A Comparison of Global Illumination Methods Using Perceptual Quality Metrics

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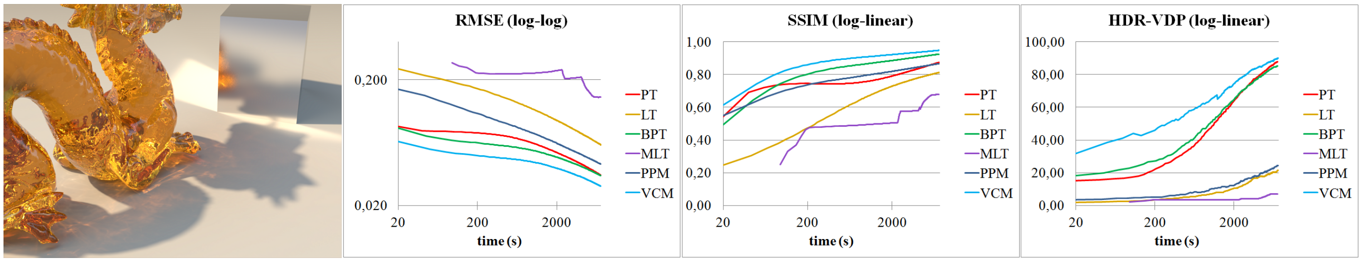


Fig. 1. Reference image and results for test scene Dragon Caustics Glossy.

***Abstract*—This work aims to build a comparison basis for analyzing global illumination methods. We have compared six state-of-the-art global illumination methods, ranging from Monte Carlo Path Tracing techniques to Density Estimation methods such as Progressive Photon Mapping, and the mixture approach Vertex Connection and Merging, using nine test scenes with very different characteristics, including many different light types, illumination conditions and BRDFs, exploring many different light scattering events. In order to compare results, the perceptual quality metrics SSIM and HDR-VDP-2 were used. We provide a complete set of convergence rate curves and results for all test scenes and all analyzed methods. We also discuss strategies to generate reference images for each case. We concluded that in general cases the overhead introduced by BPT and VCM is well compensated by the quality of the produced images, however VCM can handle more interesting effects. We also showed there are usual cases such as interiors with strong indirect illumination in which Light Tracing method excels.**

***Keywords*-Global illumination; Image synthesis; Image quality metrics; Perceptual quality; Ray tracing;**

I. INTRODUCTION

This work is a survey on Global Illumination methods, including analysis of most state-of-the-art algorithms such as Path Tracing (PT), Light Tracing (LT), Bidirectional Path Tracing (BPT), Metropolis Light Transport (MLT), Progressive Photon Mapping (PPM) and Vertex Connection and Merging (VCM), all of them implemented on Mitsuba Renderer [1]. Some of these methods were implemented by the authors of this paper, being the source code available to the public.

The goal is to analyze the efficiency and accuracy of global illumination methods in the presence of some usual and difficult light transport conditions, by the point of view of perceived quality. Given this focus, we discuss comparison metrics which measure the perceptual image quality, such as Structural Similarity Index (SSIM) and Visual Difference Predictor (VDP-HDR-2), against metrics which consider only numerical absolute differences between images, such as Root Mean Square Error (RMSE).

Since the use of reference images is an important issue when analyzing image synthesis methods, we also discuss strategies to generate them in each case, including the choice of the most suited global illumination method to accomplish this task.

In order to evaluate and compare global illumination methods in a fair basis, the results derive from rendering all the scenes on each method, during the same amount of time, and using the same comparison metrics in each case.

The test scenes used herein, also made publicly available, are all very simple and were built to explore specific light transport conditions, although they contain a variety of light types (including area lights, environment maps, delta lights such as directional lights), illumination conditions (mostly direct illumination, mostly indirect illumination, presence of caustics) and different materials (diffuse, specular mirror and glass, glossy surfaces, textures). The purpose of this wide range of scenes is to explore the strengths and weaknesses of each global illumination method.

Contributions: The main contribution of this work to the global illumination field is the analysis of convergence rates of each method in many different light transport conditions using the perceptual quality metrics SSIM and HDR-VDP-2, as well as the RMS error, which as far as we know has not been performed yet.

II. RELATED WORK

Surveying global illumination methods on CPU is something missing in the literature, since most comparisons apply just between a new proposed method and a few similar other ones, being therefore specific (normally addressing a few issues from the illumination process).

One of the firsts to publish ideas about global illumination methods precision was Arvo et al. [2], who analyzed the source of errors on rendered images due to approximations in the mathematical model, inaccuracies on the geometric description of the scene components and algorithms computational simplifications, although he has not compared different global illumination methods.

Khodulev [3] restricted the comparison to two global illumination methods: Light Tracing and Deterministic Radiosity, concluding that Monte Carlo Tracing methods are better suited to handle general scenes. In order to compare the methods, he used a simple scene which has a known analytical solution. In more complex scenes, he compared results with reference images generated by a long period of computation. Szirmay-Kalos et al. [4] extended the set of scenes with analytical solution, using different light types, different BRDFs and geometry. The work was particularly interesting because the authors created scenes with constant analytical solution at any given point on the image plane, but their work was theoretical and they did not compare different global illumination methods.

Smits and Jensen [5] proposed a set of very simple scenes, some with analytical solution, allowing the scientific community to validate new global illumination methods, but they have not proposed any metric or comparison method. Drago et al. [6] proposed a more complex scene based on a scenario from the real world, the atrium of the University of Aizu, providing many real measurements of material characteristics and incoming light at many points. They also proposed perceptual methods, using real user judgments, in order to compare the real and computer generated images. A wider set of scenes was proposed by CIE [7], but Maamari et al. [8] stated that this set needed to be extended in order to include more aspects of light propagation – e.g. not all possible light paths, especially interesting effects, were present in the proposed scenes.

McNamara [9], [10] explored some methods to compare scenes from the real world against computer generated images, analyzing metrics and perceptual differences, making many experiments. He verified that comparisons by numeric techniques not necessarily provide significant measures of perceptual quality by real observers, he thus explored perceptual comparison methods in experiments involving real users as well as using perceptual quality metrics such as Structural Similarity Index (SSIM) and Visual Difference Predictor (VDP) [11]. But this kind of comparison is only possible when there is a real scene to compare against computer generated imagery.

A complete analysis on how to validate global illumination methods was performed by Ulbricht et al. [12], in which they created some perceptual quality definitions and proposed some metrics for comparing results, but their work was a state of the art report, and they have not compared any existing global illumination method.

In the last years many advances emerged in the light transport simulation field, but low attention has been given in making a complete comparative analysis of results.

Hachisuka et al. [13] for example, proposed the method Progressive Photon Mapping (PPM), in which images were compared against ones generated by other methods, but no metric was adopted. All comparisons were made by instructing the reader to visually compare images, noticing the presence of noise and certain characteristics. When the Stochastic Progressive Photon Mapping method was proposed by Hachisuka and Jensen [14], results were compared almost exclusively against images produced using the previous method, PPM, except in only one scene in which results were also compared to images generated by Bidirectional Path Tracing. In the latter work only one comparison metric was used, the Root Mean Square Error (RMSE). In both cases, the authors showed only results from scenes in which the new methods were designed to perform better – no general scenes with more light conditions and wide range of materials were used.

When the VCM method – Georgiev et al. [15] – was proposed, they compared in the supplemental document the results from their new global illumination method against results from other state-of-the-art methods, such as Path Tracing, Bidirectional Path Tracing and Progressive Photon Mapping, using perceptual quality metrics such as SSIM and VDP-HDR. Again, they only compared the new method in scenes which had the desired characteristics. They neither showed results in general cases, nor mentioned how they generated the reference images.

Differently from previous work, the present paper analyses different global illumination methods bringing them to the same comparison basis. All methods are applied to all the proposed scenes, and the same comparison metrics are applied to images generated by all methods. This way we want to build a fair and wide comparison basis among all the methods analyzed.

III. GLOBAL ILLUMINATION METHODS

Ray Tracing [16] was the first method to trace rays going from the camera towards the virtual scene, looking for intersections with objects. At each intersection point, direct illumination was computed, determining the color of the corresponding pixel in the rendered image. This method does not account for light scattering between objects in the scene; hence, the indirect illumination has to be approximated by a constant factor. Obviously this method does not converge to the correct equilibrium of light distribution on the scene.

Distributed Ray Tracing [17] incorporated to Ray Tracing the exploration of other dimensions when tracing new rays looking for intersections with objects in the scene. It was possible to simulate effects such as motion-blur (exploring the time dimension), depth-of-field (varying the position on the camera lens), glossy surfaces (angle of the reflected ray) and area lights (position on the emitting surface). Although new effects were introduced, no indirect light could be calculated: a constant factor should thus be applied to fake the indirect illumination.

Kajiya [18] proposed the Rendering Equation – a mathematical formulation which accounts for all light scattering events in every point of the scene. In the same paper, to try to solve the equation, he proposed the Path Tracing method, which is an extension to Ray Tracing. At every ray intersecting some object on the scene, new rays are randomly traced generating new intersections until some light source is found. Path Tracing is still widely used nowadays because, being considered a brute-force method, it can generate high-quality images at the cost of a great computational effort. It is also the recommended method to generate reference images, whenever applicable.

Light Tracing [19] proposes the reverse idea of Path Tracing. Paths are started at the light sources and, when an intersection between a ray and an object is found, a ray is generated towards the observer in order to hit the image plane, adding a color contribution to the corresponding pixel. Although it is not a very useful method in general cases, because many pixels on the image may not be hit at all, Light Tracing is a very useful method to visualize pure caustics – an effect very difficult for Path Tracing to produce.

Bidirectional Path Tracing [20], [21] was introduced to take advantage of both Path Tracing and Light Tracing methods, being very useful in scenes containing hard to find light sources, such as lights inside luminaries. Paths are traced starting from lights and from the camera, and at each intersection point with objects on the scene, the method tries to connect both paths, in order to create complete paths which effectively carry light on the scene.

Metropolis Light Transport [22] was proposed to enhance the efficiency of Path Tracing algorithms by perturbing (mutating) already found paths which carry light. The idea is to concentrate the exploration around bright regions, instead of stochastically exploring the scene. When a light carrying path is found, small perturbations are generated in order to create new paths, trying to make these new paths also carrying a significant amount of light.

Photon Mapping [23], [24] is a two-stage method which first shoots photons starting at light sources and scattering through objects in the scene, creating a spatial photon map detached from geometry, and then shooting rays from the camera towards the scene: for each intersection with objects, the photon map is consulted to compute an estimation of photon density around the intersection point. This is a biased method because it computes the color of each pixel based on light arriving on its neighborhood. But as the search radius diminishes, the bias introduced decreases. Photon Mapping is a very efficient method to produce caustics and its subsequent scattering events on the scene – effects very difficult to be reproduced using unbiased methods.

Progressive Photon Mapping [13] is an extension to Photon Mapping which removes the limitation imposed by the amount of memory available in the system, and can thus generate better quality results by shooting more photons. This is also a two-stage method, but the first stage consists in tracing rays from the camera and finding the intersection points on the scene. The second stage is repeated progressively, shooting photons from the light sources and accumulating the density estimation at each intersection point found in the first stage.

Finally, Vertex Connection and Merging, introduced simultaneously and independently by Georgiev et al. [15] and Hachisuka et al. [25] is a method which combines Path Tracing and Photon Mapping concepts, bringing these methods to the same framework, being benefited from the strengths of each strategy.

A. *Choosing Global Illumination Methods to Evaluate*

In the present study we have chosen to evaluate the following global illumination methods: Path Tracing (PT), Light Tracing (LT), Bidirectional Path Tracing (BPT), Metropolis Light Transport (MLT), Progressive Photon Mapping (PPM) and Vertex Connection and Merging (VCM).

These methods were chosen so we could consider many algorithm categories: methods starting on the camera, starting on the light sources, methods based on Path Tracing, based on Density Estimation, Bidirectional strategies, methods based on Metropolis sampling and the most recent approach which is a combination of Path Tracing and Density Estimation methods.

B. *Implementation Details*

Except in the case of MLT method, all methods were implemented in Mitsuba. There were two reasons why we have chosen to do so. First, our VCM implementation is essentially a port to Mitsuba from SmallVCM – an open source reference implementation of the method by Georgiev et al. [15] – which contains also implementations of other methods: PT, LT, BPT, PPM and BPM. And second, we needed to save partial images over time, which Mitsuba is not able to natively do because in most methods images are rendered by blocks, while in SmallVCM at each iteration the complete image is rendered, being progressively refined over time.

Regarding the MLT method, we used the implementation already present in Mitsuba because it is one of the few methods which natively renders complete images at each iteration.

If using a pin-hole camera model, it is worth mention LT is not able to render directly visible specular objects – they come out completely black – so our approach was to use a PT pass after LT execution, rendering only directly visible specular surfaces.

IV. TEST SCENES

The scenes we chose are all very simple and were built to explore specific light transport conditions, whilst using many different light and material types. The scenes were made simple so we could focus on analyzing results in the presence of specific path types, and not on particular ray-tracing strategies or hierarchical structures efficiency. The following paragraphs explain each scene in more detail.

Cornell Box Diffuse. This scene is a reproduction of the classic Cornell box experiments by Goral et al. [26]. It has simple illumination conditions and only diffuse materials. It is intended to reveal the additional overhead generated by methods which do extra calculations trying to find light sources that are very easily found.

Cornell Box Mirror. It consists of the same elements as before, but the large boxs material is replaced by a perfect mirror. In this scenario we wanted to observe the behavior of each method by the introduction of a simple specular object.

Cornell Box Glossy. It is the same as the first scene, but both boxes' materials are replaced by glossy metals, generating interesting light interactions like glossy-glossy scattering.

Cornell Box Spheres. This scene was modeled after a test scene proposed by Georgiev et al. [15]. It's composed by a box with two spheres inside: one is a perfect mirror and the other is a perfect glass. In this scene it's possible to explore effects like reflected caustics and caustics generated by reflected light. There are many scattering events which are impossible to be sampled by pure Path Tracing techniques, even bidirectional ones.

Dragon Caustics. This scene is composed of a complex model – a dragon – with an orange glass material and a floor. The focus is on the blurred caustics generated by an environment light. The dragon model was downloaded from the Stanford 3D Scanning Repository.

Dragon Caustics Glossy. It is the same as previous scene, but with the addition of a glossy box, which reflects the dragon and the caustics.

Lafortune Room. Scene containing simple objects and materials but with strong indirect illumination. It was inspired in a scene from Lafortune's PhD thesis [27].

Pool Interior. The scene is a simplified version of a swimming pool provided in the work by Vorba et al. [28]. It focuses on the caustics generated inside the pool by a directional light.

Veach Door. It is a reproduction made by Lehtinen et al. [29] of the classic door scene used by Veach on his PhD thesis [22]. It is composed of a dark room illuminated only by indirect light coming through a small aperture through the door.

V. COMPARISON METHODS

Here we present the adopted comparison metrics as well as the process used to generate test and reference images for each scene.

A. Comparison Metrics

As the focus of this work is to analyze global illumination methods to generate pleasant images to the human eye, perceptual quality metrics were used. Two indices widely adopted to this purpose are used herein: Structural Similarity Index (SSIM) [30], which searches structural similarities between two gamma-corrected images, and HDR-VDP-2 [31], which focus on luminance aspects between test and reference images, ignoring the color. In order to have a comparison with a well known metric, we have also computed RMS error.

The SSIM index computation was slightly altered from its original formulation because the default implementation first converts images to grayscale. But we consider each color channel has an important role in the human perception of computer generated images, especially the case in which global illumination methods are used because these techniques compute many light scattering events between surfaces of different colors. We thus computed the SSIM index for each color channel separately, and the final score we present is the mean of each color channel SSIM index.

In order to compute the HDR-VDP-2 index, the default parameters proposed by the authors in the original paper were used, but scores were computed on already LDR (Low Dynamic Range) gamma-corrected (PNG format) images. The score corresponds to the QMOS output parameter from the algorithm, which is the mean-opinion-score prediction, measuring quality in a scale from 0 to 100.

B. Generating Test Images

In order to generate data to build convergence rate curves, Mitsuba original code was modified allowing snapshots of the rendering progress to be saved every N seconds, where N was set to 6 in simple scenes and to 20 in more complex ones. The total running times were also defined so we could observe most of methods converging to the reference solution – we have stipulated a minimum score of at least 0.9 in SSIM metric.

In simple scenes, graphs were plotted using data from the first 30 minutes of rendering. In scenes with intermediate complexity we used data from 60 minutes of computation and in the most complex cases we used data from the first 120 minutes of rendering. The simple cases are *Cornell Box Diffuse*, *Mirror* and *Glossy*, the intermediate cases are *Cornell Box Spheres*, *Lafortune Room* and *Pool Interior*, and complex cases are *Dragon Caustics*, *Dragon Caustics Glossy* and *Veach Room.*

In order to generate results in the fairest basis, all rendering parameters were kept the same among all methods and scenes. We have used the same sampling strategy – an independent sampler –, the same filter type and size as well as other optimizations like next event estimation for calculating direct illumination and the use of Russian Roulette after 4 bounces. For methods based on density estimation, radius reduction factor was set to 0.75 and initial search radius was set to 0.1% of the scene bounding sphere radius.

C. Generating Reference Images

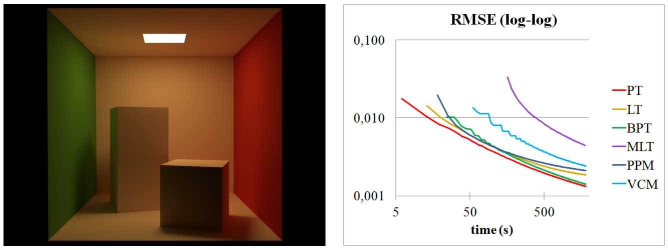
As the proposed scenes don't exist in the real world, the reference images must be also generated by computers. As we are not concerned about image exactness, but our main focus is to produce a pleasant image to the human eye, it should be enough to synthesize a reference image with all light scattering conditions being accounted for and almost free of noise. In order to achieve this, several hours of computation are needed using currently available methods and CPUs.

Another important observation is that each proposed scene was analyzed in order to reveal if a method would fail to sample important paths. For example, in the Cornell Spheres scene, because of the directional light, there are paths which are impossible to be sampled by Path Tracing – such as reflected caustics – so this method cannot be used to generate this scene's reference image.

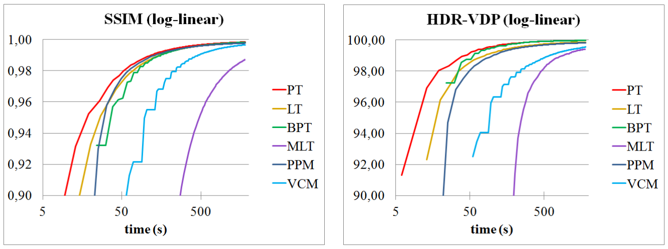
VI. RESULTS

The results in this section show some rendered images, convergence rate graphics and some computed indices for each adopted metric. All the images were rendered with resolution 1024x768, each in a single machine running Windows 7 64 bits, Intel(R) Xeon(R) CPU E5-2680 processor running at 2.7GHz with 8 cores and 16 threads, 24 Gb DDR3 at 1600 MHz RAM, using our modified version of Mitsuba Renderer via command line, which is based on official version 0.5.0, 64 bits.

A. Result Analysis



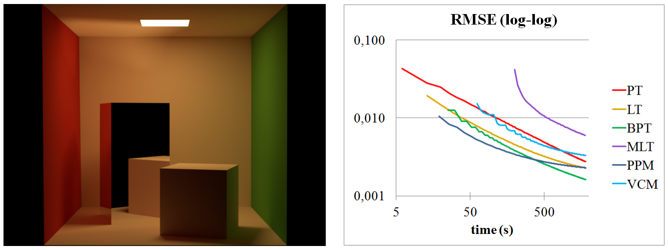
(a) Reference image (b) RMSE



(c) SSIM (d) HDR-VDP-2

Fig. 2. Reference image and results for test scene Cornell Box Diffuse.

In the *Cornell Box Diffuse* scene the reference image was generated by PT after 7 hours of rendering. We noticed all methods converge to the reference solution. Although methods based on Path Tracing have produced images with lower RMS error, we can see that when using perceptual quality metrics all methods, except MLT, converge to the highest score after 30 minutes of calculation. Path Tracing method converges faster because other methods spend time trying to find light carrying paths on the scene, but in this case these paths are easily found everywhere. We can notice in the quality curves that BPT and VCM sometimes advance in steps – this is due to the overhead of each iteration which took longer than 6 seconds to refine the image. As for the MLT method, it has a longer delay on the startup due to the calculation of the overall luminance – stage required only for MLT based methods.



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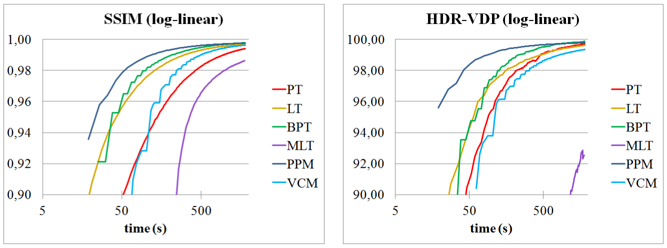
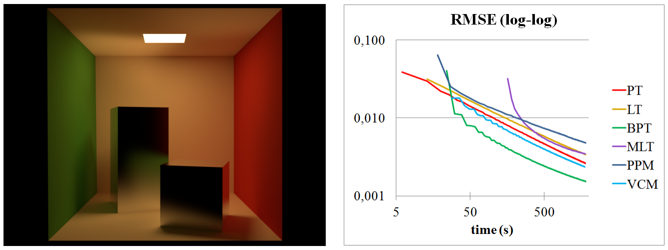


(c) SSIM (d) HDR-VDP-2

Fig. 3. Reference image and results for test scene Cornell Box Mirror.



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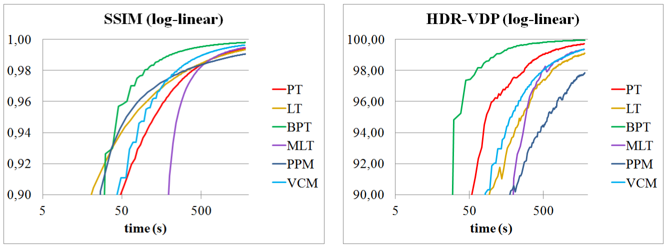
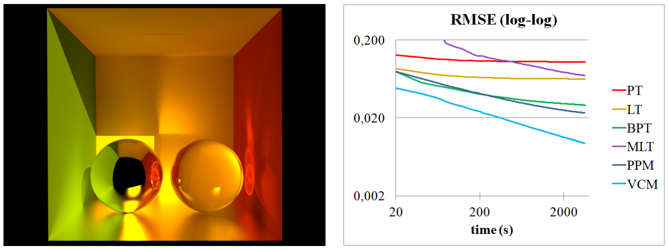


(c) SSIM (d) HDR-VDP-2

Fig. 4. Reference image and results for test scene Cornell Box Glossy.

In the *Cornell Box Mirror* scene, we can reach almost the same conclusion as in the previous example. The reference image was rendered using BPT for 9 hours. In this case, due to the introduction of a mirrored object, Path Tracing looses its superiority in all adopted metrics. BPT, PPM and LT have shown to converge faster to the reference solution. We can conclude that by introducing specular materials, methods which trace paths starting on the light sources become more efficient. It's worth mentioning that PPM in this case achieved the highest convergence rates in both perceptual metrics adopted.

In the *Cornell Box Glossy* scene, where we have two glossy boxes reflecting each other, it's clear in all metrics that BPT was the best method to render the scene, both in



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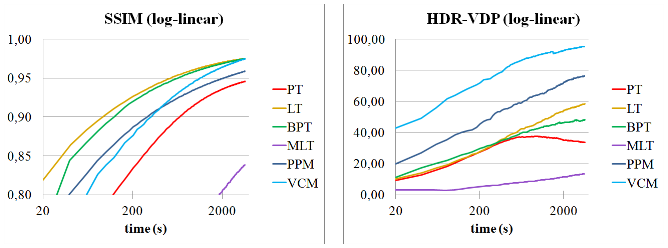
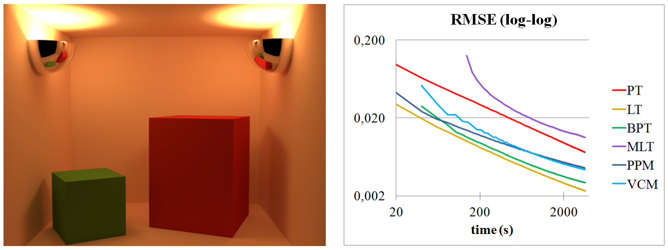
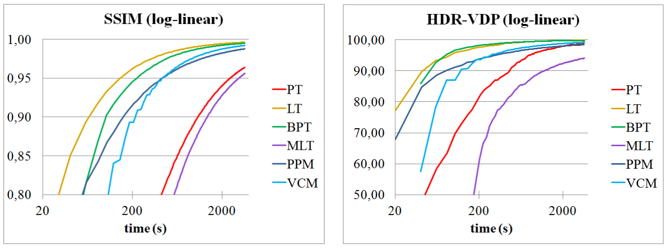


(c) SSIM (d) HDR-VDP-2

Fig. 5. Reference image and results for test scene Cornell Box Spheres.

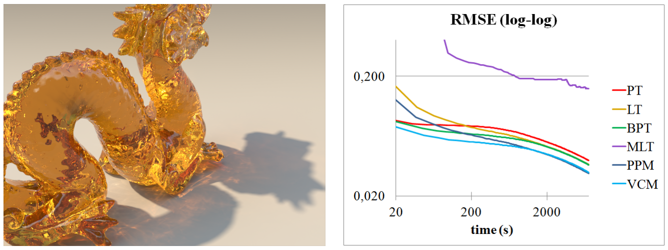


(a) Reference image (b) RMSE

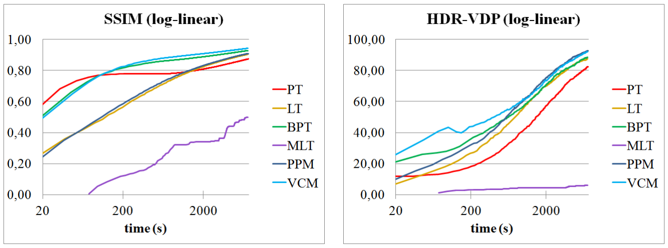


(c) SSIM (d) HDR-VDP-2

Fig. 7. Reference image and results for test scene Lafortune Room.

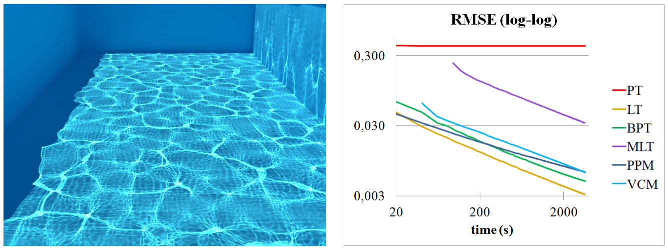


(a) Reference image (b) RMSE



(c) SSIM (d) HDR-VDP-2

Fig. 6. Reference image and results for test scene Dragon Caustics.



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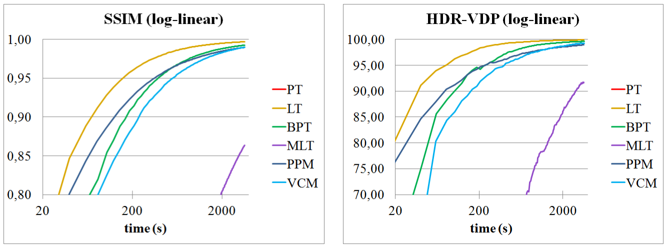


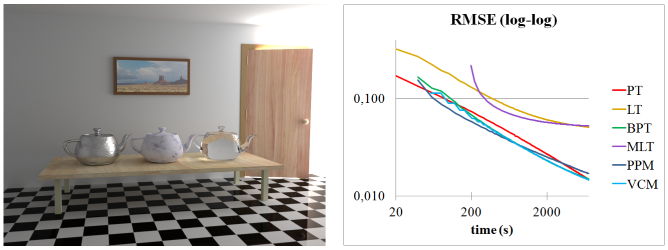
(c) SSIM (d) HDR-VDP-2

Fig. 8. Reference image and results for test scene Pool Interior.

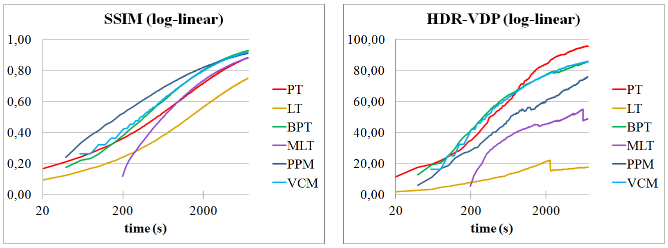
terms of absolute image error and perceived image quality. The reference image was generated by PT after 12 hours of computation. Density Estimation techniques, which were expected to perform better in cases like this, failed because of the material's high glossiness – although VCM showed very good results in the SSIM metric, having the second best convergence rate just behind BPT.

The score curves generated by the *Cornell Box Spheres* scene are more interesting because in this case not all methods converge to the correct solution. There are some effects impossible to be sample by Path Tracing methods such as caustics generated by reflected light and reflected caustics. We can clearly conclude this by examining how RMSE decays in each case. Density Estimation methods – PPM and VCM – have higher convergence rates when looking at the perceptual metrics. The VCM method particularly excels here because it handles better all light scattering types generate by diffuse-specular-glossy interactions. We notice also that MLT faces difficulties trying to find light carrying paths among so many specular and glossy surfaces. The reference image was generate by PPM after 24 hours of rendering.

In the *Dragon Caustics* scene the most difficult effect to be generated is the caustics created by an environment light through a glass material. By looking at the RMSE curves, we notice a stationary region, specially when using PT. This is because all the light paths are efficiently handled except



(a) Reference image (b) RMSE



(c) SSIM (d) HDR-VDP-2

Fig. 9. Reference image and results for test scene Veach Door.

the caustics which takes a long time to converge to the reference solution. We can see this behavior also in the perceptual metrics curves. In cases of caustics directly seen by the camera, even if Density Estimation methods are most recommended, we see that BPT and our LT approach handles the scene equally well. And again, we can see VCM converges faster and more accurately to the correct solution, and MLT extra computation does not efficiently lead to finding light carrying paths. The reference image was generate by BPT after 30 hours.

In the *Dragon Caustics Glossy* scene we can see same convergence behaviours as in the previous case, but a bit more clearly. Because of the highly glossy object directly visible – and the effects generated by light scattering on it –, light tracing methods such as LT and PPM do not perform very well. This combination of effects is better handled by VCM followed by BPT. MLT again has performed very poorly, due to the same reasons explained in the previous case. The reference image was generate by BPT after 60 hours of rendering.

The *Lafortune Room* has revealed curious results. It is a scene with strong indirect illumination, but light emitters are inside the room, which allows most of the light to bounce many times in the scenario. This combination of characteristics makes finding light sources difficult when starting paths from the camera, but benefits from emitting light from inside the room. So methods starting at light sources have performed better, in particular our LT approach had the lowest RMSE and highest convergence rate for SSIM, but only slightly better than BPT. Other good results were observed by PPM and VCM methods, although slightly worse because the presence and size of specular objects were showed to be not so relevant. The reference image was generated by BPT using 16 hours of calculation. MLT was expected to produce good results because of the strong indirect illumination, but instead generated the worst scores. The possible reason is the heavy overhead in generating path mutations, resulting in new light contributing paths that would have been found anyway by random exploration.

The *Pool Interior* scene is a direct visualization of caustics, generated by a directional light, and its subsequent bounces inside the pool. PT fails completely because it cannot connect to the light source because it is occluded by the water surface. LT excelled in this case because no computation is spend trying to find light carrying paths. PPM also showed very good results but because it computes the photon density inside a (small) search radius, the caustics looses its finest details, becoming a little blurred. Except for PT and MLT, all remaining methods converge at approximately the same rate in all metrics to the correct solution. The reference image was generated by LT with 9 hours of rendering.

The last test case is the *Veach Door*, built specifically to prove MLT strength over other methods. However this was not the case. MLT showed comparable results only in the SSIM metric. It may seem, by looking at its RMSE curve, that MLT is not converging to the correct solution. The fact is that it indeed is, but much slower than other methods. With exception of MLT and LT, the latter being very inefficient in cases like this – with strong indirect illumination and light coming from outside the scenario –, we consider all methods performed equally well in all adopted metrics. The reference image was generated by BPT after 57 hours of computation.

VII. CONCLUSIONS

We have compared six state-of-the-art global illumination methods using nine different test scenes and three metrics, being one numeric relative error and two perceptual quality metrics. The characteristics we wanted to focus our tests were specific light scattering events like caustics in the presence of diffuse, specular and glossy materials, generated by many types of lights such as area, directional and environment lights, also evaluating strong indirect illumination conditions – all cases that are very difficult to be handled by current global illumination methods. We have done all tests using our open-source patch to the popular Mitsuba Renderer, consisting of VCM and several other methods re-implementation. We have generated convergence rates curves for all methods in all scenes using all metrics and have made an in-depth analysis of results.

We conclude that bidirectional methods such as BPT and, more recently, VCM handle very well general cases, generating almost always the lowest error and highest perceptual quality scores when comparing to the reference solution, proving that the additional computation overhead is compensated by the generation of more accurate results.

We also observed, surprisingly, that LT – ignored in many recent studies – has generate best results in some common cases like strong indirect illumination inside a (partially) closed room. And it indeed excelled in the visualization of pure caustics, as showed in the Pool Interior scene, even though there were many scattering events generating indirect illumination inside the pool.

Another curious revelation is that the overhead imposed by MLT method is not justified in general cases, not even in strong indirect illumination conditions. MLT implementation in Mitsuba might not be as optimal as all other methods' implementations, but this has to be verified in depth.

Regarding the generation of reference images, special attention had to be paid to the choice of rendering methods. Each scene had to be analyzed in order to reveal if there could be paths that were impossible to be handled by some method. We found that Path Tracing is a very good method to generate reference images in general cases. If there are effects impossible to be sampled using this method, or in situations where pure random walks are extremely inefficient, we have considered next using Bidirectional Path Tracing, ensuring, this way, that reference images are generated by unbiased methods. If there are paths impossible to be sampled even by BPT, we have opted to use Progressive Photon Mapping, but we were extremely careful by manually choosing a very small photon search radius to minimize the bias introduced by the method. In all cases, the recommended running time should be high enough in order to completely eliminate visual noise on the synthesized images.

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