Stochastic distances for patch-based ultrasound image despeckling

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***Abstract*—Ultrasound image despeckling is an important research field since it can improve the interpretability of one of the main categories of medical imaging. Many techniques have been tried over the years for ultrasound despeckling, and more recently, a great deal of attention has been focused on patch-based methods, such as non-local means (NLM) and block-matching collaborative filtering (BM3D). A common idea in these recent methods is the measure of distance between patches, originally proposed as the Euclidean distance, for filtering additive white Gaussian noise. In this work, we derive several new similarity measures based on the statistics of the speckle and apply them for despeckling both radio frequency (RF) and log-compressed US signals. State-of-the-art results in filtering simulated, synthetic, and real ultrasound images confirm the potential of the proposed approach.**

***Index Terms*—despeckling; ultrasound imaging; patch-based filtering; stochastic distances; geodesic distances; BM3D; NLM**

I. Introduction

ULTRASONOGRAPHY is an important modality of medical imaging since it is non-invasive, harmless, portable, low cost and is conducted in real time. The main issue affecting ultrasound images is a random granular pattern, the speckle, which is a phenomenon arising from the coherent nature of the acquisition system. The speckle, a form of multiplicative noise, affects the interpretability of the image, both by specialists or automated tools, and should be attenuated as much as possible. This kind of noise is also present in other types of coherent imaging systems, such as the laser, sonar, and synthetic aperture radar (SAR).

A. Related work

Several techniques have been proposed in the literature for speckle removal, including the well-known Lee [1], Frost [2], and Kuan [3] adaptive filters, wavelets [4] [5], and anisotropic diffusion [6] [7]. More recently, attention has focused on patch-based approaches, such as NLM [8] [9] and BM3D [10] [11]. A common concept underlying these new techniques is the patch similarity measure, which indicates the similarity of two patches. In NLM, the patch similarity measure is used to determine the weights of the filtering process, while it is used in BM3D to aggregate the blocks of similar patches for subsequent filtering. As in the original proposals, both NLM [12] and BM3D [13], which were designed for additive white Gaussian (AWG) noise, use the Euclidean distance (L2 norm) as the patch similarity measure.

In [14], the authors extend the NLM approach for speckle noise using a Bayesian framework. They use the Pearson distance as a new statistical distance for patch comparison. In [15], the NLM is adapted to SAR imagery using statistical inference in an iterative approach. The patch distance measure is obtained from the image statistics, which are considered to fit a Nakagami-Rayleigh distribution. In [16], a non-local approach is used for filtering intensity SAR images of homogeneous areas, modeled to fit a gamma distribution. It uses statistical hypothesis testing based on stochastic divergences derived from the (h, ϕ)-divergence class. In [17], the stochastic distances coming from the (h, ϕ)-divergence class are used to replace the Euclidean distance in a non-local filter for intensity SAR images, modeled by the G_0 distribution. In addition, working with stochastic distances derived from (h, ϕ)-divergence class and a non-local filter, in [18], the approach is applied to polarimetric SAR images, adopting the complex Wishart distribution statistics.

In [10], the BM3D algorithm is adapted to SAR imagery by replacing the Euclidean distance in the block-matching step by a distance derived from the SAR image statistics, as suggested by [15]. Additionally, the hard thresholding step is replaced by a local minimum mean squared error (MMSE) shrinkage strategy, as proposed originally in [19].

Contributions: In this work, we deduce several new patch similarity measures, based on well-known entropies and statistical divergences. These measures are derived for Rayleigh, Fisher-Tippett and Nakagami distributions, covering the noise model of RF and log-compressed US images. Two approaches are used to generate the measures. First, we use symmetrized versions of statistical diverges based on the (h, ϕ)-divergence class. Second, we find the geodesic distances induced in a probabilistic space and based in the (h, ϕ)-entropy class. We name these two types of measures as stochastic distances throughout this manuscript. Finally, these similarity measures are used to propose stochastic filters derived from NLM and BM3D techniques. Although there are approaches using stochastic distances as new patch similarity measures in [16] and [17], they are valid only for the statistics of SAR imagery and are applied only to NLM. Furthermore, the first attempt to apply stochastic distances with the BM3D method is derived from this research and given in [20]. In the same way, the first attempt to use geodesic distances based on probabilistic spaces for despeckling is also based in this research and given in [21].

II. Theoretical background

A. Speckle statistics

The ultrasound signal is acquired by transmitting high-frequency sound waves into the body and collecting the returning echoes. The interference among the returned echoes gives

This manuscript is related to a PhD dissertation.

rise to a phenomenon known as the speckle, which is responsible for the granular aspect of the ultrasonic images. The way the returned echoes are modeled depends on certain factors, such as the placement (regular or random) and density of scatterers and the presence of strong specular reflections. The various combinations of these factors yield different scattering regimes and consequently different statistical models for the radio frequency (RF) ultrasound envelope signal and related speckle pattern. Destrempes and Cloutier (2010) provide a deep review of such statistical models in [22].

Considering a resolution cell with a high density of scatterers (>10) that are randomly distributed and without strong specular reflections, we have the regime referred to as the fully developed speckle. For such a regime, the amplitude of the RF ultrasound signal envelope is known to be well described by a Rayleigh random variable (Y):

$$P\left(Y|\sigma\right) = \frac{Y}{\sigma^2} exp\left(-\frac{Y^2}{2\sigma^2}\right),\ Y \geq 0,\ \sigma > 0 \tag{1}$$

where Y is the amplitude of the RF signal and σ is the tissue reflectivity for the specific resolution cell.

Defining a new random variable by applying the logarithm transform to equation (1), we can model the log-compression phase, which is carried out by any commercial ultrasound equipment, to adjust the large dynamic range of the RF image to the dynamic range of the equipment display:

$$Z = log(Y + 1) \tag{2}$$

Notice that the Rayleigh random variable Y has support in interval $[0, +\infty)$. By adding one in equation (2), we conveniently keep the new Z random variable in the same support interval since there should be no negative amplitudes in the log-transformed image.

The distribution of the transformed random variable Z will be given by

$$P(Z) = \left|\frac{dY}{dZ}\right| P(Y) \tag{3}$$

From equation (2), we have

$$Y = \exp\left(Z\right) - 1 \tag{4}$$

and

$$\frac{dY}{dZ} = \exp\left(Z\right) \tag{5}$$

Replacing equation (4) in (1) and using the result, together with (5), in equation (3), we have the distribution for the log-compressed ultrasound image, modeled by Z in equation (2). This distribution has a double exponential or Fisher-Tippett shape, of which the only parameter is the tissue reflectivity σ:

$$P(z|\sigma) = \left(\frac{\exp(z) - 1}{\sigma^2}\right) \exp\left(z - \frac{\left(\exp(z) - 1\right)^2}{2\sigma^2}\right) \tag{6}$$

Let z_1, z_2, $\ldots$, z_3 be samples of independent and identically distributed observations coming from the distribution of Z in equation (6). It can be easily shown that the maximum likelihood estimator for the parameter σ of this distribution is given by

$$\hat{\sigma}^2 = \frac{1}{2n} \sum_{i=1}^{n} (\exp(z_i) - 1)^2 \tag{7}$$

The fully developed speckle regime leading to the Rayleigh and Fisher-Tippett models is valid only in the homogeneous areas of the image. To take into account other scattering regimes, other statistical models have to be considered, as the K-family of distributions. The issue about working with these distributions is their analytical complexity, which makes them unsuitable to apply in this paper. One possible solution for this situation is to use the Nakagami distribution. Nakagami is a two parameters distribution, known to be an approximation for one of the distributions of the K-family [22], the K-distribution.

The probability density function of the speckle under Nakagami model is given by

$$f(x; m, \Omega) = \frac{2m^m x^{2m-1}}{\Gamma(m)\Omega^m} \exp\left(-\frac{m\, x^2}{\Omega}\right),\ x, \Omega > 0,\ m \geq \frac{1}{2}, \tag{8}$$

where m is the shape and Ω the scale parameters, and Γ denotes the gamma function, as defined in [23, p.255]. Notice that for $m = 1$, Nakagami becomes the Rayleigh distribution.

For a sample $\{x_1, x_2, \therefore, x_n\}$, with n points, the MLE estimator of the scale parameter (Ω) of the Nakagami distribution is given by

$$\hat{\Omega} = \frac{1}{n} \sum_{i=1}^{n} x_i^2, \tag{9}$$

and the MLE estimator of the shape parameter (m) is given by the solution of the equation

$$\Psi(m) - log(m) - \frac{1}{n} \sum_{i=1}^{n} log(x_i) + \log\left(\frac{1}{n} \sum_{i=1}^{n} x_i^2\right) = 0, \tag{10}$$

where Ψ denotes the digamma function, as defined in [23, p.258]. Details on these estimators can be found in [24].

B. Multiplicative model

Assuming a fully developed speckle model and, consequently, Rayleigh statistics for the speckle, if we take a homogeneous region of the image (σ constant), the ratio of the standard deviation to the expected value is constant, meaning that they are proportional. The fact that the standard deviation is proportional to the mean in homogeneous areas suggests a multiplicative model for the speckle. Such a model has been widely used for deriving speckle reducing schemes and may be expressed as:

$$I(i, j) = S(i, j) * Y(i, j) \tag{11}$$

where $Y(i, j)$ is the multiplicative speckle noise in the spatial position (i, j), with statistics given by equation (1). Moreover, $S(i, j)$ represents the noiseless pixel value to be recovered for the same spatial position.

One possibility to deal with speckle under the multiplicative model is to apply a homomorphic approach. It is based on taking the logarithm to transform the multiplicative noise into additive noise:

TABLE I
(h, ϕ)-DIVERGENCES AND RESPECTIVE $h(x)/\phi(x)$ FUNCTIONS.

$(\mathbf{h}, \phi)$-**Divergences**	$\mathbf{h(x)}$	$\phi(\mathbf{x})$
Bhattacharyya	$-\log(1-x)$	$-\sqrt{x} + \frac{x+1}{2}$
Hellinger	$\frac{x}{2}$	$(\sqrt{x}-1)^2$
Kullback-Leibler	x	$x \log(x)$
Havrda-Charvát	x	$\frac{(x-x^s)}{1-s}$
Sharma-Mittal	$\frac{\exp[(s-1)x]-1}{s-1}$	$x \log(x)$
Rényi$^{\beta}$	$\frac{\log((\beta-1)x+1)}{\beta-1}$	$\frac{x^{1-\beta}+x^{\beta}-\beta(x-1)-2}{2(\beta-1)}$
Triangular	x	$\frac{(x-1)^2}{x+1}$
Harmonic-Mean	$-\log(1-\frac{x}{2})$	$\frac{(x-1)^2}{x+1}$

$$\begin{aligned} log[I(i,j)] &= log[S(i,j)] + log[Y(i,j)] \\ log[I(i,j)] &= log[S(i,j)] + Z(i,j) \end{aligned} \tag{12}$$

where $Z(i, j)$ is now an additive log-transformed speckle noise.

In this work, we adopt the multiplicative model for speckle in US images, using the relation in (11) for the RF images, and the relation in (12) for the log-compressed images.

C. Stochastic Distances based on symmetrized divergences

Divergence measures play an important role in statistical inference and discrimination since they are measures of the statistical distance between probability distributions [25]. We use the term 'stochastic distances' for symmetrized versions of divergence measures, as suggested by [26].

Let V_1 and V_2 be random variables defined on the same probability space Λ, with densities $f(x;\theta_1)$ and $f(x;\theta_2)$ of the same parametric distribution, and with parameter vectors θ_1 and θ_2 respectively. The (h, ϕ)-divergence between the densities will depend only on the parameter vectors and can be expressed by

$$D^{(h,\phi)}(\theta_1, \theta_2) = h\left(\int_{\Lambda} \phi\left(\frac{f(x;\theta_1)}{f(x;\theta_2)}\right) f(x;\theta_2)\, dx\right), \tag{13}$$

where ϕ is a convex function and h is a strictly increasing function [26].

Some h and ϕ functions that give rise to well-known divergences are listed in Table I. Reference to these functions can be found in [26] and [27].

Since the measures defined by equation (13) are usually non-symmetric, we define here our stochastic distances as in equation (14). This is a typical procedure to symmetrize divergence measures, as in [26]:

$$d^{(h,\phi)}(\theta_1, \theta_2) = \frac{D^{(h,\phi)}(\theta_1, \theta_2) + D^{(h,\phi)}(\theta_2, \theta_1)}{2}. \tag{14}$$

Let *X* and *Y* be two noisy patches of size *N x M*, whose statistics can be described by the Rayleigh ((1)), Nakagami ((8)) or Fisher-Tippett ((6)) distributions, with parameter vectors θ_1 and θ_2 and defined in the interval Λ. Replacing $f(x;\theta)$ in equation (13) by one of these parametric family of distributions, using a pair of $h(x)$ and $\phi(x)$ functions given in Table I, and applying the result in equation (14), it is possible to deduce several stochastic distances. The explicit expressions for the distances derived for Rayleigh, Fisher-Tippett and Nakagami distributions, based on all the divergences listed in Table I, may be found in [28]. This work uses the divergences shown in Table I since those are the divergences that resulted in closed-form expressions for the stochastic distances when using Rayleigh, Fisher-Tippett and Nakagami distributions.

D. Geodesic distances based on probabilistic spaces

Considering $f_P(x;\theta)$ as a parametric family of probability distributions, where $\theta = \{\theta_1, \theta_2, \ldots, \theta_M\}$ is an M-dimensional parameter vector, the set of all possible values of this parameter vector form an M-dimensional Riemannian space, as shown by Rao in [29]. Also in [29], Rao studied the geometric structure of this space, introducing a quadratic differential metric that allows the geodesic distance between two points in this space to be calculated. In summary, the geodesic distance proposed by Rao is a measure of distance between two parameterizations from a family of probability distributions

Additionally, the studies [30] and [31] introduced a generic method to calculate Rao's geodesic distance based on the ϕ-entropy class. In [32] this previous method is extended and considers the (h, ϕ)-entropy class. Entropy classes are generalizations that allow the representation of a set of entropies. The (h, ϕ)-entropy class can be expressed as

$$H_{(h,\phi)}(s) = h\left(\sum_i \phi(p_i)\right), \tag{15}$$

where choosing adequate functions h and ϕ result in well known entropy measures. Table II shows a set of entropies and the related h and ϕ functions used to obtain them based on the (h, ϕ)-entropy class.

The method proposed by the study in [32] allows the calculation of generalizations of Rao's geodesic distance, based on all entropy measures that can be represented by the (h, ϕ)-entropy class. Considering a family of distributions with only one parameter, this method can be summarized as: consider P a random variable with density $f_P(x;\theta)$ and support interval *I*, where $\theta = \{\theta_1\}$ is the parameter vector. The geodesic distance between the points θ_a and θ_b in the space generated by the parameter vector of the density $f_P(x;\theta)$, expressed in terms of the (h, ϕ)-entropy class, is given by [32]

$$d(\theta_a, \theta_b) = \left|\int_{\theta_a}^{\theta_b} [g(\theta)]^{\frac{1}{2}}\, d\theta\right|, \tag{16}$$

where

$$\begin{aligned} g(\theta) = &\left[h''\left[\int_I \phi\left(f_P(x;\theta)\right) dx\right] 2\int_I \phi'\left(f_P(x;\theta)\right) \frac{\partial f_P(x;\theta)}{\partial\theta} dx\right. \\ &+ \left[h'\left[\int_I \phi\left(f_P(x;\theta)\right) dx\right] \int_I \phi''\left(f_P(x;\theta)\right) \frac{\partial f_P(x;\theta)^2}{\partial\theta^2}\, dx\right], \end{aligned}$$

assuming that the integrals exist and are finite. The symbols h'/ϕ' and h''/ϕ'' denote the first and second derivatives of the functions $h(y)$ and $\phi(x)$ with respect to y and x respectively.

Consider P_1 and P_2 as two noisy patches that can be statistically described by the Rayleigh (equation (1)) or Fisher-Tippett (equation (6)) distributions, and with parameter vectors $\theta_1 = \{\sigma_1\}$ and $\theta_2 = \{\sigma_2\}$, respectively. Replacing $f_P(x;\theta)$ in equation (16) by one of these parametric family of distributions, using a pair of $h(y)$ and $\phi(x)$ functions given in Table

TABLE II
ENTROPIES-(h, ϕ) AND RESPECTIVE $h(x)$ AND $\phi(x)$ FUNCTIONS.

$(\mathbf{h}, \phi)$-**Entropy**	$\mathbf{h}(\mathbf{y})$	$\phi(\mathbf{x})$
Arimoto	$\frac{(y^s-1)}{s-1}$	$x^{\frac{1}{s}}$
Havrda-Charvát	y	$\frac{(x^s-x)}{1-s}$
Rényi	$\frac{\log(y)}{(s-1)}$	x^s
Shannon	y	$-x\log(x)$
Sharma e Mittal	$\frac{\exp[(s-1)y]-1}{1-s}$	$x\log(x)$
Tsallis	$\frac{y-1}{1-s}$	x^s
Varma	$\frac{log(y)}{m-r}$	x^{r-m+1}

II, and solving the integrals in the interval $[0, +\infty]$, we can deduce several geodesic distances. The explicit expressions for the distances derived for Rayleigh and Fisher-Tippett, based on Arimoto, Havrda-Charvát, Rényi, Shannon, Sharma-Mittal and Varma entropies may be found in [28] or [21]. This work uses the entropies shown in Table II since those are the entropies that resulted in closed-form expressions for the geodesic distances when using Rayleigh and Fisher-Tippett distributions.

E. Stochastic patch-based filters

In this section, we propose four types of stochastic despeckling filters, two based on NLM and two based on BM3D.

For the stochastic filters derived from the NLM, we use as base implementation, the pixel-wise NLM approach described in [33]. In that implementation, designed for Gaussian noise, the image is divided in overlapping windows, named search windows, with a central pixel p. The estimation of the pixel p in the filtered image is a weighted average using all the pixels (q_i) in the search window, where the weight is based on an exponential kernel depending on the Euclidean distance (d) between the patches:

$$w_i = \exp\left(-\frac{d(B(p,f), B(q_i,f))^2}{h^2}\right), \tag{17}$$

where $B(x, f)$ denotes a patch centered in the pixel x and size $(2f + 1) \times (2f + 1)$. The parameter h controls the degree of smoothness the filter will apply to the image.

To define stochastic versions of such a filter, we replace the Euclidean distance in equation (17) by one of the stochastic distances that can be derived in Section II-C. These filters are named NLM-SIM. In the same way, replacing the Euclidean distance in equation (17) by one of the geodesic distances that can be derived in Section II-D we define the filters named NLM-GEO.

For the stochastic filters derived from the BM3D, we use as base implementation, the BM3D algorithm found in [34]. The BM3D algorithm performs three core operations: block-matching, 3D collaborative filtering in a sparse domain, and reconstruction. For the block-matching operation, the image is scanned in overlapping windows and for each of these windows, a reference patch is compared to all other (overlapping) patches within the window. All similar patches, according to Euclidean distance, are stacked to form a 3D block. These 3D blocks undergo a linear transformation and are filtered in a sparse domain. After inverse transformation, there are many estimates for the same pixel, and the patch is reconstructed by combining those estimates. The *BM3D* algorithm executes the core operations twice. In the first step, the *3D* blocks are filtered using a sparse coefficient threshold; during the second step, the 3D blocks are filtered using a Wiener filter with coefficients estimated from the result of the first step. To define stochastic versions of such a filter, we replace the Euclidean distance in the block-matching step by one of the stochastic distances that can be deduced in Section II-C. These filters are named BM3D-SIM. In the same way, replacing the Euclidean distance by one of the geodesic distances that can be derived in Section II-D we define the filters named BM3D-GEO.

III. EXPERIMENTS AND RESULTS

To validate the filters defined in section II-E, we use simulated, synthetic, and real ultrasound images. The experiments with synthetic images use the database of realistic breast phantoms found in [35]. The phantoms mimic the breast tissue echogenicities, tumor shapes, and lesion histology. There are three types of lesions (carcinoma, cyst, and fibroadenoma), with three possible tumor shapes each. For each combination of lesion/tumor shape, there are 50 phantoms, where the position of the tumor varies. We used 10 phantoms from each combination, totaling 90 tested phantoms. These phantoms are the ground truth or noiseless images. Next, we define a noise pattern as an image with Rayleigh-distributed amplitudes of mean 1 and of the same size as the ground truth. Since we consider uncorrelated noise, the noisy image must be obtained by the pixel-by-pixel multiplication of the noiseless image and the noise pattern. These noisy images are referred to as synthetic RF noisy images. We note that this noisy image corresponds to the US multiplicative noise model in equation (11), where the speckle noise corresponds to the Rayleigh distribution in equation (1). The synthetic log-compressed images are obtained by log-compressing the synthetic RF noisy images using equation (2).

The experiments with real images use a set of RF frames of breast lesion recorded from a malignant and a benign tumor, available from [36]. The sets are composed by 183 RF frames for each type of tumor. We used 50 RF frames from each set for our experiments. These images are in RF format, thus we extracted the envelope of the signal using the Hilbert transform, as suggested in [37]. These are the images referred to as the real RF US images. The real log-compressed images are obtained by log-compressing the real RF images, as the model in equation (2).

For the simulated ultrasound images, we use the tool Field-II [38], and we simulate images for the cyst phantom, as in [39] and [8]. The US RF signal envelope is extracted with the Hilbert transform exactly as shown in the sample code provided with the tool. These are the simulated RF images. The simulated log-compressed images are generated by log-compressing the simulated RF images using equation (2).

Besides the original NLM and BM3D algorithms, we compare our approach with other despeckling filters that have publicly available versions. These filters are the PPB [15],

TABLE III
DESPECKLING PERFORMANCE OF NLM-SIM, NLM-GEO, NLM, PPB AND OBNLM.

Image	Simulated		Synthetic		Real	
	RF	Log	RF	Log	RF	Log
NLM-SIM	**1.0**	0.79	**0.99**	0.99	**1.0**	0.93
NLM-GEO	0.93	**0.85**	0.98	**1.00**	0.98	**0.94**
NLM	0.88	0.76	0.95	0.84	0.87	0.86
OBNLM	-	0.70	-	0.85	-	0.72
PPB	0.87	-	0.90		0.85	-

TABLE IV
DESPECKLING PERFORMANCE OF BM3D-SIM, BM3D-GEO, BM3D, SAR-BM3D AND OBNLM.

Image	Simulated		Synthetic		Real	
	RF	Log	RF	Log	RF	Log
BM3D-SIM	**1.00**	0.92	**1.00**	**0.96**	**1.00**	**0.94**
BM3D-GEO	**1.00**	**0.97**	**1.00**	0.72	**1.00**	0.90
BM3D	0.93	0.68	0.84	0.68	0.94	0.77
OBNLM	-	0.64	-	0.65	-	0.64
SAR-BM3D	0.90	-	0.71		0.86	-

OBNLM [8] and SAR-BM3D [40]. We use PPB and SAR-BM3D as references for filtering RF images and OBNLM as reference for filtering log-compressed images.

To quantify the performance of the filtering experiments we use the speckle suppression index (SSI) [41], the mean preservation speckle suppression index (MPSSI) [42], the peak signal-to-noise ratio (PSNR) [43], the structural similarity index (SSIM) [44], the edge preservation index (EPI) [45], and the resolution index(RI) [46]. To simplify the comparison among the filters, since we have a large number of filters, we define the average value of normalized metrics (AVNM). It is calculated by normalizing each metric by its maximum value, and calculating the average value. AVNM ranges from zero, low filter performance, to one, maximum filter performance.

To be able to compare the performance of the filters, we fix the RI metric. With RI fixed we can analyze the performance based on the other metrics. To have the RI fixed, we run one filter, measure the RI value and then run the other filters, with a small variation in the parameters, until the same RI is found.

In the Tables III and IV we show the performance of the stochastic filters when compared with the original methods and other despeckling filters, for NLM and BM3D respectively.

IV. DISCUSSIONS

The result in the Tables III and IV show that both NLM and BM3D benefit from replacing the Euclidean distance by one of the distances suggested in sections II-C and II-D. This is valid for simulated, synthetic and real US images, both in RF and log-compressed formats. The filters defined in section II-E have better performance than the originals NLM and BM3D respectively, and also have better performance than the references used for comparison, when considering the average value of the normalized metrics.

With the full set of data shown in [28], it is also possible to list the following findings:

- When using the NLM algorithm:
  - the stochastic distances derived from the symmetrized divergences of Havrda-Charvát and Rényi for Fisher-Tippet distribution have the better performance for filtering log-compressed images. This is valid for simulated, synthetic and real images;
  - the geodesic distances derived from the Havrda-Charvát, Sharma-Mittal and Tsallis entropies for Rayleigh distribution have better performance for filtering RF images. This is valid for simulated, synthetic and real images;
- When using the BM3D algorithm:
  - there is no difference using any of the stochastic distances derived from symmetrized divergences for filtering log-compressed images;
  - the stochastic distances derived from the symmetrized divergences of Havrda-Charvát and Rényi for Nakagami distribution have the better performance for filtering RF images. This is valid for synthetic and real images;

Finally, it also can be verified by the data in [28], that when filtering RF images with the NLM stochastic filters, it is better to adopt the Rayleigh model for the speckle.

V. CONCLUSIONS

This work presented a set of new stochastic distances for Rayleigh, Fisher-Tippett and Nakagami distributions based on the (h, ϕ)-divergence class and in the (h, ϕ)-entropy class. Patch-based US image filters derived from the NLM and BM3D approaches were defined using these stochastic measures. Experiments with simulated, synthetic and real US images suggest that these stochastic distances may be used to improve the performance of patch-based methods filtering US images.

VI. RESULTS

As a result of the dissertation, the work in [20] was published and the work in [21] is accepted for publishing. Additionally, a manuscript entitled ”Patch similarity in ultrasound images with hypothesis testing and stochastic distances” is under major review by the journal ”Computerized Medical Imaging and Graphics”.

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