete wave atom transforms, see also [9].

Using Neumann's boundary conditions, we calculated $u_{ij}^{n+1}, n = 1,2,...,N$, by discrete formula of (8)

$$u_{ij}^{n+1} = u_{ij}^n + \Delta t\,\Gamma(u_{ij}^n), \quad \forall\,(i,j) \in \Omega \qquad (13)$$

with $u_{ij}^0 = I(x_i,y_j)$, N denoting the iteration number and $\Gamma(u)$ representing the right side of (8), except for calculation of h, which is computed previously for each iteration of (13).

Algorithm

Input: I (noisy image), Δt, N, α, β, θ, γ, κ, μ.
Output: $\widetilde{u}$ (reconstructed image).
1. *Compute SI by equation* (11) *and set $v^1 = SI$.*
2. *Set initial boundary conditions for u.*
3. *For $n = 1 : N$ do*
   - *Compute $O_1(i,j) = \sqrt{\delta}|\nabla v_{ij}^n|$, $O_2(i,j) = = \sqrt{\kappa}|\nabla u_{ij}^n|$ and $h = h_{ij}$ according to expression* (9), $\forall\,(i,j) \in \Omega$;
   - *Update u_{ij}^{n+1} using equation* (13), with h *has been computed in the previous step*;
   - *Update $v^{n+1} = Sv^n$ by equation* (11);

   End For

5. Experimental Results

To certify the good performance of the proposed PDE-based filter, we compared it to some recent models that have appeared in literature. In all cases, images were used on a gray scale defined with a standard range of $[0, 255]$ represented by matrices of dimension 256×256, except Figure 3. In our model it was adopted $\beta = 1$ and $\Delta t = 0.1$ in all experiments considered.

Our first experiment mentions the real known image of Barbara Fig.2(a). Here the image contaminated with noise Fig.2(b) (SNR $\cong 9.97db$) contains important features to be preserved and recuperated, among them: the texture above the neck, in the background, and the intrinsic details on the face and proximities (eyes, nose, hand). Fig.2(c) and 2.(d) were obtained using fast discrete curvelet transform [6], [5] and $2D$ discrete wave atoms threshold-technique combined by cycle spinning [8], respectively. The hard- thresholding function mentioned above was used in the coefficients of these two transforms. In the image obtained with curvelet, the texture was not appropriately recovered. Moreover, Gibbs artifacts are present. On the other hand, a wave atom transform restored all the texture but produced a blurred image. Fig.2(e) shows the image restored with the model based on adaptive fidelity term λ (5), with $C = 0$, while Fig.2(f) shows the version obtained by diffusion-combined curvelet transform (3), with step 0.0005 and 11 iterations. Although the first method recovered texture, some important details of the image were excessively smoothed, such as face and hand. In contrast, the second method did not produce any excessive smoothing, but kept part of the noise in the reconstructed image. Fig.2(g)) is the restored image using the proposed filter, with 20 iterations. The parameters used were $\alpha = 0.8$, $\theta = 0.13$, $\delta = 0.09$, $\kappa = 0.001$ and $\mu = 0.25$. In this case, both texture and details of the image (intrinsic data) are recovered, besides minimizing noise level without there being excessive smoothing.

Fig.3 shows a close up of right-eye Barbara taken from Fig.2(c)-(g). We can clearly see that in images 3.(c), 3.(d) and 3.(f) the texture is erroneously extended to the face. Furthermore, Fig.3(e) is quite smoothed, besides the loss of part of the oriented texture. Fig.3(g) shows the result obtained by our model. It can be seen that both the texture and intrinsic details of the face are recovered. The parameters adopted in the used methods in this paper were chosen according to the best visual quality obtained by applying each one of these models.

In the second experiment we evaluated the performance of the proposed model on quite deteriorated images. We took an image of a fingerprint that was excessively contaminated with noise. Fig.4(a) shows the original image (without noise) and Fig.4(b) its version with noise ($SNR = 1db$). Fig.4(c) shows the result obtained by our model, where 30 iterations were used. As parameters, we took $\alpha = 0.3$, $\theta = 0.15$, $\delta = 0.01$, $\kappa = 0.005$ and $\mu = 0.7$. It can be seen that the restored image Fig.4(c) is considerably close to the original image Fig.4(a). Moreover, the residue shown in Fig.4(d) between the noise image and the restored image, only the noise stood out, with no type of texture vestige showing, that is, shows that the method only removed noise, fully achieving the proposed objectives. Finally, Fig.4(e) shows a map of the edges generated by our regularizer term h in the last iteration of this experiment. In this case, we can see that the diffusion is being guided by h and thus the method will smooth the regions close to scale of the color white and reconstruct parts where there are textures and edges, that is, blackened parts.

6. Conclusion

In this paper we propose a new filter dedicated to the treatment of images contaminated by noise and having a high concentration of oscillatory characteristics such as texture and irregular details. The method combines an anisotropic diffusion filter with harmonic analysis techniques, specifically with wave atom system. Moreover, we propose a new term regularizer which can detect certain oscillatory patterns on the image. The proposed model has a selective character, that is, it applies diffusion and recovers edges and textures according to an auxiliary image generated by the term regularizer. Filters to treat images having texture can be used on a whole range of practical applications, for example: fingerprint processing, engineering surfaces, geophysical data, image processing in security, etc. Experimental tests show the efficiency of the new filter when compared to the recent models in literature.

7. Acknowledgements

The authors would like to express their gratitude to the State of São Paulo Research Foundation (FAPESP) and Brazilian Commission for Higher Education (CAPES) for the financial support of this work. We thank Prof. Célia A. Barcelos for valuable discussions and suggestions, and also the reviewers for useful comments.

References

[1] L. Alvarez, P.L. Lions, and J.M. Morel, “Image selective smoothing and edge detection by nonlinear diffusion”, SIAM J. Numer. Anal, pp. 845-866, 1992.

[2] C.A.Z. Barcelos, M. Boaventura, and E. Silva Jr, “A well-balanced flow equation for noise removal and edge detection”, IEEE Transactions on Image Processing, Vol. 12, pp. 751-763, 2003.

(a)

(b)

(c)

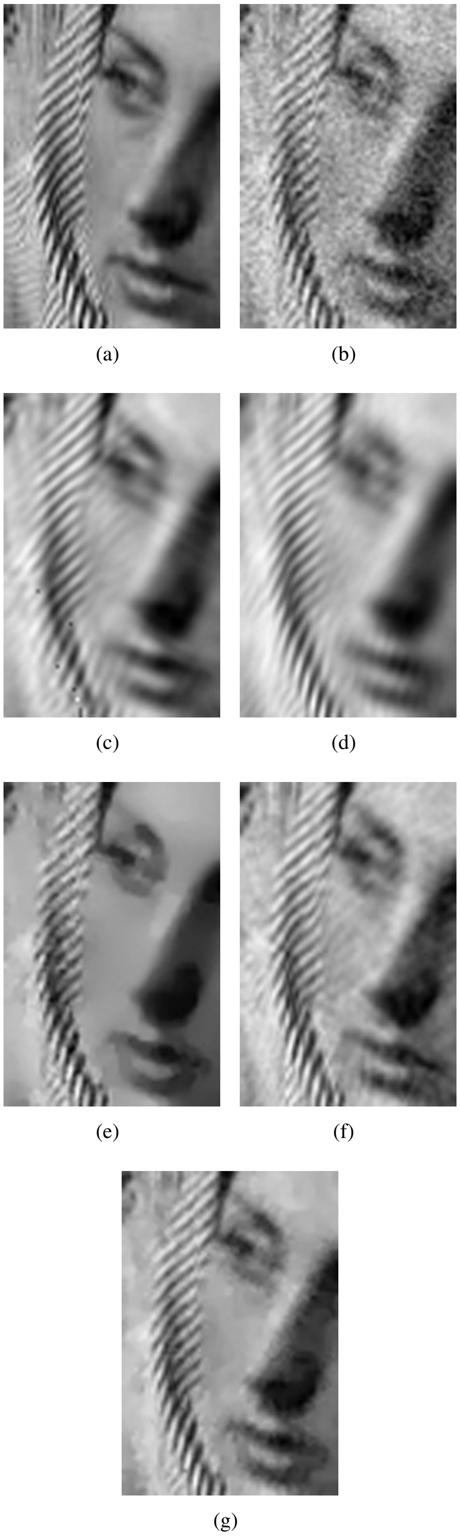
(d)

(e)

(f)

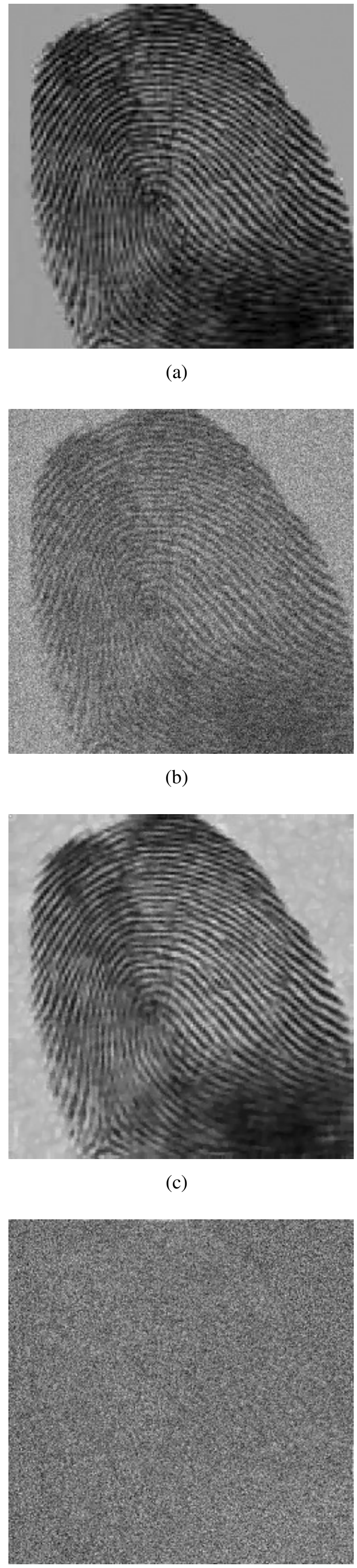
(g)

Figure 2. Photography of Barbara's face. (a) Original image; (b) version with noise (SNR = 9.97 db); (c) generated by curvelets; (d) by wave atoms; (e) by diffusion with adaptive-fidelity term; (f) by TV diffusion combined with curvelet; (g) by proposed model.

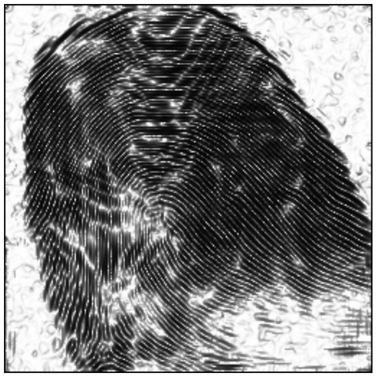


(a) (b) (c) (d) (e) (f) (g)

Figure 3. Close up on the Barbara face. (a) original image; (b) version with noise; (c) generated by curvelets; (d) by wave atoms; (e) by diffusion with adaptive-fidelity term; (f) by TV diffusion combined with curvelet; (g) by proposed model.



(a) (b) (c) (d)



(e)

Figure 4. Heavy noisy image. (a) original image; (b) version with noise (SNR = 1 db); (c) obtained by proposed model; (d) residual; (e) diffusivity term h.

[3] M. Burger, G. Gilboa, S. Osher, and J. Xu, "Nonlinear inverse scale space methods", Communications in Mathematical Sciences, Vol. 4, pp. 179-212, 2006.

[4] M. Burger, S. Osher, J. Xu, and G. Gilboa, "Nonlinear inverse scale space methods for image restoration", VLSM, LNCS, Vol. 3752, pp. 85-96, 2005.

[5] E. J. Cands, L. Demanet, D. L. Donoho, and L. Ying, "Fast discrete curvelet transforms", SIAM J. on Multiscale Model. Simul., Vol. 5, pp. 861-899, 2006.

[6] E. J. Cands, and D. L. Donoho, "New tight frames of curvelets and optimal representations of objects with piecewise C^2 singularities", Comm. Pure Appl. Math., Vol. 57, pp. 219-266, 2004.

[7] T. F. Chan, S. Osher, and J. Shen, "The digital TV filter and nonlinear denoising", IEEE Transactions on Image Processing, Vol. 10, pp. 231-241, 2001.

[8] L. Demanet, and L. Ying, "Wave atoms and sparsity of oscillatory patterns", Appl. Comput. Harmon. Anal., Vol. 23, pp. 368-387, 2007.

[9] L. Demanet, and L. Ying, "Curvelets and wave atoms for mirror-extended images", Proceeding of SPIE Wavelet XII, San Diego, Vol. 6701, 2007.

[10] L. Demanet, "Curvelets, wave atoms and wave equations", Ph.D. thesis, California Institute of Technology, 2006.

[11] G. Gilboa, N. Sochen, and Y. Z. Zeevi, "Variational denoising of partly textured images by spatially varying constraints", IEEE Transactions on Image Processing, Vol. 15, pp. 2281-2289, 2006.

[12] J. Gilles, "Noisy image decomposition: a new structure, texture and noise model based on local adaptivity", Journal of Mathematical Imaging and Vision, Vol. 28, pp. 285-295, 2007.

[13] F. Li, C. Shen, C. Shen, and G. Zhang, "Variational denoising of partly textured images", Journal of Visual Communication and Image Representation, Vol. 20, pp. 293-300, 2009.

[14] G. Liu, X. Feng, and J. Bai, "Variational image decomposition using wave atoms", Current Development in Theory and Applications of Wavelets, Vol. 2, pp. 277-291, 2008.

[15] J. Ma, "Image assimilation by geometric wavelet based reaction-diffusion equation", Proceeding of SPIE Wavelet XII, San Diego, Vol. 6701, 2007.

[16] J. Ma, and G. Plonka, "Combined curvelet shrinkage and nonlinear anisotropic diffusion", IEEE Transactions on Image Processing, Vol. 16, pp. 2198-2206, 2007.

[17] K.N. Nordström, "Biased anisotropic diffusion: a unified regularization and diffusion approach to edge detection", Image and Vision Computing, no. 8, pp. 318-327, 1990.

[18] S. Osher, M. Burger, D. Goldfarb, J. Xu, and W. Yin, "An iterative regularization method for total variation-based image restoration", SIAM Multiscale Model. and Simu., Vol. 4, pp. 460-489, 2005.

[19] P. Perona, and J. Malik, "Scale-space and edge detection using anisotropic diffusion", IEEE Transactions Pattern Analysis and Machine Intelligence, Vol. 12, no. 7, pp. 629-639, 1990.

[20] G. Plonka, and J. Ma, "Nonlinear regularized reaction-diffusion filters for denoising of images with textures", IEEE Transactions on Image Processing, Vol. 17, pp. 1283-1294, 2008.

[21] L. Rudin, S. Osher, and E. Fatemi, "Nonlinear Total Variation Based Noise Removal Algorithms", Physica D 60, pp. 259-268, 1992.

[22] J. L. Starck, E. J. Cands, and D. L. Donoho, "The curvelet transform for image denoising", IEEE Transactions on Image Processing, Vol. 17, pp. 670-684, 2002.