Can Fractal Dimension be used in textile imperfection identification?

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Abstract. Fractal dimension (FD) is an interesting feature proposed to characterize roughness in image. This feature has been used in texture classification, segmentation, shape analysis and other problems. An approach based on FD to estimate differences between a picture and its original pattern is proposed in this note. Experiments on real textile failings are presented.

1 Introduction

Many applications of fractal concepts rely on the ability to accurately estimate the fractal dimension (FD) of objects. This concept can be useful in the measurement, analysis, and classification of texture. [Pentland (1984)] noticed that the FD of surfaces can be used to obtain shape information and distinguish between smooth and rough regions. Some other applications involve image segmentation [Rigaut (1988)], particle morphology [Orford and Whalley (1983)] and image data compression [Barnsley (1988)]. We present here a new range of application: the use of FD to identify image deviation from a standard pattern, which leads to the question that became the title of this paper. We report on some experiments that contribute to answering such questions.

2 Estimation Approaches of FD

[Mandelbrot (1982)], stated that the FD of a set A in Euclidean n-space can be derived from the relation

$$1 = N_r\ r^{FD} \text{ or } FD = \log(N_r) / \log(1/r) \qquad (1)$$

where N_r is the union of non-overlapping copies of A scaled down by a ratio r. However, it is difficult to compute FD using equation (1) directly. [Peleg et al (1984)] extended Mandelbrot's idea to image, that can be viewed as a terrain surface whose height is proportional to the image gray value. The reticular cell counting estimator has been proposed by [Gangepain and RoquesCarmes (1986)]. But this estimator can not be used when the range the actual FD of an image is 2.0 -2.5.

Voss (1986) refers to box counting as the process of estimating the probability that m points lie in the box. [Keller et al. (1989)] proposed a modification of the method due to Voss, which presents satisfactory results up to FD=2.75. [Pentland (1984)] suggested a method of estimating FD by using Fourier power spectrum of image intensity surface, such method gives satisfactory results but, since Fourier transformation computation is included, it is slower than the others. [Sarkar and Chaudhuri (1994)] described an efficient box-counting approach, named Differential Box-Couting (DBC), that uses differences on computing N_r , and gives satisfactory results in all range of FD.

The FD in DBC method is given by

$$FD = \lim_{r \to 0} \log(N_r) / \log(1/r) \qquad (2)$$

N_r is counted in a different manner from the others box-counting methods. Consider the image of MxM pixels has been partitioned into grids of sxs pixels and scaled down to r=s/M. If G is the total number of gray levels then G/s'=M/s. On each grid there is a column of boxes of size sxsxs' (for example see fig 1 where s=s'=3). Assign number 1, 2, ...n to the boxes as shown. If the minimum gray level of the image in the grid (i,j) fall in box number k, and the maximum gray level of the images (i,j)th grid fall in the box number l , then in DBC approach.

$$n_r\,(i,j)=l-k+1, \qquad (3)$$

is the contribution of n_r from the grid (i,j) (for example in fig 1: $n_r(2,3)=3-1+1$). Taking contributions from all grids:

$$N_r = \sum n_r(i,j) \qquad (4)$$

N_r is counted for different values of r and s. Then using (2) FD can be estimate from the least square linear fit of $\log(N_r)$ against $\log(1/r)$.

3 DBC Modification

Although the DBC method gives a very good estimate of FD some simplifications in computations and improvements in efficiency is possible with the following changes in the original method. The Box Couting Theorem [Barnsley (1988)] states that if a set A $\in \Re^3$ is covered by just-touching boxes of side length $(1/2^n)$, then

$$FD = \lim_{n \to \infty} (\log N_n) / (\log 2^n) \qquad (5)$$

where N_n denote the number of boxes of side length $(1/2^n)$ which intersect the set A. In our proposed method, the image division in box of different length is processed in a new manner from others box counting variations and the original DBC method. Consider the image of size MxM pixels , we take M to be a power of 2 and take the range of light intensity to be the integers from 0 to 255. All images are enclosed in a big box of size MxMx256. We consider the image divided into box of side length nxnxn' for n=2,4,8,...2m ($2^{(m-2)}$=M) and n'=2,4,8...2m' ($2^{(m'-2)}$=256) for each image division , N_n is counted as

$$N_n = \sum n_n\,(i,j), \qquad (6)$$

$$n_n = \mathrm{int}(Gray\_max(i,j)/n') - \mathrm{int}(Gray\_min(i,j)/\,n') + 1$$

where int(./.) is the integer part of a division. For example, in figure 1 Gray_max(i,j)=8, Gray_min(i,j)=1, n'=3 and $n_n(i,j)$=(3 - 1 + 1). These changes turn the implementation faster and simpler than of the DBC original algorithm. The image file is read only once, in the first image division in boxes, the bitmap of MxM pixels can not be saved, when the image is read we saved only two matrices of M/2xM/2 , Gray_max and Gray_min (saving MxM/2). This first calculation of n_n , using equation (6), correspond to divided the image in boxes of 2x2 pixels. For boxes of 4x4 pixels there will be M/4xM/4 elements in Gray_max and Gray_min , and each new element (i_new,j_new) is obtained from consulting only the four element (i,j), (i+1,j), (i,j+1) and (i+1,j+1) of the after Gray_max and Gray_min matrices . If the algorithm begin in i=j=0 in each new iteration the Gray_max and Gray_min matrix elements (i_new=i/2 and j_new=j/2) for the next division of the image can be saved in the same space. Then using (5) we estimate FD from mean of $\log(N_n)/\log(2^n)$. It can be easily showed that computation complexity of others approach, including the original DBC, is much more high than that of this approach.

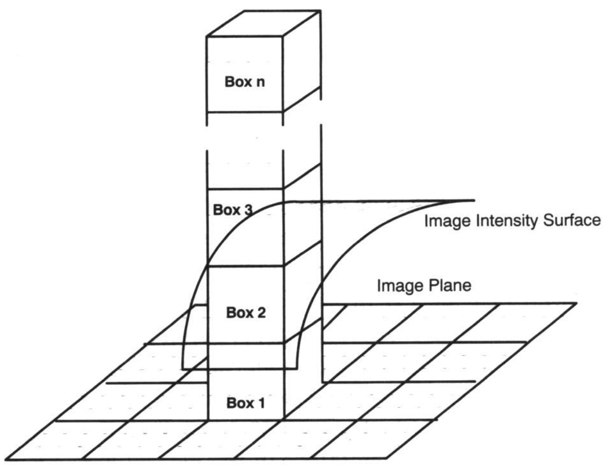


Figure 1. - Determination of n_r by original DBC.

4 Experiments in Estimating FD

The proposed DBC modifications of section 3 programmed in C language for TIFF image files have been used in some experiments in this section. Our first goal is to examine the accuracy of the proposed approach for FD estimation so we use first only images having known fractal dimensions. Test data for this first group of experiments came from synthetic textures [Qian Huang et al (1994)] (9 Brownian images and 9 Takagi surfaces generated on a 256x256 grid with 256 gray level and FD varied from 2.1 to 2.9 on steps of 0.1), it is not reported here [Campos (1995)]. For our experiment with natural images we took 9 images presented in [Sarkar and Chaudhuri (1994)]. The results of this experiments are summarized in table 1, while images are shown in fig 2 . The fact that our approach return accurate values on images with known dimension turns possible ask the heading question: the validity of the use of different values FD to identify variations in images. The remainder of our experiments investigates this possibility.

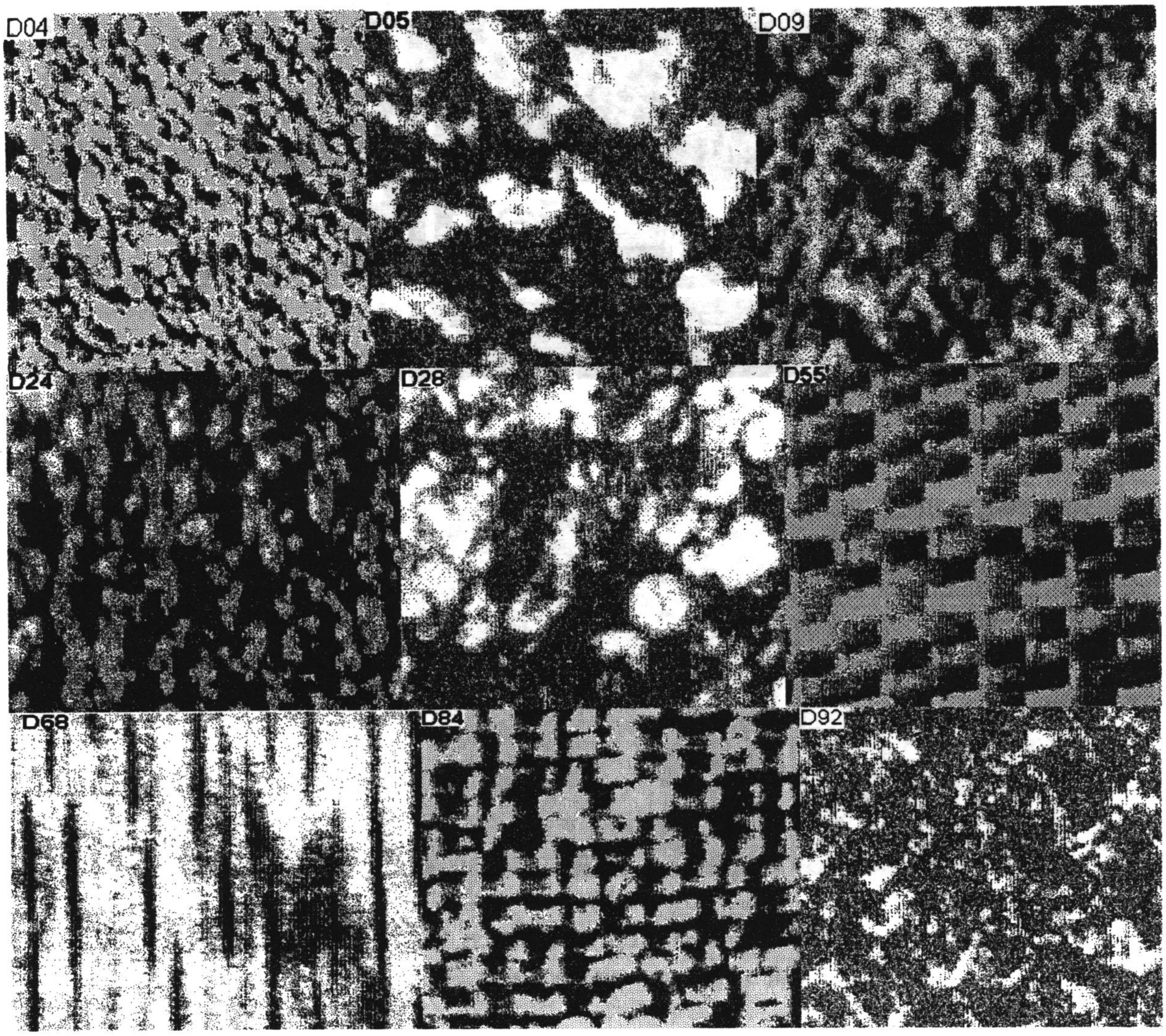
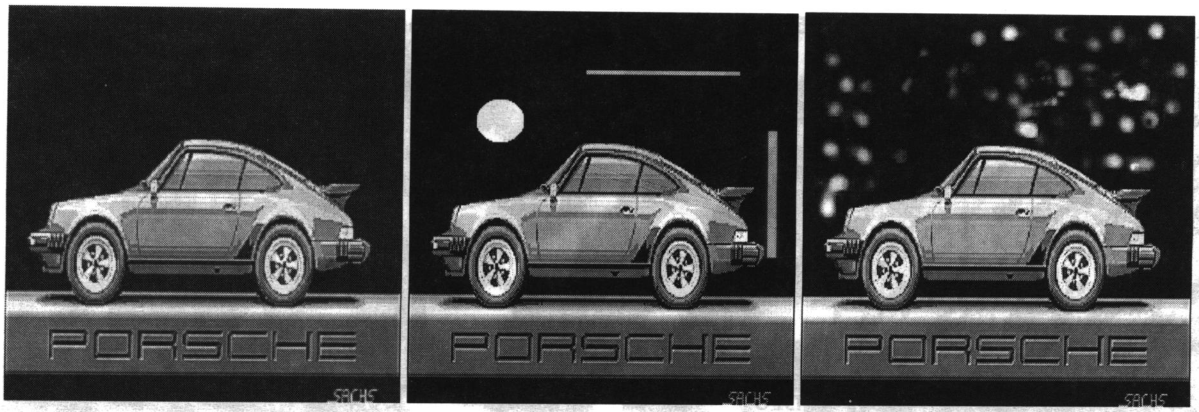


Figure 2. - Brodatz natural textures

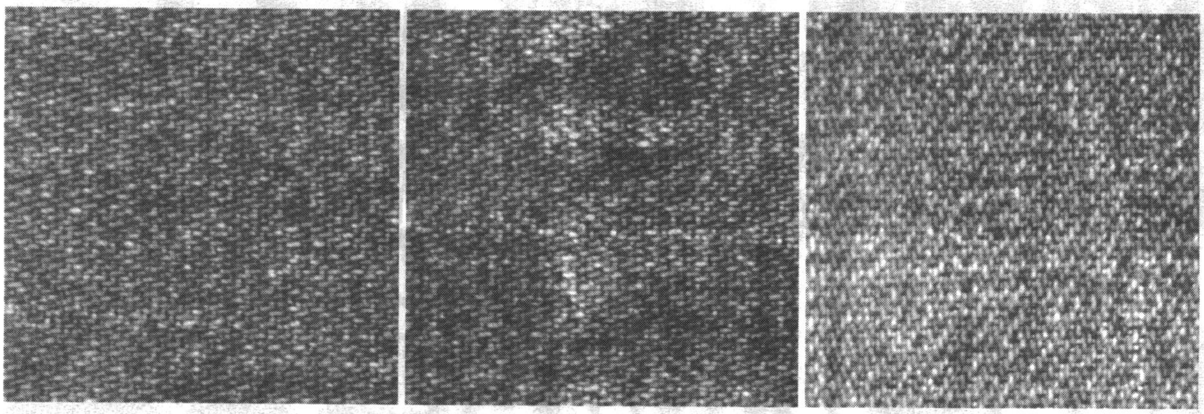
Texture Images	Original DBC	Proposed DBC
D04	2,66	2,66
D05	2,45	2,45
D09	2,59	2,58
D24	2,45	2,44
D28	2,55	2,55
D55	2,48	2,48
D68	2,52	2,53
D84	2,60	2,61
D92	2,50	2,49

Table 1. - Fractal Dimension of Natural Textures (Image numbers correspond to Brodatz's Book)



(a) (b) (c)

Figure 3. - Variations: FD = 2,258(a); FD = 2,304(b) and FD = 2,322(c)



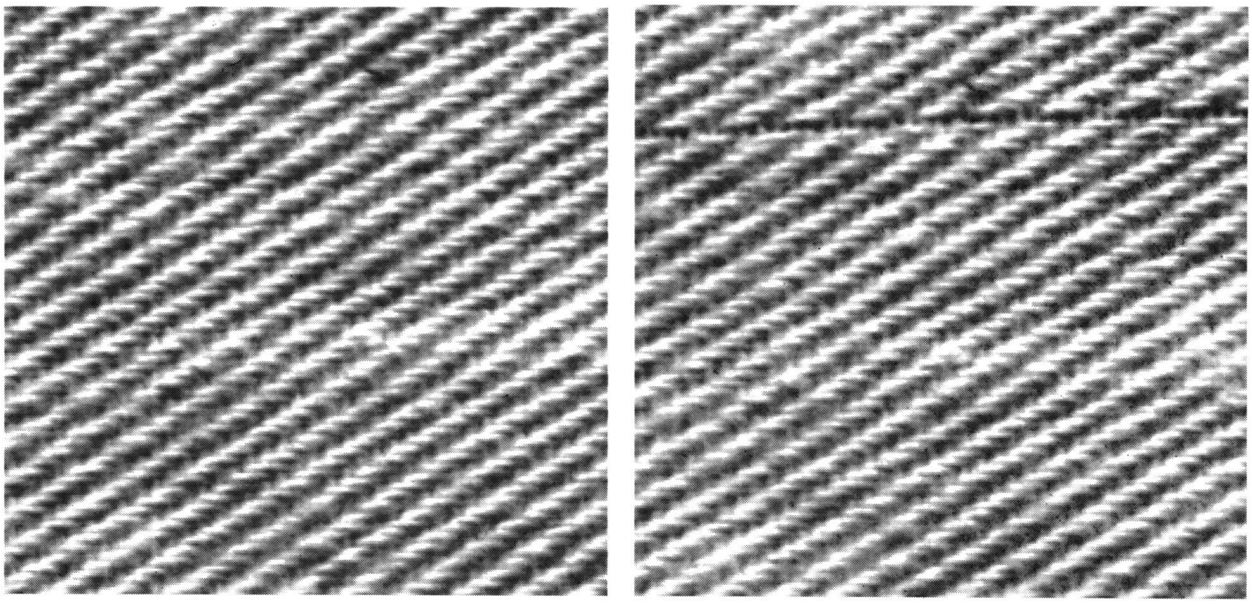
(a) (b) (c)

Figure 4. - Usual jeans imperfections: without defects, FD = 2,428(a); stained jeans, FD = 2,414(b) and non-uniform dyeing, FD = 2,467(c).

Experiments on estimates changes in images are shown in fig 3 to 9. Fig 3 represents an image with 2 modification. The background noise as well as the others new elements introduced in the original turns the image more complex and it is reflected in the increase of FD. On figure 4 two common types of jeans imperfections are analyzed: gradual and local color modification. These light changes in the image modified the FD computed for each image. On figure 5 to 9 different fabric woven with different patterns and kind of defect are shown and have theirs FD changes compared. For all case differences in FD can be find.

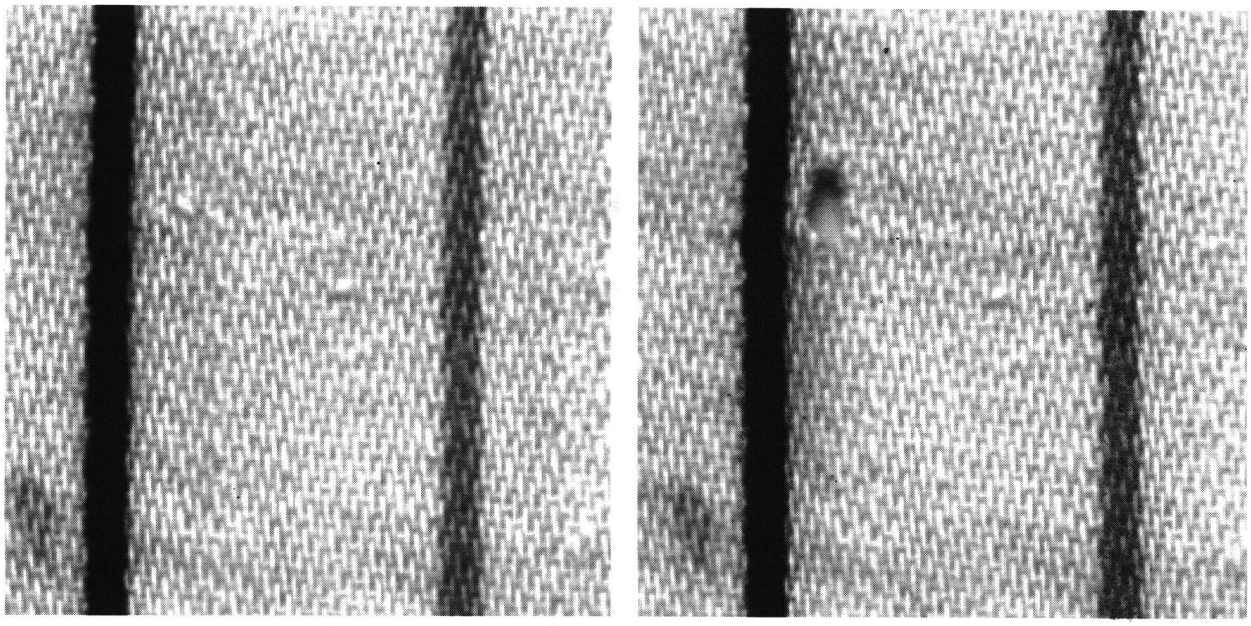
5 Conclusions

Our study covered a wide range of situations in estimating fractal dimension and its use in identification of image changes. The main goal leads to a simple approach to compute FD on images. Elementary experiments demonstrated that the variations between an original image and its reproductions can significantly affect the respective FD. Practical considerations related to specific class of images seem necessary to improve the quality and ensure best image failing recognition. A statistical analysis should be carried out in order to assess the applicability of the FD as a means for finding imperfections in textiles [Campos (1995)]. The encouraging conclusion is that this approach is simpler than the usual methods . However , we have accomplished our objective of highlight a new use of FD and introduce a new way for image modification recognition. Future work in textile and fabrics failings, will be directed toward further improvement the relationship between interruption and/or beginning on the box-length division and the expected imperfection dimensions.



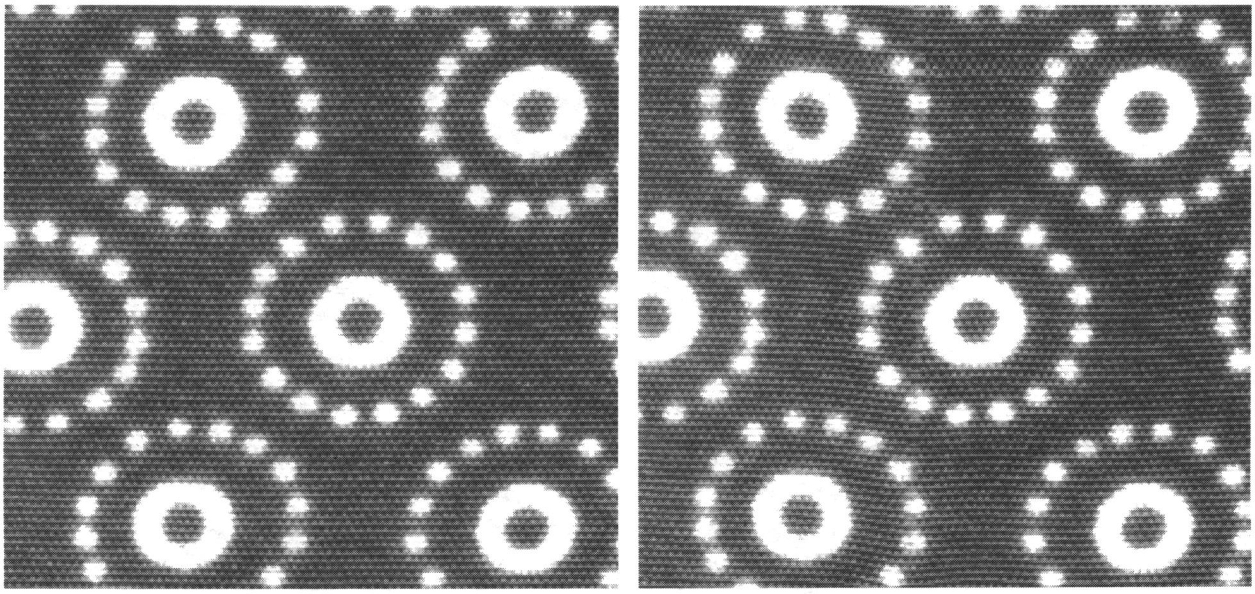
(a) (b)

Figure 5. - Example of drill imperfection: (a) without defects, DF = 2,618 and (b) with a thread pull off DF = 2,605



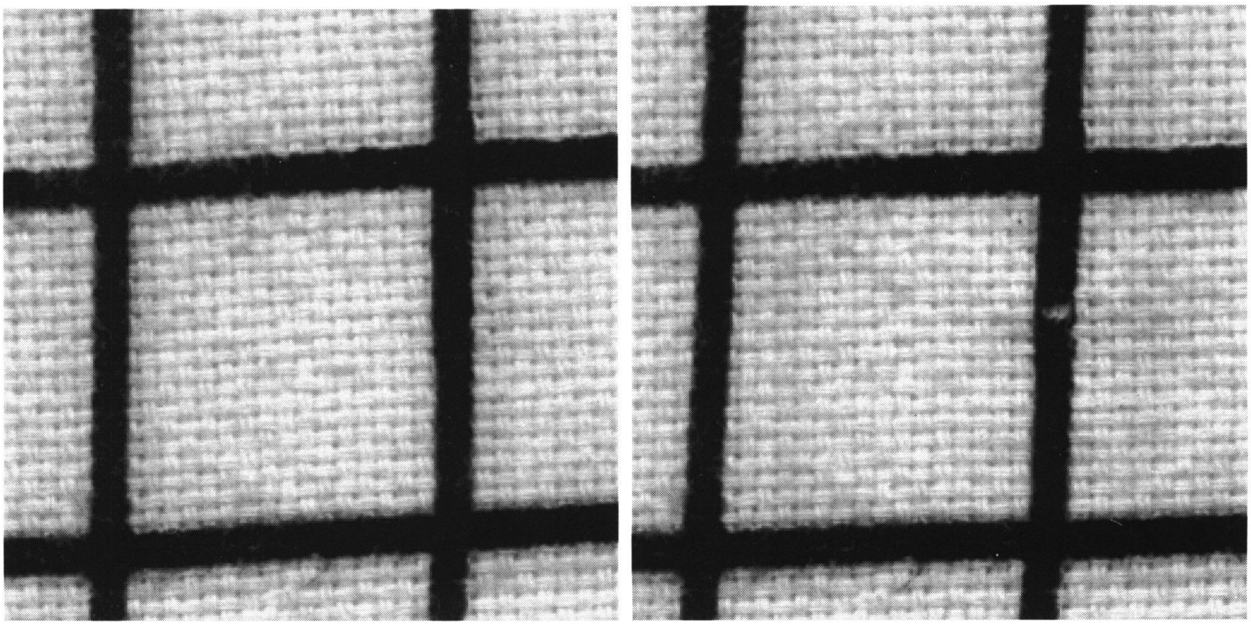
(a) (b)

Figure 6. - Example of satin imperfection: (a) without defects DF = 2,473 and (b) with hole DF = 2,465



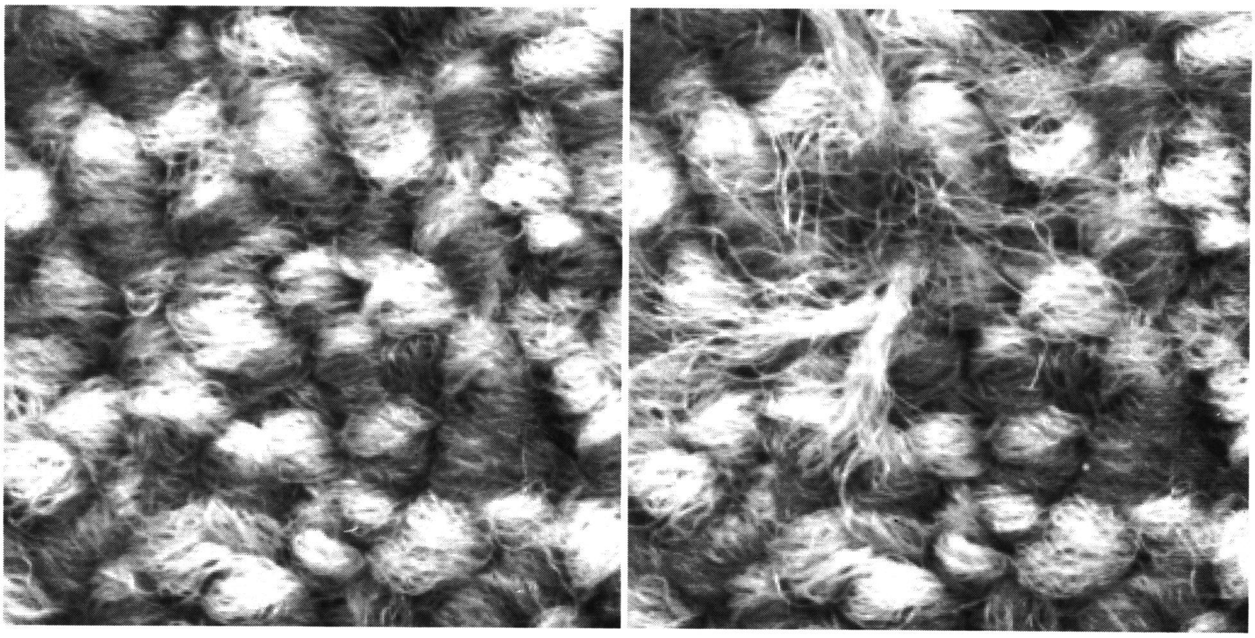
(a) (b)

Figure 7. - Example of silk imperfection: (a) without defects DF = 2,552 and (b) with four thread pull off DF = 2,548



(a) (b)

Figure 8. - Example of cotton imperfection: (a) without defects DF = 2,526 and (b) with imperfection DF = 2,513



(a) (b)

Figure 9. - Example of carpet imperfection (a) without defects DF = 2,578 and (b) with imperfection DF = 2,592.

6 References

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