Adaptive Alternating Sequential Filters

ULISSES DE MENDONÇA BRAGA NETO

Laboratório de Reconhecimento de Padrões
Departamento de Eletrônica e Sistemas
Universidade Federal de Pernambuco
C.P. 7800 50711-970 Recife, PE
e-mail: ulisses@npd.ufpe.br

Abstract. Morphological filtering is a very important branch of Mathematical Morphology, where theoretical advances are made and new useful practical applications are discovered at a fast pace. On the other hand, adaptive-neighborhood techniques have been utilized with success for some time in the image processing field. In this work we propose the extension of a well-known class of morphological filters, the Alternating Sequential Filters (ASFs), to include the paradigm of adaptive-neighborhood image processing, leading to what we have called the Adaptive Alternating Sequential Filters (AASFs). By using synthetic and real images to which Gaussian noise was added, we demonstrate the better performance of the open-close and close-open AASFs against the correspondent ASFs, both from a quantitative and qualitative point of view.

1 Introduction

Morphological filtering is a traditional and important branch of Mathematical Morphology [8, 11]. Its importance derives not only from the many useful practical applications it finds in image processing problems, but also from the theoretical richness of the subject.

One of the best-known classes of morphological filters are the Alternating Sequential Filters (ASFs) [10, 12]. ASFs are based on compositions of increasingly more severe openings and closings, and have been utilized for a long time by practitioners of Mathematical Morphology as an effective tool for the removal of noise in corrupted images. However, though the structuring elements used in the classical ASFs can be selected heuristically to match the global characteristics of the image, it is assumed the latter is spatial stationary, which is not always true. Image processing techniques which have this feature are often called fixed or non-adaptive.

In this work, we propose the construction of morphological filters which are based on the concept of adaptive-neighborhood image processing (ANIP). This is a well-known paradigm in the general image processing field [6], but we suspect that its aplication to morphological image processing has been overlooked so far. In this paper we show how to apply the ANIP principle in the definition of Adaptive Alternating Sequential Filters (AASFs). We define the AASFs rigorously as the composition of the basic (adaptive) operations of erosion and dilation, and demonstrate that they constitute valid morphological filters. We also offer some comments on the implementation of AASFs.

Finally, we utilize synthetic and real images to which Gaussian noise was added in order to show the application of AASFs to noise-removal. By using test images, we demonstrate the better performance of the open-close and close-open AASFs against the correspondent ASFs, both from a quantitative and qualitative point of view.

2 Morphological Filters

In this section we will develop briefly the theoretical aspects of Morphological Filters which will be needed to the definition of AASFs in section 4. For the proofs of all the propositions in this and the next sections, please refer to [4].

To formalize the concept of gray-level morphological filters, it is necessary first to define gray-level images in the context of *lattice theory* [3], the mathematical underpinning of morphological operations.

Definition 2.1 *We define a gray-level image as an element of the set K^E of mappings $f : E \rightarrow K$, where $E \subset \mathbf{R}^2$ is the usually rectangular domain of definition of images and $K = [0, k]$ is a closed interval of $\mathbf{Z}$. The set K^E is provided with the partial ordering relation $\leq$, defined in terms of the usual ordering relation $\leq$ of integers:*

$$f \leq g \Leftrightarrow f(x) \leq g(x), \quad \forall x \in E \qquad (1)$$

for $f, g \in K^E$.

The partially-ordered set $(K^E, \leq)$ is a *complete lattice*, that is, there is a least and a greatest element, which are respectively the constant functions $f \equiv 0$ and $f \equiv k$.

Now we can define the concept of *morphological filter* [11], one of the most important themes in Mathematical Morphology, both from the theoretical and practical point of view.

Definition 2.2 *A morphological filter is any operator ψ defined in K^E which satisfies the following requirements:*

1. *ψ is increasing: $f \leq g \Rightarrow \psi(f) \leq \psi(g)$*
2. *ψ is idempotent: $\psi[\psi(f)] = \psi(f)$*

for all $f, g \in K^E$.

The first requirement makes very clear the disticion between morphological filters and classical linear filters. The latter preserve *addition*, which is very appropriate when dealing, for instance, with acoustic or electrical signals. The visual world, however, is not translucid, but rather composed of opaque objects which hide one another. This situation is best handled by morphological filters, which are increasing, that is, preserve *inclusion*, rather than addition. On the other hand, increasing operators generally produce a loss of information, that is, as opposed to the linear case, the original image cannot be recovered after the filter is applied. The second requirement controls this loss of information, by demanding that the simplification action of a morphological filter stops at the first iteraction. Morphological filters will be henceforth referred to as simply *filters*.

The first step towards defining the filters we are interested in is the notion of *structuring functions*, which in turn allow us to define the specific kind of *erosion* and *dilation* operators we utilize in this work.

Definition 2.3 *Let $\mathcal{P}(E)$ denote the set of all subsets of E. A structuring function is an element of the set Δ of mappings $E \longrightarrow \mathcal{P}(E)$. In other words, a structuring function assigns to each point $x \in E$ a subset of E, which we call a structuring region.*

We will restrict our attention in this work to symmetrical structuring functions:

Definition 2.4 *A symmetrical structuring function Γ is an element of the set $\Delta^* \subset \Delta$, such that for all $x, y \in E$,*

$$y \in \Gamma(x) \Leftrightarrow x \in \Gamma(y) \tag{2}$$

Now we utilize symmetrical structuring functions to define the basic operators of erosion and dilation.

Definition 2.5 *The erosion and dilation of an image $f \in K^E$ by a symmetrical structuring function $\Gamma \in \Delta^*$ are given respectively by:*

$$\epsilon_\Gamma(f)(x) = \min\{f(y),\ \forall y \in \Gamma(x)\} \tag{3}$$
$$\delta_\Gamma(f)(x) = \max\{f(y),\ \forall y \in \Gamma(x)\} \tag{4}$$

for each $x \in E$.

Next we define an important kind of *duality*.

Definition 2.6 *We say that two operators ψ and φ are morphological duals if:*

- *ψ is an increasing operator*
- *φ is an increasing operator*
- *The composition $\varphi\psi$ is anti-extensive, that is, $\varphi\psi \leq \imath$, whereas the composition $\psi\varphi$ is extensive, that is, $\imath \leq \psi\varphi$, where $\imath$ denotes the identity operator.*

The pair $< \psi, \varphi >$ is also called an *adjunction*, or still they are said to form a *Galois connection* between $(K^E, \leq)$ and its dual lattice $(K^E, \geq)$.

Proposition 2.1 *For a given symmetrical structurig function Γ, the erosion ϵ_Γ and dilation δ_Γ are morphological duals.*

The symmetry of the structuring function Γ is essential in establishing the morphological duality of the erosion and dilation operators, for they guarantee the anti-extensiveness and extensiveness of the compositions $\delta_\Gamma\epsilon_\Gamma$ and $\epsilon_\Gamma\delta_\Gamma$, respectively. Actually, these compositions are so important in Mathematical Morphology that they receive special names:

Definition 2.7 *Let $\Gamma \in \Delta^*$. A morphological opening is the operator defined by*

$$\gamma_\Gamma = \delta_\Gamma\epsilon_\Gamma \tag{5}$$

whereas the morphological closing is given by:

$$\phi_\Gamma = \epsilon_\Gamma\delta_\Gamma \tag{6}$$

For the sake of brevity, we shall refer to morphological openings and closings as simply openings and closings.

The most important property of openings and closings is stated in the following proposition.

Proposition 2.2 *Openings γ_Γ and closings ϕ_Γ are filters.*

The following proposition limits (and guides) the way in which we can combine openings and closings by composition in order to generate other useful filters (see also [9])

Proposition 2.3 *Given a structuring function* $\Gamma \in \Delta^*$, $\phi_\Gamma\gamma_\Gamma$, $\gamma_\Gamma\phi_\Gamma$, $\gamma_\Gamma\phi_\Gamma\gamma_\Gamma$ *and* $\phi_\Gamma\gamma_\Gamma\phi_\Gamma$ *are filters. Further, these are the only four distinct ways of combining openings and closings by composition.*

Hereafter, the operators $\phi_\Gamma\gamma_\Gamma$, $\gamma_\Gamma\phi_\Gamma$, $\gamma_\Gamma\phi_\Gamma\gamma_\Gamma$ and $\phi_\Gamma\gamma_\Gamma\phi_\Gamma$ will be called respectively the *open-close*, *close-open*, *open-close-open* and *close-open-close* filters.

3 Alternating Sequential Filters

Alternating sequential filters (ASF) have been utilized for a long time by practitioners of Mathematical Morphology as an effective tool for the removal of noise in corrupted images [10, 12]. ASFs constitute a class of filters which are based on compositions of increasingly more severe openings and closings. In what follows, we shall define ASFs in the context of the filters we have developed in section 2.

Definition 3.1 *Let* $\Omega \subset \Delta^*$ *be any finite family of symmetrical structuring functions* $\{\Gamma_i\}$, *for* $i = 0, 1, 2, \ldots, N$, *such that*

$$\Gamma_i \leq \Gamma_j, \quad \forall i, j \text{ with } 0 \leq i \leq j \leq N \tag{7}$$

in the sense that $\Gamma_i(x) \subset \Gamma_j(x), \forall x \in E$. *The following operators are the Alternating Sequential Filters (ASFs):*

$$N_\Omega = n_{\Gamma_N} n_{\Gamma_{N-1}} \cdots n_{\Gamma_0} \tag{8}$$
$$M_\Omega = m_{\Gamma_N} m_{\Gamma_{N-1}} \cdots m_{\Gamma_0} \tag{9}$$
$$S_\Omega = s_{\Gamma_N} s_{\Gamma_{N-1}} \cdots s_{\Gamma_0} \tag{10}$$
$$R_\Omega = r_{\Gamma_N} r_{\Gamma_{N-1}} \cdots r_{\Gamma_0} \tag{11}$$

where

$$n_{\Gamma_i} = \phi_{\Gamma_i}\gamma_{\Gamma_i}$$
$$m_{\Gamma_i} = \gamma_{\Gamma_i}\phi_{\Gamma_i}$$
$$s_{\Gamma_i} = \gamma_{\Gamma_i}\phi_{\Gamma_i}\gamma_{\Gamma_i}$$
$$r_{\Gamma_i} = \phi_{\Gamma_i}\gamma_{\Gamma_i}\phi_{\Gamma_i}$$

The operators N_Ω, M_Ω, S_Ω *and* R_Ω *are called respectively the open-close, close-open, open-close-open and close-open-close ASFs.*

Theorem 3.1 *The ASFs defined in (8)-(11) are filters.*

For the proof of the above theorem, see [10, pages 205–206].

The working principle of ASFs may be described by the following argument. The definition of the family of structuring functions Ω implies that ASFs are compositions of openings and closings of increasing "size", that is, increasing severity. The "largest" openings and closings are the most effective in suppressing noise, but on the other hand they introduce the greatest distortion in the image features. The application of the "smaller" openings and closings before the "larger" ones has the property of minimizing the distortion caused by the latter.

In practice, the most usual and almost only considered case of ASFs is the one in which each structuring function $\Gamma_i \in \Omega$ is constant over the image, that is, for all $x \in E$:

$$\Gamma_i(x) = B_i + x \tag{12}$$

where "+" denotes the translation operation and $B_i \in \mathcal{P}(E)$ (for $i = 0, 1, \ldots, N$) are *symmetrical structuring elements* (see defs. 4.5 and 4.6 below), with

$$B_i \subset B_j, \quad \forall i, j \text{ with } 0 \leq i \leq j \leq N \tag{13}$$

Usually, the family Ω is generated from a single small convex symmetrical structuring element $B \in \mathcal{P}(E)$ by letting $B_0 = B$ and B_i given by i *Minkowski additions* (denoted by the symbol "$\oplus$") of B by itself:

$$B_i = \underbrace{B \oplus \ldots \oplus B}_{i \text{ times}} \tag{14}$$

Therefore, the structuring regions (see def. 2.3) correspondent to the usual ASFs are "fixed" all over the image. Although the primitive structuring element B in (14) can be selected heuristically to match the global characteristics of the image [12], usual ASFs assume a spatial stationarity of the image which is not always true. Image processing techniques which have this feature are often called fixed or non-adaptive.

4 Adaptive Alternating Sequential Filters

In this work, we propose a new approach to ASFs, based on the concept of *adaptive-neigborhood* image processing (ANIP). This is a well-known paradigm in the general image processing field [6], but we suspect that its aplication to morphological image processing has been overlooked so far. According to the ANIP principle, image operations should not assume spatial stationarity, but rather be fitted to the local çontextual details of the image.

Applied to morphological operators like the ones we defined in section 2, the ANIP principle means that the structuring regions should be defined adaptively, taking into account the local features of the image. The adaptive structuring regions should not transcend natural edges of the image, so that these edges are not degraded by the filter, and should be overlapping, so that artificial edges are not introduced. Applying the ANIP principle to the construction of ASFs yields the operators that we have called Adaptive Alternating Sequential Filters (AASFs).

The approach we have adopted in this particular paper is to build adaptive structuring functions basically by a *region growing* process [5]. For a given point of the image, the corresponding adaptive structuring region is grown by aggregating adjacent points to the given point according to a similarity criterion.

In order to make these notions formal, we must define the concept of connected regions, which in turn needs the concept of neighborhood. As is pointed out in [1], the easiest way to do this is to assume that there is a bijection between a retangular subset of $\mathbf{Z}^2$ and the domain of definition of images E, so that we can can associate a pair of *coordinates* $(a,b) \in \mathbf{Z}^2$ to each point $x \in E$.

Definition 4.1 *Let $x = (a,b) \in E$. The sets $N_4(x)$ and $N_8(x)$ defined as:*

$$N_4(x) = \{(a,b-1),(a+1,b),(a,b+1),(a-1,b)\} \cap E$$

$$N_8(x) = \{(a,b-1),(a+1,b-1),(a+1,b),(a+1,b+1),(a,b+1),(a-1,b+1),(a-1,b),(a-1,b-1)\} \cap E$$

are called respectively the 4- and 8-neighborhood associated to x.

For convenience of notation, we will denote an either 4- or 8-neighborhood by simply $N(x)$.

Definition 4.2 *A path $P_L(x_0, x_l)$ of length L ($L \in \mathbf{Z}^+$) is a sequence of $L+1$ points $\{x_0, x_1, \ldots, x_L\} \in E$ such that $x_i \in N(x_i - 1)$, for $i = 1, \ldots, L$*

Of course, there can be either 4- or 8-paths, depending on the kind of neighborhood $N(x)$ considered.

We are now able to define the following important concept of connectedness for gray-level images:

Definition 4.3 *Let $f \in K^E$ be a gray-level image and $x, y \in E$. We say the points x and y are connected of order m ($m \in \mathbf{Z}^+$), which we denote by $x \overset{m}{\longleftrightarrow}_f y$, if and only if there is a path $P_L(x_0, x_L)$ with $x_0 = x$ and $x_L = y$, such that*

$$|f(x_i) - f(x_0)| \leq m, \text{ for } i = 1, \ldots, L$$

where the $|\cdot|$ operator denotes the usual absolute function for integers. If two points x and y are connected of order m, we also say simply that they are connected, and write $x \longleftrightarrow_f y$.

Two points x, y can be therefore either 4- or 8-connected, depending on the neighborhood considered. We remark also that if f is a binary image, $f \in [0,1]^E$, then connectedness of order 0 reduces to the ordinary binary connectedness case.

Note that the gray-level connectedness relational operator $\longleftrightarrow_f$ *is not* a class of equivalence. It is obviously reflexive, that is, $x \longleftrightarrow_f x$, and symmetrical, $x \longleftrightarrow_f y \Leftrightarrow y \longleftrightarrow_f x$, but it is not in general transitive, $x \longleftrightarrow_f y,\ \ y \longleftrightarrow_f z \not\Rightarrow x \longleftrightarrow_f z$.

Based on the concept of connectedness, we define the connected regions associated to a given image:

Definition 4.4 *Letting $f \in K^E$, to each $x \in E$ it is associated a connected region $R^f_m(x)$ of order m, the set of all points $y \in E$ for which $x \overset{m}{\longleftrightarrow}_f y$.*

Again, we can have either 4- or 8-connected regions, depending on the neighborhood considered. We note also that the connected regions of order 0 in a binary image f correspond to the usual connected components of f. Moreover, it is obvious that for all $f \in K^E$, $x \in E$,

$$R^f_m(x) \subset R^f_n(x), \text{ for } n > m \qquad (15)$$

Note that if $y \in R^f_m(x)$ and $f(y) = f(x)$, then obviously $R^f_m(y) = R^f_m(x)$. This means a significant storage space savings in the implementation, for it is not necessary to allocate distinct lists to hold the regions for all the pixels in the image.

As a last step before the definition of the AASFs proposed in this work, we must define the concepts of translation and symmetrical structuring element.

Definition 4.5 *The translation of a set $B \in \mathcal{P}(E)$ by a point $x = (c,d) \in E$ is an operation between $\mathcal{P}(E) \times E$ and $\mathcal{P}(E)$ defined in terms of the usual sum opereration "+" between integers:*

$$B + x = \{(a+c, b+d),\ \forall (a,b) \in B\} \cap E$$

By abuse of notation, we also denote by "+" the translation operator, since it can be viewed as an extended sum operator. The neutral element of this extended sum is an arbitrary point $o = (0,0) \in E$ called the *origin*. Further, the complementary operation $B - x$ can be defined as

$$B - x = B + (-x) = \{(a-c, b-d),\ \forall (a,b) \in B\} \cap E$$

Definition 4.6 *A symmetrical structuring element is a set $B \in \mathcal{P}(E)$ for which*

$$\{a\} - b \subset B \Leftrightarrow \{b\} - a \subset B\,, \quad \forall a, b \in E \qquad (16)$$

We are now equipped to give the following definition:

Definition 4.7 *Let $f \in K^E$ be a gray-level image, $B \in \mathcal{P}(E)$ be a symmetrical structuring element and $I = \{t_1, t_2, \ldots, t_n\}$ be a sequence of integers such that $t_i > t_{i-1}$, for $i = 1, 2, \ldots, N$. We define the (adaptive) structuring functions $\Gamma_i \in \Omega^* \subset \Delta$ as:*

$$\begin{aligned} \Gamma_0(x) &= B + x & (17) \\ \Gamma_i(x) &= R^f_{t_i}(x) \cup (B + x) & (18) \end{aligned}$$

for $i = 1, \ldots, N$. *The following operators are the Adaptive Alternating Sequential Filters (AASFs):*

$$\begin{aligned} AN_{\Omega^*} &= n_{\Gamma_N} n_{\Gamma_{N-1}} \cdots n_{\Gamma_0} & (19) \\ AM_{\Omega^*} &= m_{\Gamma_N} m_{\Gamma_{N-1}} \cdots m_{\Gamma_0} & (20) \\ AS_{\Omega^*} &= s_{\Gamma_N} s_{\Gamma_{N-1}} \cdots s_{\Gamma_0} & (21) \\ AR_{\Omega^*} &= r_{\Gamma_N} r_{\Gamma_{N-1}} \cdots r_{\Gamma_0} & (22) \end{aligned}$$

where the operators n_{Γ_i}, m_{Γ_i}, s_{Γ_i} *and* r_{Γ_i} *are defined in the same way as in def. 3.1. The operators* AN_{Ω^*}, AM_{Ω^*}, AS_{Ω^*} *and* AR_{Ω^*} *are called respectively the open-close, close-open, open-close-open and close-open-close AASFs.*

Theorem 4.1 *The AASFs defined in (19)-(22) are filters.*

The proof of the above theorem is based on showing that the structuring functions Γ_i match definition 3.1, that is, Γ_i is symmetrical and $\Omega^* \subset \Delta^*$. and

$$\Gamma_i \leq \Gamma_j, \quad \forall i, j \text{ with } 0 \leq i \leq j \leq N$$

and then applying theorem 3.1 (See [4] for more details).

The set B in the definition of the structuring functions Γ_i is selected as a small symmetrical structuring element, which function is to avoid the problem of connected regions consisting of a single point (by the symmetry property of $\leftrightarrow_f$, those are undesirable isolated non-overlapping regions)

We also point out that the lackness of transitivity of the connectedness operator, far from being a drawback, is very important to make sure that the structuring regions will overlap and also to reduce the possibility that they may extend over natural edges of the image.

5 Experimental Results

We have based our implementation of the ASFs and AASFs on the Khoros image processing system, running in a Sun SPARC10 workstation under Unix and X11R5. Khoros is a very popular open platform developed at New Mexico University and freely available through anonymous ftp.

For implementing the ASFs, we have used the opening and closing operators of MMach, a Khoros toolbox for Mathematical Morphology [2]. MMach is also freely available through anonymous ftp at São Paulo University. For the AASFs, however, we had to write a specific program, which was integrated to our local Khoros system, in order to implement the operators of erosion and dilation by adaptive structuring functions.

We have used two test images. The first one is an artificial image consisting of particles of various shapes and sizes on a dark background (fig. 1-a), which we call the "blobs" image, while the second is a real image, a photograph of the surface of the moon (fig. 2-a), the "moon" image.

We evaluated the performance of the AASFs againt the usual ASFs qualitatively and also quantitatively, by utilizing two similarity criterions: the RMS and the (difference) entropy. These functionals are applied on the absolute difference between the filtered image and the original image, and are given respectively by:

$$\begin{aligned} \text{RMS} &= \sqrt{\frac{\sum_{i=0}^{N} f(x_i)^2}{N}} & (23) \\ \text{H} &= -\sum_{i=0}^{k} d_h(i) \log_2 d_h(i) & (24) \end{aligned}$$

where N is the total number of pixels in the image, and d_h is the normalized histogram [5] of the absolute difference image. We remark also that we define $0 log_2 0$ as zero, in case the histogram is zero for one or more points in the interval $[0, k]$.

The RMS applied to the absolute difference of the images gives a value proportional to the pixel-by-pixel euclidean distance between them, so it is zero if and only if the images are equal, and it gets larger as the difference between the images increases. Since the ASFs and AASFs are iterated approximative filters, sometimes they converge to values slightly different than in the original image, specially in the case of uniform regions, although the overall aspect of the image is usually improved. As the RMS penalizes these variations very much, we use also the entropy, a well-known concept of information theory [7] which, in spite of not being a metric like the RMS, gives a very good measure of the similarity between images when applied to the normalized histogram of the absolute difference. It is easy to see from (24) that $H = 0$ if and only if the absolute difference is a constant image, that is a simple global shift of the image, which is immaterial for visual inspection purposes. Likewise, the entropy will be small when the difference between the images consists simply of a few uniform regions, but it will increase as the difference show more and more variation. From (24), we see that the entropy takes its largest value for

$$d_h(i) = \frac{1}{k+1}, \text{ for } i = 0, 1, \ldots, k$$

that is, all gray levels are present in the same proportion (when the image "disorder" is greatest), which corresponds to a value $H_{\max} = \log_2(k+1)$, that

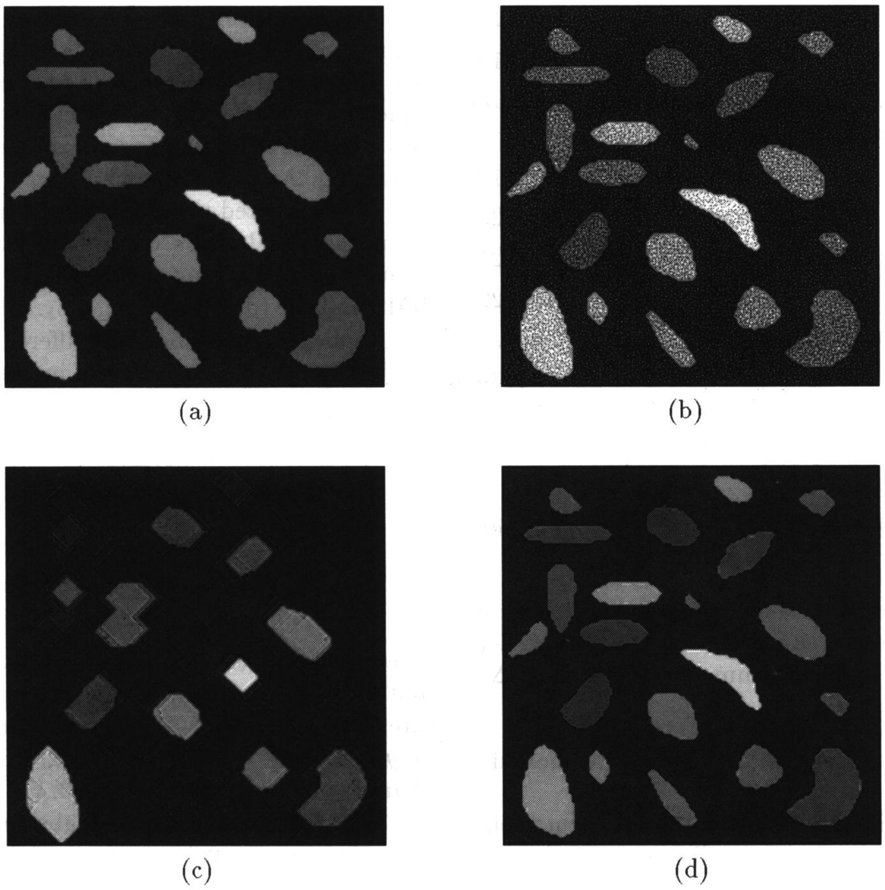

Figure 1: Evaluation of the performance of ASFs and AASFs for the blobs image: (a) original image, (b) noisy image, (c) ASF open-close filtered image (IT=9) and (d) AASF open-close filtered image (IT=9)

is, the number of bits of the image (for instance, for $k = 255$, $H_{\max} = 8$).

For carrying out the experiment of noise-removal, we first added zero-mean Gaussian noise to each of the images (figs. 1-b and 2-b) and then compared the performance of the ASF and AASF open-close filters (eqs. 8 and 19) for the blobs image, and the ASF and AASF close-open filters (eqs. 9 and 20) for the moon image. The results obtained by the computation of the RMS and entropy for these filters are presented in tables 1 and 2. As both images are of type byte, the entropy is in the range $[0, 8]$. We have set the number of iterations as $N = 10$ for the blobs image and $N = 6$ for the moon image. In the case of the blobs image we show the partial results for some values of IT, due to limitation of space. We have utilized as primitive structuring element for the ASFs (see eqs. 12 and 13) the small set $N_4(o) \cup o$, which is the same set used for the AASFs (def. 4.7). Hence, the first iteration ($IT = 0$) gives obviously the same result for both ASFs and AASFs.

The set of values I (def. 4.7) used for computing the connected regions should start with a small value, and the separation between the values should not be large, for then the principle of minimizing the distortion of the "larger" filters by application of the "smaller" ones first would be destroyed. In our case, we have adopted the simple set $I = \{1, 2, 3, \ldots, N\}$.

In the case of the blobs image, we note that for the AASF the entropy was always decreasing for larger IT (for IT=9 it is very small), while for the ASF it decreased at the beginning, and then increased. As to the RMS, the AASF managed to stay reasonably close to the original noisy image, but the RMS for the ASF rises very fast from the fourth iteration on, indicating that the ASF filtered image becomes very distant from the original image. The best results for both criterions are achieved by the AASF, for IT=1 (RMS) and IT=9 (entropy). In figs. 1-c and 1-d we see respectively the ASF and AASF filtered blobs images for IT=9. Note how the AASF, unlike the ASF, preserved very well the edges of the constant regions,

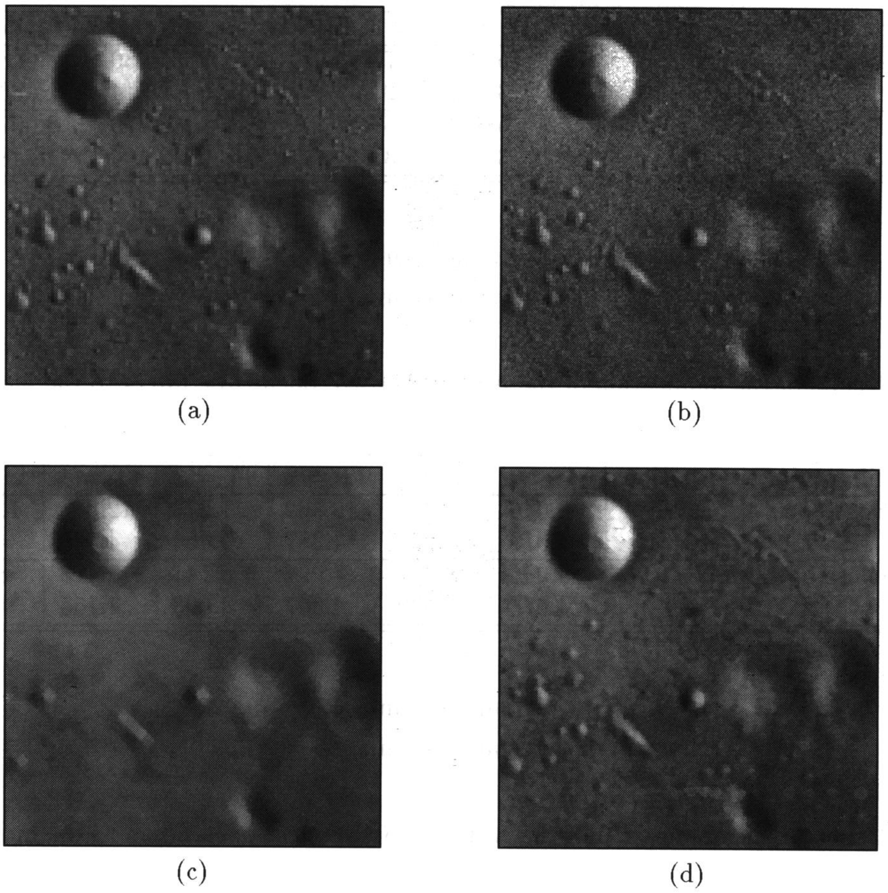

Figure 2: Evaluation of the performance of ASFs and AASFs for the moon image: (a) original image, (b) noisy image, (c) ASF close-open filtered image (IT=2) and (d) AASF close-open filtered image (IT=2)

yielding an image very close to the original, at least by visual inspection (that is why the entropy is so low, though the RMS is greater than the original).

For the moon image, we see that the RMS and the entropy for the AASF were always below the original values, with both criteria achieving a minimum for IT=2. For the ASF, however, the values of RMS and entropy exceeded the original for IT$\geq$2 and IT$\geq$ 3, respectively. The best global result is again achieved by the AASF, in the case IT=2. In figs. 2-c and 2-d we see respectively the ASF and AASF filtered moon images for IT=2. See how the fine details (the little craters and hills) were almost completely removed by the ASF (the degradation for larger values of IT is even worse), while they were much better preserved by the AASF.

6 Conclusion

We have shown that the application of the adaptive-neighborhood image processing paradigm to Mathematical Morphology can give good results. We have developed a rigorous definition of AASFs, the Adaptive Alternating Sequential Filters, and have demonstrated their superior performance against the usual ASFs, for the synthetic and real images utilized. We have evaluated the two techniques not only qualitatively, but have employed also numerical measures of the similarity between the filtered images and the original ones. We think that the idea of building morphological operators which locally adapt to the features of an image is very promising, and there is certainly much work to be done on this subject.

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Filter	Criterion	IT = 0	IT = 1	IT = 2	IT = 3	IT = 5	IT = 7	IT = 9
ASF open-close	RMS	5.372	5.394	7.863	13.079	24.080	35.668	45.648
	Entropy	3.452	3.102	2.869	2.785	2.814	2.983	3.038
AASF open-close	RMS	5.372	5.333	6.557	8.350	9.870	8.097	14.859
	Entropy	3.452	3.432	3.240	3.465	2.164	1.748	1.276

$$\begin{aligned} \text{RMS of original noisy image} &= 7.942 \\ \text{Entropy of original noisy image} &= 4.081 \end{aligned}$$

Table 1: Results of the application of the ASF and AASF open-close filters to the blobs image

Filter	Criterion	IT = 0	IT = 1	IT = 2	IT = 3	IT = 4	IT = 5
ASF close-open	RMS	8.046	9.340	10.462	11.273	12.080	12.545
	Entropy	4.078	4.256	4.387	4.464	4.536	4.590
AASF close-open	RMS	8.046	8.014	7.991	8.141	8.475	8.939
	Entropy	4.078	4.070	4.067	4.093	4.155	4.235

$$\begin{aligned} \text{RMS of original noisy image} &= 9.927 \\ \text{Entropy of original noisy image} &= 4.398 \end{aligned}$$

Table 2: Results of the application of the ASF and AASF close-open filters to the moon image

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