Face Recognition Using LBP on an Image Transformation Based on Complex Network Degrees

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***Abstract*—Automated visual face recognition involves acquiring descriptive features from the image. Local Binary Patterns (LBP) is a powerful method to that end, capably characterizing local features. An crucial limitation of LBP, however, is that the feature vector's size becomes unmanageable when the method employed on even moderately large regions. In order to describe larger scale features, this work proposes a descriptor based on applying the LBP histogram applied to an image transformation based on node degree data derived from a complex network representation of the original image. The complex network generation heuristic and parameters are discussed. The complex network representation is shown to be able to condense larger scale image patterns into a local value that can be handled by LBP. LBP applied to this image transformation yields results that outperform LBP. We validate our proposed approach by applying our method to a face recognition task using three challenging databases. Results demonstrate that, for a large enough complex network generation radius, our method consistently outperforms LBP, while using a feature vector of the same size.**

I. Introduction

Characterization of facial features is an important challenge in Computer Vision, and a necessary step to enable tasks such as face detection and recognition. As computers approaches human performance levels, more complex and demanding data sets have emerged, and with many modern applications, interest in the face characterization problem remains high, and it is tackled through a variety of approaches.

Statistical methods are a class of image description methods that focus on the distribution and statistical behavior of image features for characterization [1]. Local binary patterns (LBP) are a category of statistical methods for image description that consist of describing local patterns in the neighborhood surrounding a pixel. This category of method has been popular as it has been successfully applied to a variety of computer vision applications, such as texture [2] and facial analysis and recognition [3] [4], often outperforming similar methods while using a comparatively small feature vector and computational time. Due to the success of LBP, many variants to the method have been developed [5]. Despite being very effective at describing a pixel in terms of the relation between the pixel's value and the values of its neighboring pixels, local binary patterns are not an easily scalable method; applying LBP to a larger neighborhood radius increases feature vectors exponentially and becomes unmanageable even for radii values as small as 2, not to mention the increased processing time, such that it is not an adequate method to describe larger scale patterns. Scaling in LBP, therefore, has been done mostly by multi-scale methods which increase the feature vector size, and incurs interpolation or loss of information [6].

One other popular statistical approach for image description has been the use of complex networks, which can be defined as graphs with non-trivial topology. Complex networks can be used to organize a variety of information and describe many different types of data configurations and relationships. Arranging data as a complex network can make evident underlying patterns in the data, and allow for unique metrics both for local and global topological analysis. A comprehensive and domain-independent review of these metrics can be found in Costa *et al.* [7]. Other works employ agents and heuristics such as graph crawlers to statistically sample and characterize paths in the complex network applied to texture classification [8] [9] [10], [11]. In Computer Vision, specifically, complex networks have been employed to describe image textures [12] and geometrical shapes [13] [14], for example.

This work proposed a novel approach to face recognition using a combination of complex networks and local binary patterns for face recognition. Our method uses a complex network node degree metric to describe a larger pixel region and allow for LBP to be performed on data descriptive of a larger pixel patterns without the impracticable need to increase the LBP radius. We validate our proposal in three different challenging data sets. Results show that our approach outperforms the LBP results obtained without the complex network under the same classification parameters, while maintaining the same size of feature vector. Sections II and III outline, respectively, past approaches to LBP applied to image characterization, as well as state-of-the-art complex network-based approaches to Computer Vision. Section IV presents our approach for combining complex networks and LBP into a descriptor with the same dimensionality as LBP that potentially accounts for larger pixel patterns. After that, Section V presents experimental parameters and results for three challenging data sets that are widely used in correlated literature. Finally, in Section VI we discuss our proposed method and possible future research following the findings in this work.

II. Local Binary Patterns

Local binary patterns are a statistical local image descriptor [15]. There are many variants to LBP, but the original form of the method still holds up as a very well performing descriptor. The method is based on sliding a window (usually $3x3$) across a gray-scale image I and at each step evaluating each respective region in relation to the window's center pixel $p = I(x_p, y_p)$ in coordinate (x_p, y_p). The evaluation consists of simply comparing the value of every neighboring pixel to that of the center pixel on a fixed order, effectively thresholding the neighboring pixels using the center pixel's value, and assigning a binary value to the neighbors depending on whether their value is lower than that of the center pixel, or higher or equal to the center pixel. Considering the typical $3x3$ window, the pixel p would be compared each of the 8 pixels in the set of neighbors N, with coordinates $S = \{(x_{p-1}, y_p), (x_{p-1}, y_{p+1}), (x_p, y_{p+1}), (x_{p+1}, y_{p+1}), (x_{p+1}, y_p), (x_{p+1}, y_{p-1}), (x_p, y_{p-1}), (x_{p-1}, y_{p-1})\}$. This results in 8 comparisons, which yield an LBP codification with 8 binary values in total, that can be thought of as the 8 bits from a byte. The value of the resulting byte is a number between 00000000_2 and 11111111_2, or 0 and 255 in decimal. This values are then used to replace the original image's pixel values, generating an LBP transform I_{LBP} with the same dimensions as the original image I. In I_{LBP}, the value of every pixel describes the gray-scale pattern around the pixel of same coordinate from I.

Typically, the gray levels histogram from I_{LBP} is used as a feature vector for I. Considering 256 possible gray levels from the 8 bits LBP value, the resulting feature vector has 256 dimensions, which is a reasonable amount in comparison to competing methods. LBP is relatively fast to compute, with complexity $O(N \times (W-1))$, where $N = |I|$ is the cardinality of I, that is, the number of pixels in the image, and W is the size of the sliding window in pixels, which, as mentioned, is much smaller than N, resulting in complexity $O(N)$. The LBP histogram is robust against monotonic changes in brightness and contrast.

Figure 1 provides an example of the method as it is applied to a single pixel inside a $3x3$ sliding window.

Due to LBP's sensitivity to noise and low scalability, several preprocessing techniques and filters have been proposed in the past, specially regarding face recognition [16] [17] [18]. However, no complex network degree based transformations have been found in published literature, despite complex network degrees' descriptive capabilities relative to local patterns.

III. Complex Network Representation of Images

When using complex networks in Computer Vision and Image Processing applications, a particular challenge lies in the decision of how to represent an image or a video

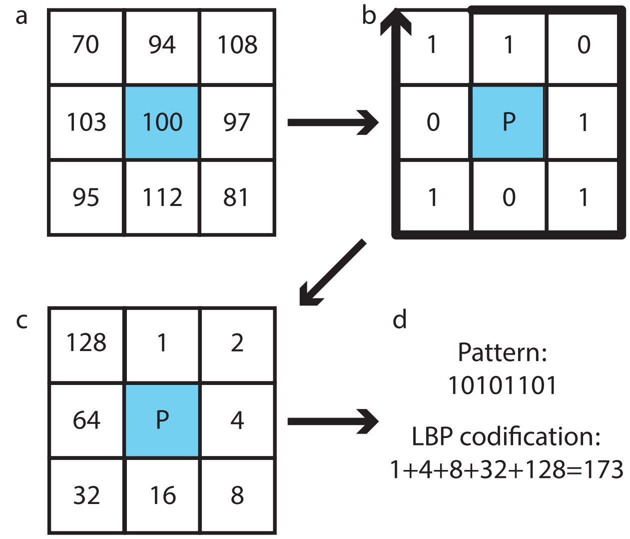

Fig. 1. Example of LBP codification from the original image's pixel and its neighborhood within the sliding window (a), forming a binary sequence (b) that is converted to its decimal equivalent (c).

as a graph. The work by Backes *et al.* [19] achieves this representation by corresponding each pixel $p = I(x, y)$ in a gray-scale image I to a vertex (or node) $v_{x,y} \in V$ in a graph $G = (V, E)$. So we have that V has as many nodes as there are pixels in the image. The most important decision, however, is how to assign edges between pairs of nodes, forming the set of edges E. Considering the desire to use this complex network as a local image descriptor, it is important to preserve local pixel relations in the graph representation. The proposed graph, therefore, has geographical properties: the closer the vertices are, the more likely their are to be paired. The first step toward achieving this is to establish a maximum connection radius, effectively a circular kernel signifying the maximum possible distance between pixels for the pair to be eligible for an edge. This preliminary set of edges E' is given by equation (1).

$$E' = \{(v_{x,y}, v_{x',y'}) \in I \times I \mid d((x,y),(x',y')) \le r_G\} \quad (1)$$

Where $d((x,y),(x',y'))$ is the Euclidian distance between the coordinates:

$$d((x,y),(x',y')) = \sqrt{(x-x')^2 + (y-y')^2} \quad (2)$$

Another purpose of the maximum radius r_G is to improve performance, since distant pixels are unlikely to be paired. Also, in the case of this work's face recognition application, this keeps the method scalable to images with arbitrarily large resolution.

Notice $G' = (V, E')$ does not fit the definition of a complex network, as its topology is regular. The next step is to prune edges from E' in order to achieve a set of edges E that is in some way descriptive of the underlying visual data. In complex networks, similarity is a commonly used criterion to determine edges. In the case of this network, a given vertex v has an edge to vertex v' (that is to say, $(v, v') \in E$) based $w(v, v')$, a measure of dissimilarity between the pixels and the vertices they represent, given by equation (3). Similarity between pixels is based on pixels' two explicit attributes: their (x, y) coordinate and their intensity $I(x, y)$, a value in the $[0, 255]$ interval, considering a gray-scale image. The closer a pair of pixels are to each other and the closer their intensities, the lower the dissimilarity value will be. So for all pairs of edges in the preliminary set of edges E', $w(v, v')$ is computed.

$$\begin{aligned} w(v_{x,y}, v_{x',y'}) = ((x - x')^2 + (y - y')^2) + \\ + (r_G^2 \frac{|I(x, y) - I(x', y')|}{255}) \end{aligned} \tag{3}$$

One of the most important attributes of a node in a graph is its degree [20]. The degree of a given vertex v is defined as the vertices connected directly to it by an edge, that is, the number of pairs in E that include v. Different thresholds values for t provide different graphs with different vertex degrees. Backes *et al.* [19] experiments with a range of values for t, settling on an optimal interval of thresholds $t \in [0.005, 0.053]$. A higher threshold is more judicious and results in lower degrees for nodes, while lower thresholds are less selective and result in higher degrees. Threshold values above or below the interval were shown to be too permissive or too strict, respectively, and do not cause significant changes in the network. Unweighted edges are attributed to pairs of vertices whose dissimilarity is below the chosen threshold value t, forming the set of edges E, finally forming the complex network $G = (V, E)$. Considering $\partial(v_t)$ the sub-set of E that denotes the neighbors of a vertex $v_{x,y}$ for a chosen t, the degree $deg(v_{x,y})$ is given by the number of elements in $\partial(v_{x,y})$, as shown in equation (5):

$$deg(v_{x,y}) = |\partial(v_{x,y})| \tag{4}$$

Where $\partial_{v_{x,y}}$, the set of $v_{x,y}$'s neighboring vertices, is:

$$\partial_{v_{x,y}} = \{v' \in V | (v_{x,y}, v') \in E \quad \text{and} \quad w(v_{x,y}, v') \leq t\} \tag{5}$$

From the graph G we can, therefore, obtain the transformed image I_{CN}. In I_{CN}, the intensity value of a pixel $I(x, y)$ is given by the degree $deg(v_{x,y}$ of the corresponding vertex $v_{x,y} \in G$. Note that I_{CN} has the same resolution as the original image I.

The degree highlights neighborhood features, which makes is capable of expressing high level taxonomic properties of an image such as period, anisotropy (direction), regularity, granularity, contrast, roughness, among others [21]–[23]. In the complex network formulation presented above, in the context of images, a high degree value means a high homogeneity between the sliding window's center pixel's intensity value and its neighbors'. Complex network degrees have been used as basis for crawler based image descriptors [12] to describe paths in networks.

IV. Proposed Approach

This work proposes to acquire the complex network degree-based transformation as a preprocessing operation before applying LBP. LBP is then applied to the network degree values in transformed image I_{CN}. The goal in doing this is, considering the complex network degree transformation's capacity to highlight in a single value (the network degree) the similarity between a pixel and its neighbors in a radius in the original image, that LBP can be applied to data that already intrinsically describes a pixel region. The major difference in this case is that in the complex network degrees' case, much larger radii are viable in comparison with LBP radii. For example, a radius of 15 yields 706 neighbors, which is manageable for the complex network degree calculation and does not change the resulting feature vector size, but for LBP would yield a feature vector with size 2^{707}, far beyond what is computationally feasible.

LBP can be applied to the complex network-based image transform I_{CN} without modification, and the resulting feature vector is the same size as it would be applied to a regular image. The complex network radius affects the maximum number of gray-scale levels the transformed image might have, since the radius determines the maximum degree possible for each node. This carries the caveat that, especially for small radii, the number of possible degrees might be rather small (12 for radius 2, for example). This will inevitably lead to a high number of ties in the values between neighbors. This is undesirable because when pixel values are the same in LBP, the decision between attributing value 0 or 1 to that position in the LBP descriptor is arbitrary. Therefore, we opted to perform the complex network transformation for the same range of threshold t values as in Backes *et al.* [19] and finally taking the average of the degree values for each threshold, which makes ties much less likely. Applying LBP to I_{CN} results in the image I's feature vector ϕ.

We also experimented with adding histogram measurements to the resulting feature vector, topological attributes to further characterize the histogram. These histogram attributes are seven in total: energy, entropy, skewness, contrast, mean, variance and kurtosis [7] [1]. They are computed for the feature vector ϕ and appended to it.

Figure 2 summarizes the proposed method with a real example (except in complex network G's case) generated with one of the images from the JAFFE database [24].

V. Experiments and Results

We applied the proposed descriptor for the task of classifying individual faces in three different data sets: the Japanese Female Facial Expression (JAFFE) Database [24], with 214 images divided into 10 classes, the Yale Faces Data Base, with 165 samples divided into 15 classes [25], and the Faces Data Set [1], with 2500 images divided into 125 classes. All three data sets present individuals photographed posing with different facial expressions, and often changes in light

[1] https://www.kaggle.com/c/face-recognition2, accessed in June 13th, 2019

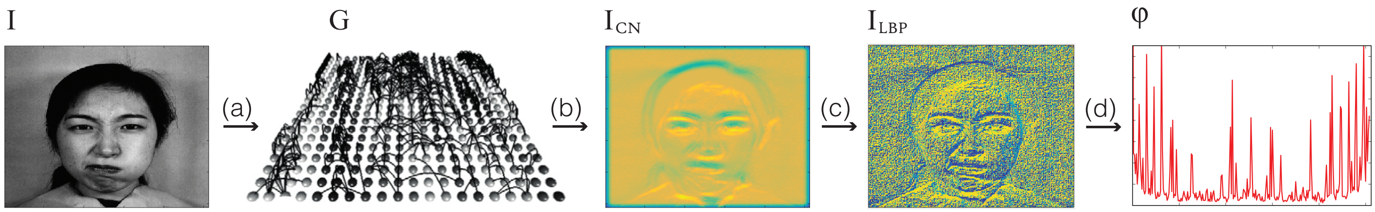

Fig. 2. Summary of the proposed descriptor. The original image I is converted to a complex network G (a), then the degrees of the network's vertices are used as the piuxel values for a transformed image I_{CN} (b). Then, the LBP opeartor is applied to eadh pixel in I_{CN} to generate I_{LBP}. A gray-scale histogram of I_{LBP} is the feature vector ϕ, which will be used for classification.

direction, presence or absence of glasses and other challenging conditions.

In all these experiments, we use the MATLAB implementation of LBP made available by the method's authors. The LBP sliding window size, a crucial parameter, is set at the recommended $3x3$ value, yielding 8 neighboring pixels and feature vector of dimension $2^8 = 256$. Furthermore, the complex network radius parameter has been evaluated using all integer values in the $[1, 15]$ range. Complex network generation was also performed with a variety of thresholds in the $[0.005, 0.53]$ range, with increments of 0.015, before averaging the resulting degrees for each pixel. Classification has been performed using the Linear Discriminant Analysis (LDA) classifier, with the same leave-one-out cross-validation setup in all cases.

Figures 3, 4 and 5 present face recognition success rates for each data set, for the range of complex network radius values. In each figure, the original LBP classification performance is denoted as a horizontal dashed line. It can be clearly seen that our proposed method outperforms LBP from a certain r_G radius and upwards, which suggests the hypothesis that the larger radius would be helpful in describing larger neighborhoods is accurate. This improvement in performance is true in all cases, but specially noticeable in the JAFFE database from $r_G = 5$ and upwards. However, from we observe a performance peak for radii around 12 or 13, and then again for radius $r_G = 17$, reaching a classification rate of 97.65%, while LBP achieved 91.55%. Further increasing the radius in JAFFE yielded worse or similar results.

Notably, the Yale Faces database yielded the worst results both for the proposed method and for LBP. This is likely due to the fact that this data set presents the starkest disparities in illumination direction and pose. Still, the proposed method consistently achieved higher correct classification rates than LBP for $r_G \geq 8$. Is also worth noting that further increases in r_G continued to yield better results up to $r_G = 20$, a parameter which yielded 81.21% correct classifications, to LBP's 68.48%. At this point, however, processing time might be an issue. In practice, an application's performance requirements would help decide if computing larger radii would be worthwhile.

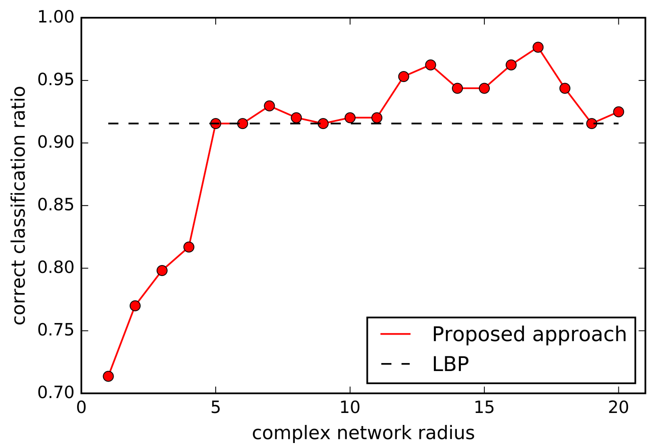

Fig. 3. Recognition results for JAFFE database. As the complex network radius is increased, the proposed method's performance tends to improve, surpassing LBP.

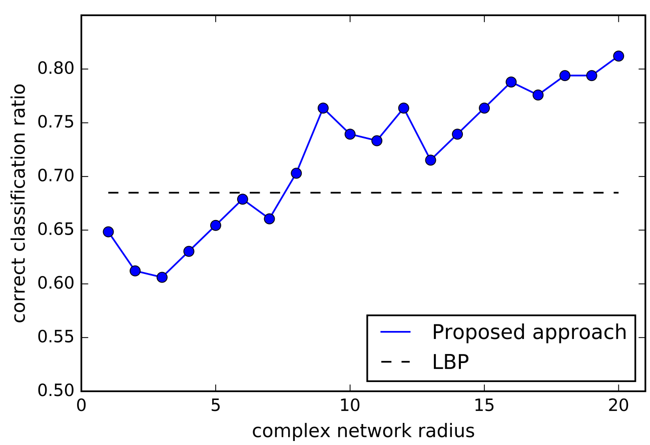

Fig. 4. Recognition results for Yale Faces database.

The difference in behavior of the correct classification rate curves for the JAFFE and Yale Faces databases is interesting, given the similarity between both data sets. Both have images and faces in a very similar resolution (JAFFE's faces being on average 8 pixels larger). It is natural to anticipate that, for faces in similar scales, the best performing radii r_G would be similar, being indicative of the type of facial feature that is captured by the complex network data, but that was not the case between these two databases. We posit that the disparity is due to texture. Images in the Yale Faces database are slightly

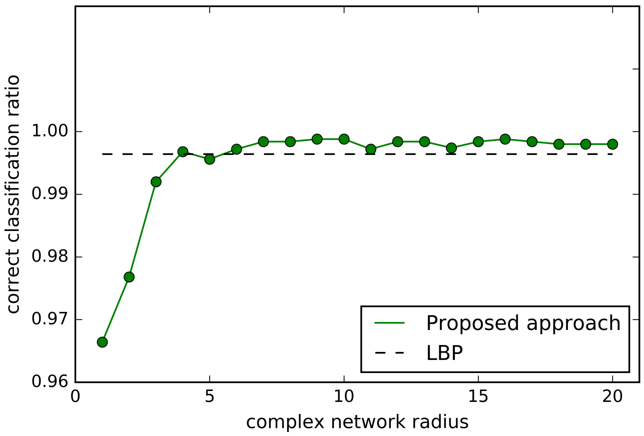

Fig. 5. Recognition results for Faces Data database.

more blurred in comparison to images in JAFFE, meaning that smaller scale patterns, highlighted by smaller radii, are missing. This could also account for the worse classification performance on the Yale Faces database.

Figures 6 e 7 show confusion matrices for the proposed method and for LBP for the JAFFE and Yale Faces databases. Below the confusion matrices there are samples of mistakenly classified faces from the most confused classes. In the case of these two databases, it is noticeable that facial expressions likely do account for the confusion to a certain degree.

Results for the Faces Data database were high, despite Faces Data being the data set with the most samples and classes in it. The proposed method achieved 99.88% for $r_G = 9$, 10 and 16 to LBP's also high classification rate of 99.64%. This is likely due to the Faces Data database's more stable poses and lighting conditions than in the other tested databases. When classification rates are very high, as is the case, the confusion matrix is predictable, but it is still interesting to observe the error cases. Figure 8 showcases the only two incorrect classification results for $r_G = 10$ in Faces Data, as two images from class 12 were incorrectly classified as class 70 and 104. It is worth noting that LBP also had trouble with these exact same classes.

Finally, we analysed the influence of histogram attributes appended to the feature vector. Table I compares results for the JAFFE database with and without histogram attributes for the range of r_G that yielded the best results across the three databases. It is noticeable that the attributes generally improve classification rates, but improvements are small and inconsistent. Yet, it is a simple to compute and small addition to the feature vector (7 dimensions). The use of histogram attributes, therefore, should be dependant on the desired application, and whether the marginal improvements are desired.

VI. Analysis and Conclusions

This work proposed a novel descriptor based on an complex network degrees-based image transformation followed by the use of the LBP operator. LBP is a powerful method, but

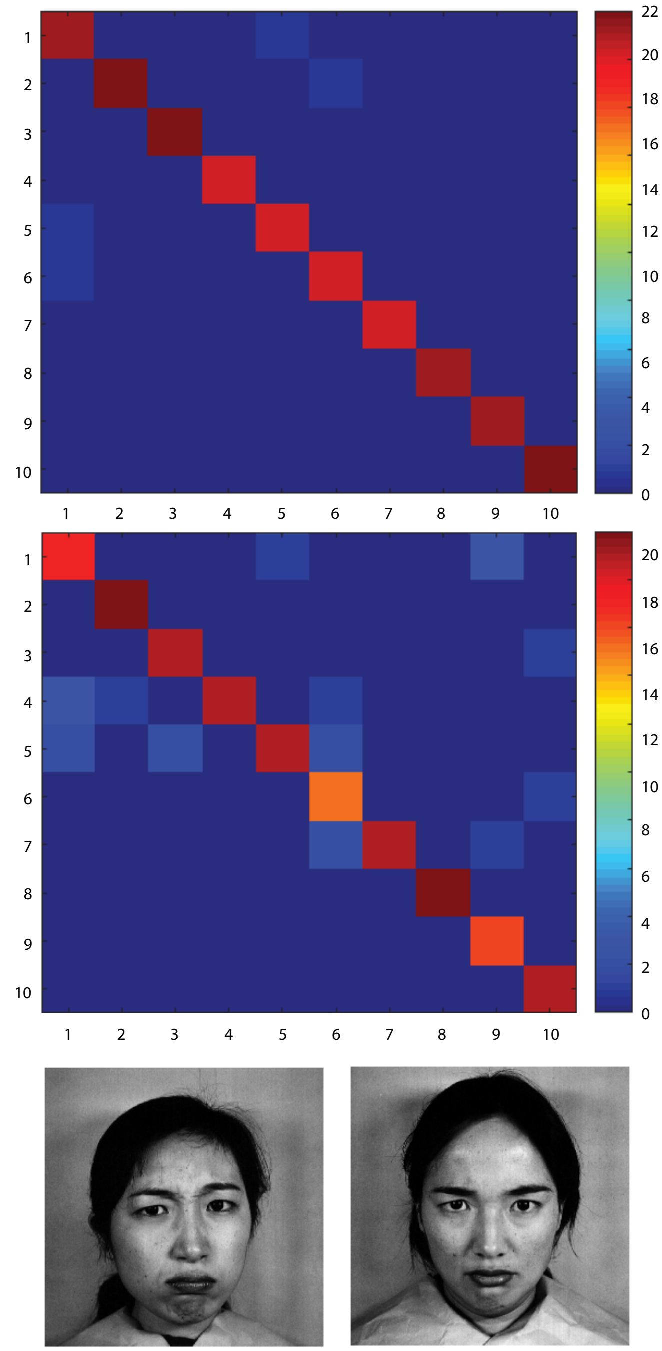

Fig. 6. Above is the confusion matrix for $r_G = 17$ in the JAFFE database. Beneath it is the confusion matrix for LBP. Beneath the matrices, there is a sample of each class from the most confused pair of classes.

it also that has a critical limitation in terms of scalability, being viable only on very small local regions. Our complex network-based image transformation condenses information about a wider region into the LBP sliding window, allowing for the description of larger scale patterns in the same amount of data. Experiments have demonstrated the effectiveness of this approach when employed to face recognition, one of the seminal and most popular LBP applications in literature.

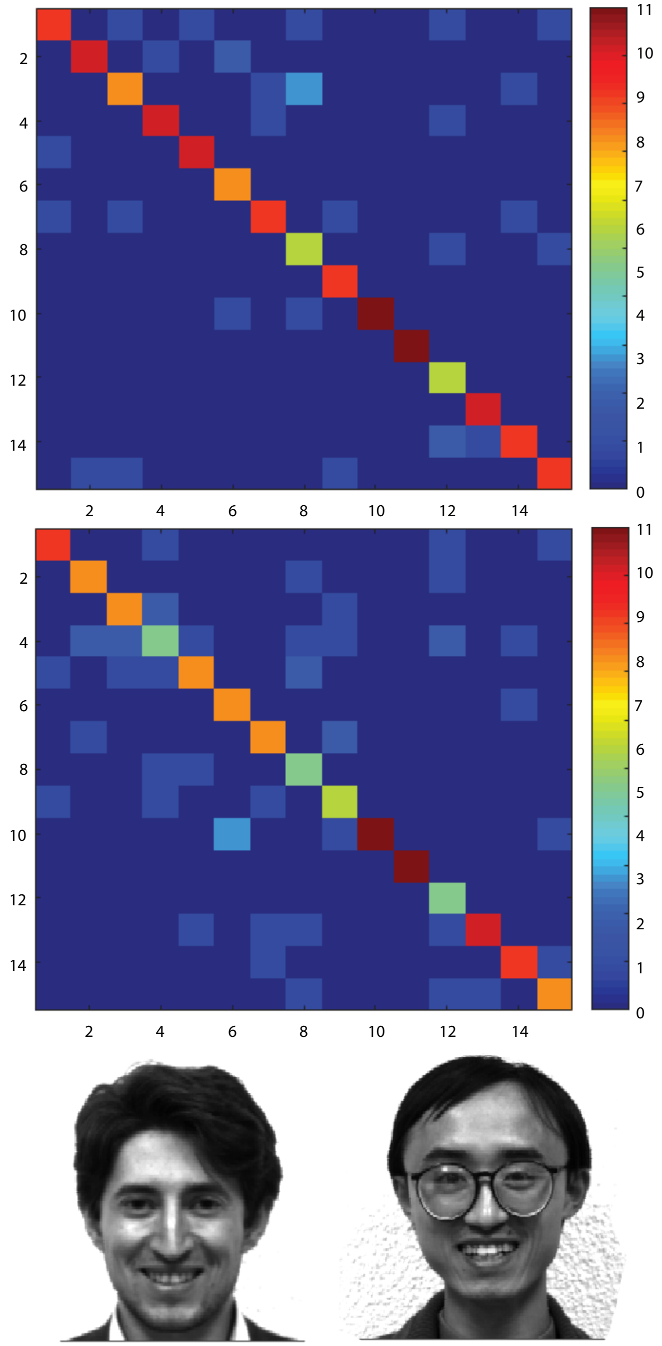

Fig. 7. Above is the confusion matrix for $r_G = 20$ in the Yale Faces database. Beneath it is the confusion matrix for LBP. Beneath the matrices, there is a sample of each class from the most confused pair of classes.

The proposed method has significantly outperformed LBP for complex network radii larger than a certain value. For the sake of a fair comparison, we also did not take into account concatenations of our complex network-based LBP feature vectors for multiple radii, as that would yield a larger, but still manageable, feature vector. Assuming that various radii for the complex network highlight facial features in different scales, combining several radii would be a useful step in composing

Fig. 8. Two images from class 12 (the left-most person) were the only ones incorrectly classified out of all 125 classes, being classified as classes 70 and 104 (the middle and right-most persons, respectively).

TABLE I
RECOGNITION RESULTS COMPARING THE USE AND ABSENCE OF HISTOGRAM ATTRIBUTES FOR THE JAFFE DATABASE.

	Success rate per r_G (%)					
Attribs.	10	11	12	13	14	15
Absent	92.02	92.02	95.31	96.24	94.37	94.37
Present	92.49	91.08	96.24	97.65	94.84	96.71

a comprehensive descriptor.

Regarding future developments of this work, it would be useful to explore the application of the method alongside some of the more recent LBP variants, such as Median Local Ternary Patterns [18] [26]. Division of the face into sub-regions is also a common approach with face recognition methods such as LBP, which despite yielding a much larger feature vector imbues the descriptor with useful visual-word information. Automated threshold t selection would lead to faster network computation. The solution could also be applied to discover larger patterns in texture. A deeper exploration of complex network statistics other than node degree, such as local clustering coefficient, transitivity, betweeness centrality [7], random paths [27] and "network motifs" (as proposed by Milo *et al.* [28] and applied to image recognition by *de Lima et al.* [29]), could help further characterize the image's local regions.

REFERENCES

[1] R. M. Haralick, "Statistical and structural approaches to texture," *Proceedings of the IEEE*, vol. 67, no. 5, pp. 786–804, May 1979.

[2] L. Nanni, A. Lumini, and S. Brahnam, "Survey on lbp based texture descriptors for image classification," *Expert Systems with Applications*, vol. 39, no. 3, pp. 3634–3641, 2012.

[3] T. Ahonen, A. Hadid, and M. Pietikainen, "Face description with local binary patterns: Application to face recognition," *IEEE Transactions on Pattern Analysis & Machine Intelligence*, no. 12, pp. 2037–2041, 2006.

[4] D. Huang, C. Shan, M. Ardabilian, Y. Wang, and L. Chen, "Local binary patterns and its application to facial image analysis: a survey," *IEEE Transactions on Systems, Man, and Cybernetics, Part C (Applications and Reviews)*, vol. 41, no. 6, pp. 765–781, 2011.

[5] M. Pietikäinen and G. Zhao, "Two decades of local binary patterns: A survey," in *Advances in independent component analysis and learning machines*. Elsevier, 2015, pp. 175–210.

[6] S. Liao, X. Zhu, Z. Lei, L. Zhang, and S. Z. Li, "Learning multi-scale block local binary patterns for face recognition," in *International Conference on Biometrics*. Springer, 2007, pp. 828–837.

[7] L. F. Costa, F. A. Rodrigues, G. Travieso, and P. R. V. Boas, "Characterization of complex networks: A survey of measurements," *Advances in Physics*, vol. 56, no. 1, pp. 167–242, 2007.

[8] A. R. Backes, O. M. Bruno, M. G. Campiteli, and A. S. Martinez, “Deterministic tourist walks as an image analysis methodology based,” in *Progress in Pattern Recognition, Image Analysis and Applications*, ser. Lecture Notes in Computer Science. Springer Berlin Heidelberg, 2006, vol. 4225, pp. 784–793. [Online]. Available: http://dx.doi.org/10.1007/11892755_81
[9] A. R. Backes, A. S. Martinez, and O. M. Bruno, “Texture analysis based on maximum contrast walker,” *Pattern Recognition Letters*, vol. 31, no. 12, pp. 1701–1707, 2010.
[10] ——, “Texture analysis based on maximum contrast walker,” *Pattern Recognition Letters*, vol. 31, no. 12, pp. 1701–1707, 2010.
[11] L. N. Couto, T. P. Ribeiro, A. R. Backes, and C. A. Z. Barcelos, “Texture characterization via improved deterministic walks on image-generated complex network,” in *Image Processing (ICIP), 2015 IEEE International Conference on*. IEEE, 2015, pp. 4416–4420.
[12] L. N. Couto, A. R. Backes, and C. A. Barcelos, “Texture characterization via deterministic walks direction histogram applied to a complex network-based image transformation,” *Pattern Recognition Letters*, vol. 97, pp. 77–83, 2017.
[13] A. R. Backes and O. M. Bruno, “Shape classification using complex network and multi-scale fractal dimension,” *Pattern Recognition Letters*, vol. 31, no. 1, pp. 44–51, 2010.
[14] A. B. de Oliveira, P. R. da Silva, and D. A. C. Barone, “A novel 2d shape signature method based on complex network spectrum,” *Pattern Recognition Letters*, vol. 63, pp. 43–49, 2015.
[15] T. Ojala, M. Pietikäinen, and T. Mäenpää, “Multiresolution gray-scale and rotation invariant texture classification with local binary patterns,” *Pattern Analysis and Machine Intelligence, IEEE Transactions on*, vol. 24, no. 7, pp. 971–987, 2002.
[16] X. Li, W. Hu, Z. Zhang, and H. Wang, “Heat kernel based local binary pattern for face representation,” *IEEE Signal Processing Letters*, vol. 17, no. 3, pp. 308–311, 2009.
[17] X. Tan and W. Triggs, “Enhanced local texture feature sets for face recognition under difficult lighting conditions,” *IEEE transactions on image processing*, vol. 19, no. 6, pp. 1635–1650, 2010.
[18] L. Ji, Y. Ren, X. Pu, and G. Liu, “Median local ternary patterns optimized with rotation-invariant uniform-three mapping for noisy texture classification,” *Pattern Recognition*, vol. 79, pp. 387–401, 2018.
[19] A. R. Backes, D. Casanova, and O. M. Bruno, “Texture analysis and classification: A complex network-based approach.” *Inf. Sci.*, vol. 219, pp. 168–180, 2013. [Online]. Available: http://dblp.uni-trier.de/db/journals/isci/isci219.html#BackesCB13
[20] M. Newman, A.-L. Barabasi, and D. J. Watts, *The structure and dynamics of networks*. Princeton University Press, 2011, vol. 12.
[21] H. Tamura, S. Mori, and T. Yamawaki, “Textural features corresponding to visual perception,” *Systems, Man and Cybernetics, IEEE Transactions on*, vol. 8, no. 6, pp. 460–473, 1978.
[22] M. Hájek, *Texture analysis for magnetic resonance imaging*. Texture Analysis Magn Resona, 2006.
[23] K. I. Laws, “Textured image segmentation,” Ph.D. dissertation, 1980.
[24] M. Lyons, S. Akamatsu, M. Kamachi, and J. Gyoba, “Coding facial expressions with gabor wavelets,” in *Proceedings Third IEEE international conference on automatic face and gesture recognition*. IEEE, 1998, pp. 200–205.
[25] P. N. Belhumeur, J. P. Hespanha, and D. J. Kriegman, “Eigenfaces vs. fisherfaces: Recognition using class specific linear projection,” *IEEE Transactions on Pattern Analysis & Machine Intelligence*, no. 7, pp. 711–720, 1997.
[26] L. Liu, S. Lao, P. W. Fieguth, Y. Guo, X. Wang, and M. Pietikäinen, “Median robust extended local binary pattern for texture classification,” *IEEE Transactions on Image Processing*, vol. 25, no. 3, pp. 1368–1381, 2016.
[27] M. Rosvall and C. T. Bergstrom, “Maps of random walks on complex networks reveal community structure,” *Proceedings of the National Academy of Sciences*, vol. 105, no. 4, pp. 1118–1123, 2008.
[28] R. Milo, S. Shen-Orr, S. Itzkovitz, N. Kashtan, D. Chklovskii, and U. Alon, “Network motifs: simple building blocks of complex networks,” *Science*, vol. 298, no. 5594, pp. 824–827, 2002.
[29] G. V. de Lima, T. R. Castilho, P. H. Bugatti, P. T. Saito, and F. M. Lopes, “A complex network-based approach to the analysis and classification of images,” in *Iberoamerican Congress on Pattern Recognition*. Springer, 2015, pp. 322–330.