A max-tree simplification proposal and applications for the interactive max-tree visualization tool

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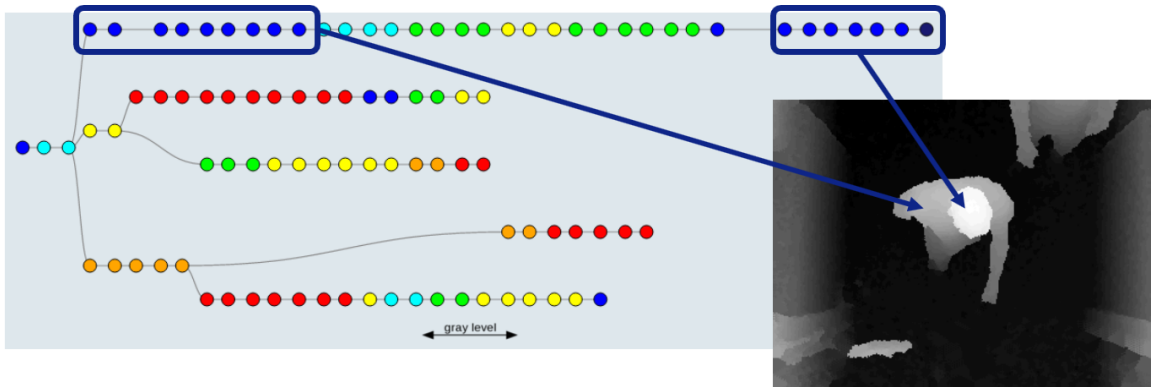
Fig. 1. Teasing of one of our proposals, the visualization of attributes for the interactive max-tree: in the dendrogram (left), we highlight two set of nodes with low eccentricity values, and we indicate their respective match in the depth image (right), the head and the body of a person.

Abstract—**The max-tree is a data structure that represents hierarchically all connected components of an image. It is a powerful structure to perform image processing and analysis tasks. The interactive max-tree visualization tool was the first proposal of an interactive graphical representation for the max-tree, a tool that can be used to navigate, visualize and manipulate the max-tree nodes. In this paper, we propose a more meaningful simplification procedure for the max-tree, we extend the interactive max-tree visualization tool to work with 3D images and we propose the visualization of attributes, a new feature using the interactive max-tree that provides insight for the development of automatic methods.**

Keywords-**max-tree visualization; interactive max-tree; attribute analysis; 3D max-tree; max-tree simplification;**

I. INTRODUCTION

The max-tree is a powerful mathematical morphology data structure proposed by Salembier et al. [1]. It represents hierarchically all connected components resulting of all possible upper thresholds of an image. Max-trees can be computed and processed fast [2], [3], [4]. They have been used in many automatic applications like object recognition [5], detection of Morse Regions [6], recognition of text in natural scenes [7], 3D segmentation [8], remote sensing [9], among many others.

The max-tree also has been used in interactive applications [10], [11], [12], in which the user either defines parameters for the processing task or selects regions within the input image, then the application shows interactively the filtering or segmentation result. In these works, the user interacts only with the image and the parameters, never directly with the max-tree, probably due the fact that max-trees of natural images have tens of thousands of nodes (Fig. 2).
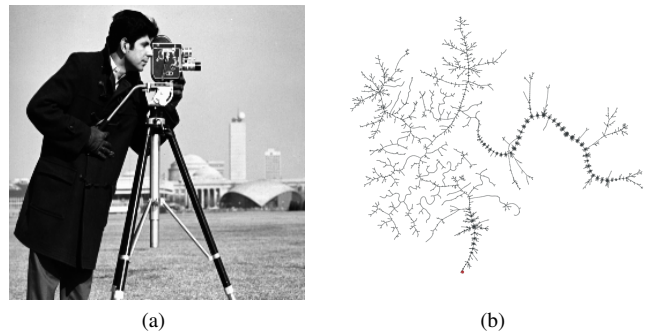

Fig. 2. Max-tree of a natural image: (a) Cameraman image (256x256 pixels), and (b) its corresponding max-tree with 11,123 nodes.

The interactive max-tree tool proposed by Tavares et al. [13] was the first tool that displayed the max-tree graph and provided interactive features that allow the user to navigate, visualize and manipulate the max-tree nodes. The main contribution of their work was the simplification procedure that selected the n first nodes resulting of the lexicographical sorting using the extinction value [14] and the area of the nodes as the attributes for the sorting. The major disadvantage of this procedure is that consecutive nodes with very similar sizes and shapes are preserved, while important tree bifurcations can be lost.

Motivation: The main motivation of this work is to develop a max-tree simplification filter, called area difference filter, capable of generating a more meaningful graphical representation of the max-tree. Also, we want to add attribute information, which provides insight for the development of automatic methods, and 3D image support to the graphical representation, since the max-tree is very useful for developing medical image applications.

Contributions: The main contribution of this work is the max-tree simplification proposal and the analysis of the area difference filter. Other significant contribution is the improvement of the max-tree graphical visualization tool [13] by extending it to work with 3D images, and adding the display of nodes' attribute values.

II. TECHNICAL BACKGROUND

A. Max-tree

The main concept related to this work is the data structure called max-tree. The max-tree (Fig. 3(b)) is a data structure in which its nodes represent hierarchically the connected components for all threshold levels of a gray-scale image (Fig. 3(a)).
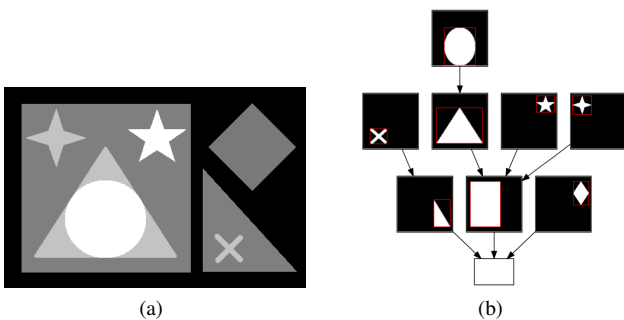

(a)   (b)

Fig. 3. Max-tree of a simple synthetic image: (a) synthetic image and (b) corresponding max-tree with 9 nodes. We can observe the hierarchy of the connected components corresponding to each max-tree node.

The direct rule defined by Salembier et al. [1] is a common rule to filter the max-tree, where the node is removed if it does not meet some criterion. In the full contraction by the direct rule (Fig. 4), the pixels of the selected node (Fig. 4(a)) merge with the pixels of its parent (Fig. 4(b)). The selected node is contracted and its children become children of its parent (Fig. 4(c)).
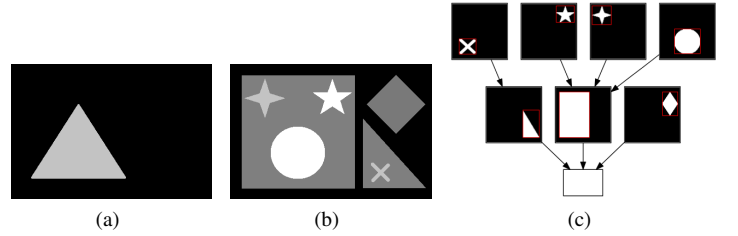

(a)   (b)   (c)

Fig. 4. Illustration of the contraction operation on the max-tree presented in Fig. 3: (a) connected component to be contracted, (b) image reconstruction after contraction, and (c) max-tree resulting from the contraction operation.

B. Extinction values

The concept of extinction values [14] is helpful to identify relevant and irrelevant extrema, which are typically artifacts. Considering any increasing attribute, the extinction value of a regional maximum, corresponding to a max-tree leaf in our context, is the maximum size for an attribute filter so that this maximum will be preserved after filtering.

The extinction values for the area attribute considering an image with only one dimension is demonstrated in Fig. 5. The input image highlighting the extinction values is shown in Fig. 5(a), its corresponding max-tree with its nodes represented by the notation ID:level[area] is depicted in Fig. 5(b). From the area attribute values are calculated the extinction values 9 and 3 for the two leaves.
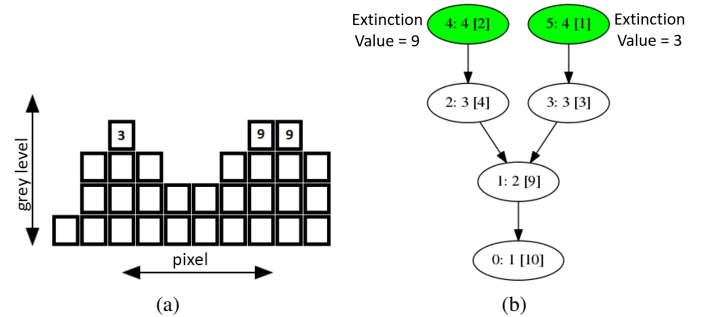

(a)   (b)

Fig. 5. Illustration of the extinction values considering the area attribute: (a) input image and its extinction values, and (b) its corresponding max-tree with area attribute highlighting its two leaves.

C. Extinction filter

Considering the extinction values EX for all the max-tree leaves, the extinction filter preserves only the NL (number of leaves) with higher extinction values (Fig. 6(a)) and the nodes in the path from the preserved leaves to the root (Fig. 6(b)), removing the other nodes (Fig. 6(c)). We denote the input max-tree as $\mathcal{T}_0$ and the resulting max-tree as $\mathcal{T}_1$:

$$\mathcal{T}_1 = \Psi_{NL,EX}(\mathcal{T}_0) \qquad (1)$$

We can observe the application of the extinction filter to an image in Fig. 7. Considering the extinction values for the volume attribute and preserving 3 leaves ($NL = 3$), we apply
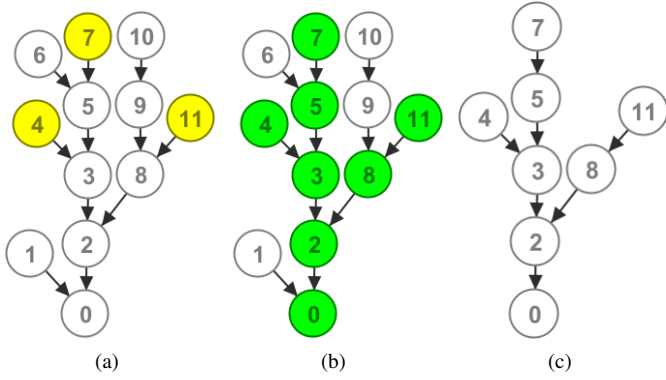
Fig. 6. Illustration of the extinction filter preserving three leaves: (a) max-tree indicating three leaves with higher extinction values, (b) max-tree indicating nodes that will be preserved by the filter in green, and (c) the filtering result.
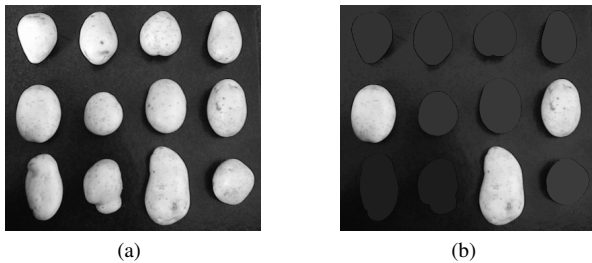

Fig. 7. Applying the extinction filter with parameter $NL = 3$ and using extinction values for the volume attribute: (a) input image and (b) the filtering result.

the filter on the image seen in Fig. 7(a) and the result is shown in Fig. 7(b).

D. Area Open Filter

For all nodes of the input max-tree $\mathcal{T}_0$, the area open filter [15] removes all nodes with area less than a certain threshold value. This filter can be defined by the equation:

$$\mathcal{T}_1 = \Psi_A(\mathcal{T}_0) = \{i \in \mathcal{T}_0 | Area(i) \geq A\}, \qquad (2)$$

where i is the max-tree node considered, $Area$ is the area attribute, and A is the area threshold criterion.
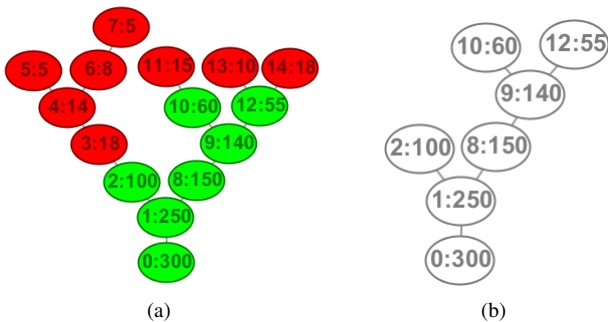

Fig. 8. Illustration of the area open filter using the threshold value 20: (a) max-tree identifying the nodes that will be filtered in red and the nodes that will be preserved in green, and (b) the result of the area open filter.

The area open filter is illustrated in Fig. 8, where each node is identified using the notation ID:area. Considering the area criterion 20, the nodes in red in Fig. 8(a) will be filtered from the max-tree, as they have area smaller than the threshold criterion. The filtering result is shown in Fig. 8(b).
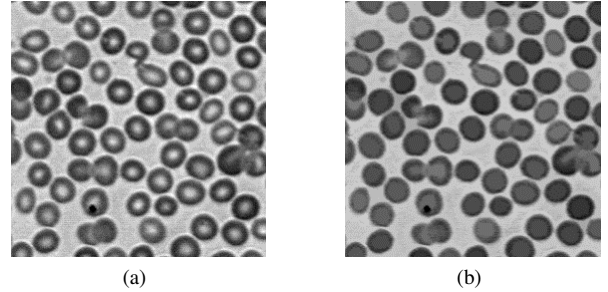

Fig. 9. Applying the area open filter with parameter $A = 200$: (a) input image, and (b) the filtering result.

We can observe the area open filter, with criterion 200, applied to an image in Fig. 9. The input image with some small lighter regions is shown in Fig. 9(a). The connected components corresponding to these regions have area smaller than 200, so they will be filtered (Fig. 9(b)).

III. INTERACTIVE MAX-TREE REVIEW

The interactive max-tree was proposed by Tavares et al.[13], as an educative tool for the max-tree data structure and a tool to perform image segmentation and filtering tasks. The main idea of the interactive max-tree was to create a way to navigate, visualize and manipulate the max-tree nodes. In order to do so, it was necessary to perform a simplification of the max-tree, preserving its most meaningful nodes.
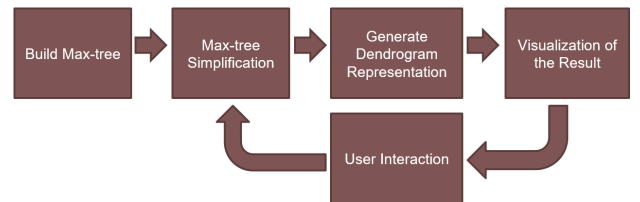

Fig. 10. Interactive max-tree methodology.

The main steps of this methodology are illustrated in Fig. 10. Given an image, the complete max-tree is built and then simplified. A graphical dendrogram for the simplified max-tree is generated, so the user can visualize the simplification result and interact with the tool. The user, when using some navigation features or modifying interactive parameters triggers a new simplification operation on the max-tree, emphasizing some selected region or new parameters for simplification. In all steps this technique always keeps two instances of the max-tree: the original max-tree and its simplified version. The following subsections give more details of each mentioned step.
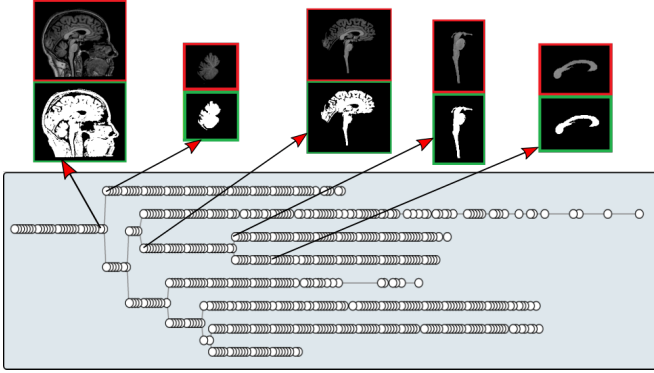
Fig. 11. Interactive dendrogram and some node samples with their respective connected component (binary image) and the reconstruction of the node and all its descendants nodes (gray-scale image).

A. Max-tree simplification

In the simplification procedure proposed by Tavares et al.[13] the attribute propagated extinction value and the attribute area are computed for all max-tree nodes, then the tuples of attributes $(extinction, area)$ are lexicographically sorted, with the propagated extinction value having higher weight. After the sorting, the n first nodes of the sorting result are defined to be preserved and the remaining nodes are removed, resulting in the simplified max-tree. The simplification procedure aims to preserve only the most relevant nodes.

The propagated extinction value is a node attribute that was proposed as an adaptation of the extinction value, which is an attribute of the max-tree leaves. To transform a leaf attribute in a node attribute, the extinction values are propagated from the leaves to the ancestor nodes, and when the node already had a propagated value, the higher one prevails.

B. Dendrogram and interactive features

A dendrogram representation was designed to display the simplified max-tree, it provides an easy way to analyze the evolution of the connected components through the bifurcations. After generating the dendrogram, there are a set of interactive features for the user to interact with the tool. Some features can be used to navigate through the max-tree nodes and some features can be used to perform operations on the max-tree. When the user selects some node in the interactive dendrogram, it is possible to see its connected component (binary image) and its gray-scale reconstruction of the node and all its descendant nodes. A dendrogram with node samples can be seen in Fig. 11.

While browsing the max-tree interactive representation, the user can perform the zoom descendants operation to observe more details of a particular connected component through the increasing threshold levels. This is a recursive feature that allows the user to perform a new simplification on the max-tree emphasizing some region of interest and also allows the user to browse all nodes in the original max-tree.

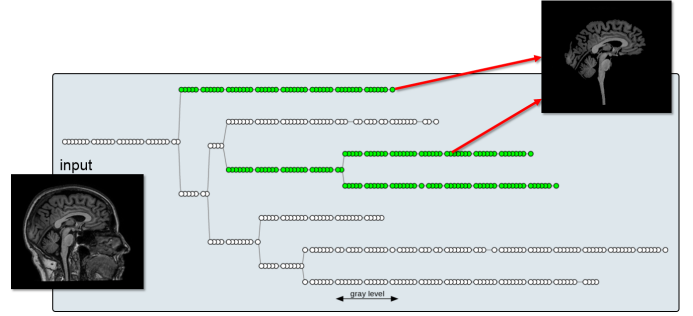In order to perform image processing tasks on the max-tree,

Fig. 12. Skull-stripping segmentation: in the dendrogram of the MR image (left) we include two nodes and theirs descendants in a sub-set of nodes, performing the segmentation (right).

a subset of max-tree nodes can be created using the interactive features inclusion and exclusion.

C. Applications

The interactive max-tree has potential to be used in different applications. A skull-stripping segmentation on a MR image is illustrated in Fig. 12. It was performed using the inclusion of two nodes and their descendants in a subset of nodes.

It's also possible to perform segmentation using subsets of nodes as markers for the watershed transform [16]. This is an example of using the tool in association with other technique.

Other tasks such as filtering and collection of training samples can be performed using the interactive max-tree representation, which is also an educative tool for the learning of the max-tree data structure.

IV. AREA DIFFERENCE FILTER

In this section we define the area difference attribute for the max-tree and we propose the area difference filter.

The area difference attribute is computed for each couple of successive max-tree nodes, as shown in equation:

$$\Delta_A(i) = Area(p_i) - Area(i), \qquad (3)$$

where i is the max-tree node being considered, p_i is its parent and $Area$ is the area attribute. In our representation the parent of the root p_0 is the root itself, the node 0.

The new filter we propose, called area difference filter, is based on the area difference attribute. Considering Δ_A the area difference attribute, and considering D some threshold criterion, the result of the area difference filter Ψ_D are all the max-tree nodes i with corresponding area difference Δ_A higher than D or nodes which have some descendant with Δ_A higher than D. This filter can be defined by the equation:

$$\mathcal{T}_1 = \Psi_D(\mathcal{T}_0) = \{i \in \mathcal{T}_0 | \Delta_A(k) > D, k \in i \cup d_i\}, \qquad (4)$$

where $\mathcal{T}_0$ is the input max-tree, Δ_A is the area difference attribute, D is the threshold criterion and d_i is the set of descendants nodes of i. As in this case the node is only removed if neither the node nor any of its descendants meets

the criterion, this case is considered a filtering by the max rule defined by Salembier et al. [1].
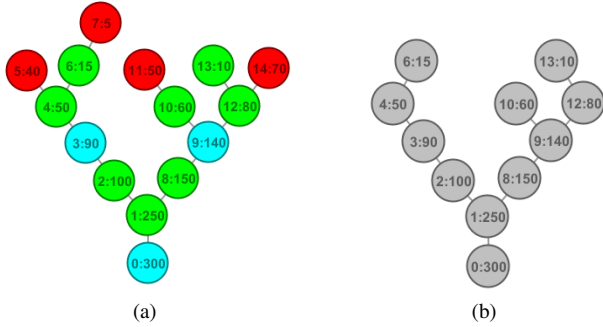

Fig. 13. Illustration of the area difference filter: (a) input max-tree where the nodes in red will be filtered and (b) filtering result.

The application of the filter considering the threshold value $D = 20$ is illustrated in Fig. 13. Each node is represented by the notation ID:area. The nodes in green in Fig. 13(a) have area difference higher than the threshold value 20, so are nodes that will be preserved. The nodes in cyan have descendants which meet the criterion, and in red are the nodes that will be removed by the filter, as we can see in Fig. 13(b).
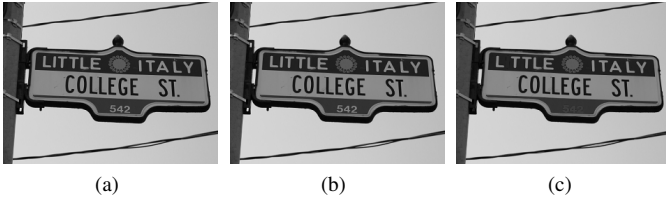

Fig. 14. Applying the area difference filter: (a) input image with 6,981 nodes, (b) result from extinction filter keeping 200 leaves with 2,978 nodes, and (c) result from area difference filter using parameter $D = 10$ with 1,659 nodes.

We can observe the result of the area difference filter applied to an image in Fig. 14. Using the input image shown in Fig. 14(a), with $6,981$ nodes, we first applied the extinction filter with $NL = 200$ to remove some irrelevant extrema, resulting the Fig. 14(b), with $2,978$ nodes. Then we used the area difference filter with criterion $D = 10$, resulting in Fig. 14(c) with $1,659$ nodes. In this case the filter removed some smaller characters from the image.

The inspiration for the development of the area difference filter began when we analyzed the signature [17] of the area attribute on some images and we realized that the nodes which occur a great change of area values correspond to relevant information in the image. So, the main idea of the area difference filter is to simplify the max-tree while preserving nodes that have a great difference of area considering its parent node, because this is typically related to relevant information.

We chose to use the max rule [1] in our filter because this rule preserves all the ancestors of nodes which directly meet the area difference criterion, preventing the loss of any information regarding the connected components with relevant difference of area.

V. IMPROVEMENTS FOR THE INTERACTIVE MAX-TREE

In this section, we present the new simplification method, the feature to visualize attributes of the max-tree nodes, and the tool extension to work with 3D images.

A. New simplification method

The simplification process proposed in [13] did not obtain satisfactory results for some cases, in which many of the preserved nodes were not relevant. In other cases the simplification removed relevant details from the image max-tree. This motivated us to improve the simplification procedure.
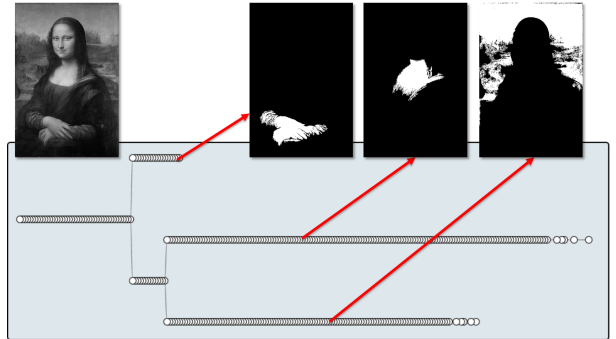

Fig. 15. Result of the first simplification proposal on the Monalisa image (left) preserving 1% of the max-tree nodes (362 nodes).

In the original simplification method it is used only one interactive parameter, which defines a percentage of nodes to be preserved. But the number of max-tree nodes varies widely depending on the image, and so the desired percentage of nodes to be preserved, which is not proportional to the number of max-tree nodes.

The simplified max-tree of the Monalisa image using the original proposal is illustrated in Fig. 15. In this example we performed a drastic simplification, preserving only 1% of the max-tree nodes, resulting in a simplified tree with 362 nodes. As can be observed in the dendrogram, the simplified tree has few bifurcations and much of the relevant information was removed by the simplification.

The new simplification we propose is a sequence of three filters: extinction filter, area open filter and area difference filter, as shown in equation:

$$\begin{aligned} \mathcal{T}_b &= \Psi_{NL,EX}(\mathcal{T}_a), \\ \mathcal{T}_c &= \Psi_A(\mathcal{T}_b), \\ \mathcal{T}_d &= \Psi_D(\mathcal{T}_c), \end{aligned} \tag{5}$$

where $\mathcal{T}_a$ is the original max-tree and $\mathcal{T}_d$ is the simplified max-tree. The filters parameters NL, A, D are interactively set by the user through interactive sliders.

The new simplification proposal has some advantages, it is easier for the user to adjust the appropriate simplification level. The three filters complement each other: the extinction filter preserves the relevant extrema, the area open filter removes
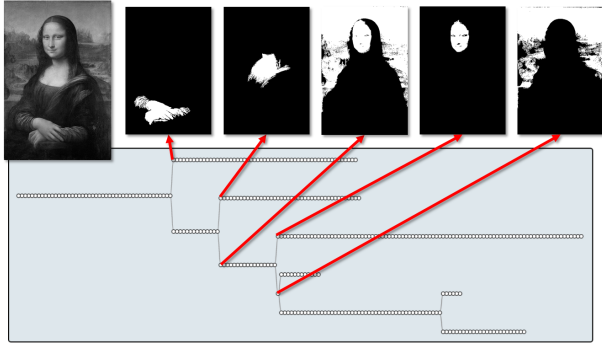
Fig. 16. Result of the new simplification proposal on the Monalisa image (left) with parameters $NL = 6$, $D = 340$, $A = 50$, preserving 357 nodes.
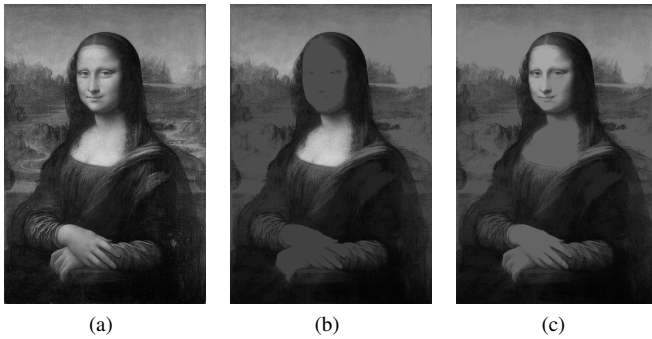

Fig. 17. Comparing the two simplification proposals: (a) Monalisa image (344x512 pixels), (b) first simplification proposal preserving 362 nodes, and (c) new simplification proposal preserving 357 nodes.

nodes with small area and the area difference filter preserves relevant changes in the image.

The simplified tree of the same Monalisa image is shown Fig. 16. The new simplification with the parameters $NL = 6$, $D = 340$, $A = 50$, preserving 357 nodes, achieved a more meaningful simplified tree, because it is possible to notice that the dendrogram has more relevant bifurcations and it has a similar number of nodes in relation to Fig. 15

The simplified tree reconstruction of both simplification approaches is shown in Fig. 17. The input image is displayed in Fig. 17(a), the reconstruction of the simplified max-tree after the original simplification procedure is shown in Fig. 17(b) and the reconstruction of the simplified max-tree after the new simplification procedure is shown in Fig. 17(c). We can see that the new simplification procedure even filtering more nodes, preserved more significant information of the input image.

The simplification we propose can also be used for other purposes. For instance, it can be a pre-processing step to max-tree operations in cases which the amount of nodes is too high.

To better illustrate our simplification proposal we analyze the Structural Similarity Index [18] of the simplification result in a set of two different classes of images, a set of ten images used in car license plate recognition and a set of ten slices of MR images. In this experiment we vary the parameters of

(a)   (b)

Fig. 18. Samples of the images used to measure the Structural Similarity Index: (a) car image and (b) MR image.
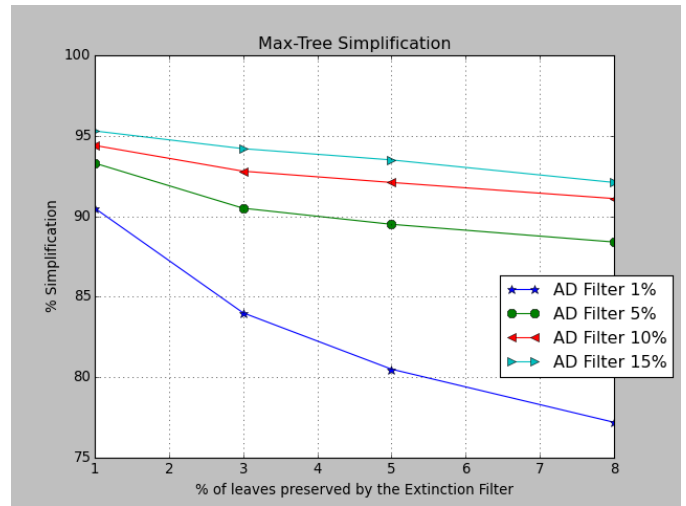

Fig. 19. Max-tree simplification rate varying the simplification parameters: the axis of ordinates represents the percentage of max-tree simplification, the axis of abscissas represents the percentage of leaves preserved by the extinction filter, and each of the plotted series corresponds to a threshold value used in the area difference filter.

the two main filters of our proposal: the extinction filter and the area difference filter, we did not use the area open filter. Samples of each set of images are displayed in Fig. 18.

The max-tree simplification rate for the combination of the filters parameters is shown in Fig. 19. The axis of ordinates represents the percentage of max-tree simplification, the axis of abscissas represents the percentage of leaves preserved by the extinction filter, and each of the plotted series corresponds to a threshold value used in the area difference filter. The threshold value used in the area difference filter was a percentage of the average area of all connected components from the image max-tree. We can observe the Structural Similarity Index of the same parameters values in Fig. 20. As we decrease the percentage of preserved leaves and we increase the percentage of the area difference criterion, it results in greater simplification and a loss of similarity compared to the original image.

B. New applications

The new proposed applications for the interactive max-tree[1] are the visualization of nodes attributes and the extension of

[1]Demo available at: http://adessowiki.fee.unicamp.br/adesso/wiki/code/view/
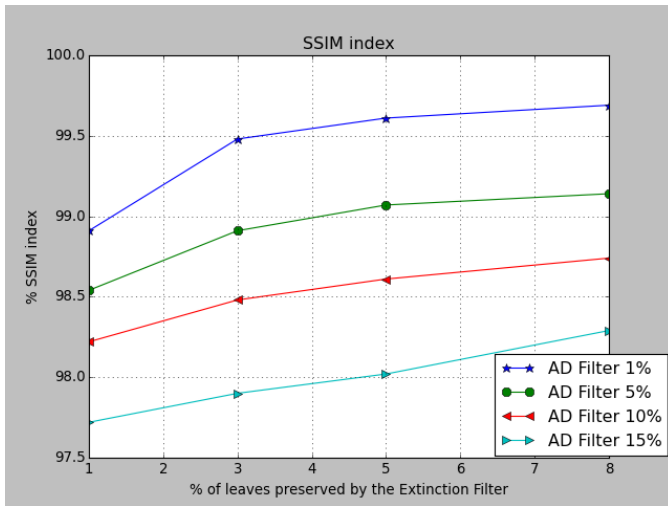
Fig. 20. Structural Similarity Index varying the simplification parameters: the axis of ordinates represents the percentage of the Structural Similarity Index, the axis of abscissas represents the percentage of leaves preserved by the extinction filter, and each of the plotted series corresponds to a threshold value used in the area difference filter.
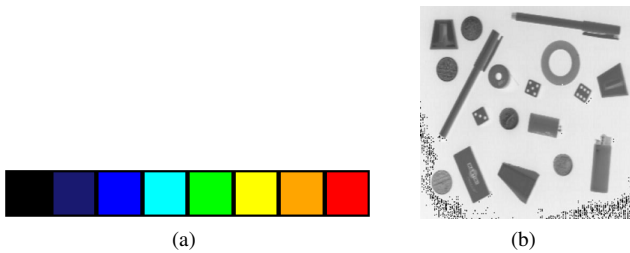

(a)   (b)

Fig. 21. Visualization of attributes: (a) color scale used to display the attribute values, where red is the highest value and black is the lowest, and (b) image containing small objects used to demonstrate eccentricity.
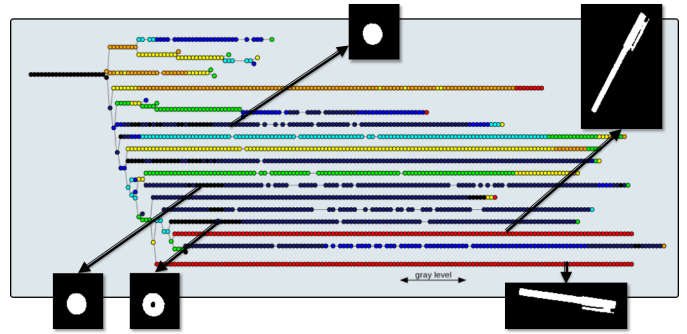

Fig. 22. Illustration of the visualization of the eccentricity attribute: in the dendrogram are indicated two objects with high eccentricity value and three objects with low eccentricity value.
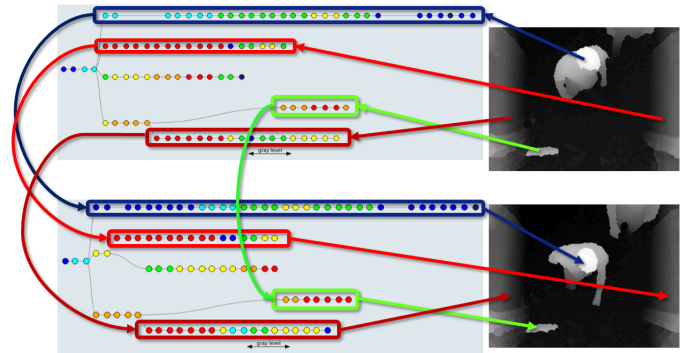

Fig. 23. Illustration of eccentricity in depth images: two depth images, its dendrograms with eccentricity values, and the corresponding match between the objects and the dendrograms branches.

the tool to work with 3D images.

1) Visualization of attributes: This new feature allows to visualize attributes values through the variations in the fill color of each node presented in the dendrogram. We use a color scale (Fig. 21(a)) to represent the attribute values.

To demonstrate this feature we use the eccentricity attribute, which is a measure that basically says how elongated the object is. It is calculated from the concept of image moments [19]. The eccentricity for the max-tree of the image presented in Fig. 21(b) is illustrated in Fig. 22. We can observe that the two indicated pens correspond to red sub-branches in the dendrogram. The red color corresponds to high values of eccentricity, so the two pens are elongated objects. The three rounded objects have corresponding nodes in the dendrogram denoted by the color black or blue, so they have low eccentricity values.

The analysis of depth images is demonstrated in Fig. 23. In this figure are shown two depth images, its dendrograms with the eccentricity values of each node, and correspondences between the objects and the dendrogram nodes.

It is possible to notice that in both cases the person in the

images presents low eccentricity values while the other objects present high eccentricity values. We also can observe that the sub-branches have other similarities like number of nodes and height of its nodes. This is an example of problem which the interactive max-tree can provide insight for the development of automatic methods. Others attributes like height, area, volume, rectangularity ratio, and MSER [20] can be used to analyze the images using the interactive max-tree.

2) Extension to 3D images: When working with different classes of images using the interactive max-tree tool it is possible to notice the tool is more suitable for some types of images, for others however, the tool shows inadequate due to factors such as low contrast, high luminance variation, homogeneous features of nearby objects, among others.

Computed tomography is a case in which the tool is adequate for analysis and segmentation. A computed tomography imaging (CT) of the lungs displayed in 3 views, one slice at a view, is shown in Fig. 24. Through the displayed cursors the user can browse the slices on each axis and the application automatically updates the images displayed in the views.

The dendrogram of the CT image shown in Fig. 24 is displayed in Fig. 25. In this dendrogram are highlighted two samples of nodes and their 3D projection in 3 views. The first node corresponds to a candidate node for the lungs
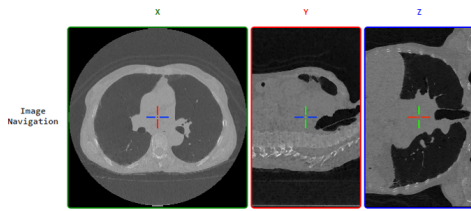
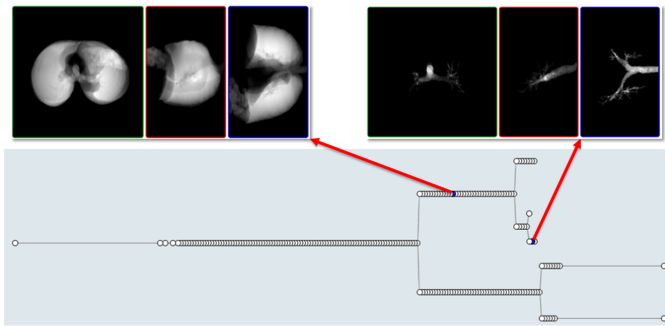Fig. 24. Example of the visualization of 3D Lungs CT.

Fig. 25. Illustration of Lungs CT Dendrogram: a dendrogram highlighting two 3D connected components.

segmentation and the last node corresponds to a node that can be used to segment the trachea. This example illustrates the extension of the tool to work with 3D images.

VI. Conclusion

In this paper, we proposed a new simplification procedure for the interactive max-tree, we also proposed a new attribute filter called area difference filter, we extended the tool to work with 3D images and we proposed a new application for the tool, the visualization of attributes of the max-tree nodes.

The new simplification proposal shows potential to preserve important changes in the image, allowing to perform a drastic simplification on the max-tree while preserving the most relevant information. It can also be a pre-processing step for operations on the max-tree.

The use of the interactive max-tree for segmentation of lungs in a 3D computed tomography showed good results. The visualization of attributes of the nodes illustrated a new perspective of the tool. This feature helps to analyze the nature of the image providing insight to develop automatic methods.

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