

The Development of a Tool for the Air Data System Calibration Flight Tests Campaign using Image Processing

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Abstract— The first Flight Test Campaign (FTC) carried out for an experimental aircraft is the calibration of its Air Data System (ADS). In this case, the altitude and airspeed measurements of the aircraft are provided by both anemometric along with the Flight Tests Instrumentation (FTI) systems that are corrupted by installation errors. Therefore, a tool was developed using techniques such as image processing in order to minimize these errors. A tool is described for detecting the position of the aircraft during a FTC through a video of the high-speed camera. The tool detects the aircraft and reference points, calculates altitude and airspeed of the aircraft and determines validity of test point in FTC. The preliminary tests of this tool showed satisfactory results compared to the current method. This paper will discuss such scenario, its preliminary development and the results through performed flights with EMBRAER Jet XAT-26 aircraft and HELIBRAS HB-350L-1 helicopter.

Keyword - Image Processing, Computer Vision, Computer Application, Aircraft.

I. INTRODUCTION

Images frames have been used as information source to clearly pinpoint the aircraft behavior at the Flight Tests Campaigns (FTC) [1]. Although data extraction from motion picture has been a relevant issue, such technology was the only information source that matched the desired requirements (i.e. Resolution, Sensitivity and Speed). Therefore several systems were integrated and used. This includes the Optical Tracking Systems and the Hi-speed film camera that was mainly used to develop, certify and integrate weapon systems. Nowadays, with the development of the electronics, a video camera can produce hi-resolution (e.g. 1024i or 720i Format), hi-speed (e.g. up to 1,000 fps) images with a very good sensitivity, so the film camera became obsolete. Today the state-of-the-art technology are the digital video cameras, but the required bandwidth to transmit and processing power to extract real-time quantitative information from such source still requires further research.

The most usual process used to the execution of the Air Data System (ADS) Calibration FTC is the tower fly-by method [2] that requires the knowledge of the exact aircraft reference altitude. This method consists of comparing the altitude indicated by the aircraft pitot-static system with the actual pressure altitude to determine the static pressure error.

The determination of this pressure difference (generally known as static-system position error) throughout the aircraft flight envelope can be complicated, time consuming, and expensive. Moreover, the use of other positioning sensors such as Global Navigation Satellite System (GNSS) based systems are not adequate because they cannot reach the desired sample rate (i.e. 400 frames/s). In addition, data synchronization and correlation with GNSS is not so simple [3]. Most GNSS receivers are not synchronized with Global Positioning System (GPS) time, so they correct their time base only when the deviation reaches ± 1 ms [3]. For some Ashtech receivers (e.g. Z-FX), this error could occur once every 10 min [3]. On the other hand, inertial sensors with low bias are too expensive for this application.

An alternative solution is to obtain knowledge of the exact aircraft reference altitude from image frames. Currently, an off-line video processing application that computes the aircraft altitude from a snap-shot picture taken from a hi-resolution, hi-speed video camera is used for this purpose. The main disadvantage of this solution is the fact that measurement accuracy is jeopardized. The natural solution to improve accuracy and mechanize processing is to bring this application to the Near Real-Time environment. In order to do so, an application to process 720i video frames at up to 400 fps was developed, which is integrated into the Flight Test Dataset acquired at the Ground Telemetry System (GTS) for the execution of the ADS Calibration FTC [1]. Besides, these features (i.e. 400 fps and 720i) improve operational safety for the certified aircraft and ameliorate the flight safety this FTC because some tests points are executed with the aircraft flying at 470 kts and extremely low altitude. This is very dangerous and very close to a disaster. Thus the use of real time systems becomes essential.

The preliminary tests of this application showed satisfactory results compared to the GTS.

The remainder of the paper is organized as follows. In section 2, the concepts and the scenario of the FTC are presented. In section 3, the tool developed is presented. Experimental results are given in section 4. Finally, this paper is concluded.

II. ADS OVERVIEW

The aircraft speed and altitude are the primordial information as far as flight safety is concerned. The aircraft

altitude and speed are derived from the aircraft static and dynamic pressures provided from the pitot-static anemometric system. Under ideal conditions, the True Airspeed (V_t) and altitude (Z_p) [4] are computed with the knowledge of the free stream impact (p_i) and static (p_a) pressures as well the air compressibility correction data (ΔV_c) and the density ratio (σ).

But calibration lag [5] and installation errors corrupts p_t and p_a , so, the aircraft Air Data System (ADS) receives at its inputs the corrupted pitot total (p_p) and basic static (p_b) pressures instead of p_t and p_a .

The determination of the relation between p_p and p_t , as well p_b and p_a , requires the execution of the ADS Calibration FTC, which is normally the first evaluation for a new experimental aircraft.

To compute the pitot-static system error model, the aircraft true-reference altitude and airspeed should be compared to the equivalent measurement provided by their anemometric [6] and Flight Test Instrumentation (FTI) systems that are corrupted by installation errors.

The determination of the pressure error model using the tower fly-by method compares the aircraft reference pressures with their current values [2]. The reference pressures are computed from the tarmac reference pressure, provided by a ground based wind reference and the aircraft altitude that can be measured by different sensors, including: an optical tracking system and a differential GPS. To improve the model accuracy, the pressure difference (i.e. static-system position error) should be determined over the entire flight envelope. This process could be complex, time consuming, and expensive.

Various methods are available for obtaining in-flight calibration models. The execution of such FTC is performed with fully instrumented aircrafts, where all required parameters are acquired, stored and transmitted over the Telemetry Link. During the test flights, real-time FTI data is received and processed at the GTS to be merged with complementary measurements that arise from other sensors (i.e. Ground based wind reference system).

Normally, the GTS is a monitoring station related to flight safety issues. But signal noise and dropouts, which are inherent characteristics of the Telemetry link, limit the GTS reliability, which represents a major technical problem faced at all test ranges. Moreover, post mission data reductions require long processing time. Consequently, the efficiency of the FTI is not optimum in most cases.

The typical behavior of an installed sensor on an instrumented aircraft with FTI data received and processed at the GTS can be seen in Figure 1. The parameter displayed is the aircraft altitude (in feet) XAT-26, which is parked, but with engine running. In this case, the signal received by the telemetry presents a variation of about 14 feet caused by installation errors from FTI.

To improve the GTS efficiency, a novel concept was developed, in which quasi-real time data processing tools execute data processing while the aircraft is still flying [1, 7], so the final tests results and reports are generated at the end of the last valid test point.

Among developed tools there is the ADS Calibration application that uses still picture frames to compute the aircraft altitude. The next challenge now is to process video frames to improve data accuracy.

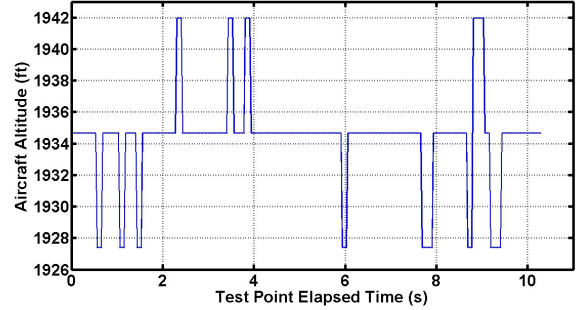


Figure 1. Aircraft Altitude measured at Telemetry

A. ADS Calibration

The ADS calibration flight consists of several test points ranging from 1.2 times the aircraft stall speed (i.e. V_s) up to its maximum horizontal speed (i.e. V_h).

At each test point, according with the test order [8], the aircraft altitude (Z_{pb}) and basic speed (V_b) should be stabilized as follows:

$$\bar{V}b_i \leq Vt_i \pm 5kts \quad (1)$$

$$\Delta Zpb_i \leq \pm 20 ft \quad (2)$$

$$\Delta Vb_i \leq \pm 2kts \quad (3)$$

Where:

- ΔVb_i is the maximum deviation of the aircraft basic speed at the i^{th} test point (kts);
- $\bar{V}b_i$ is the mean basic speed at the i^{th} test point (kts);
- Vt_i is the scheduled basic speed for the i^{th} test point (kts);
- ΔZpb_i is the maximum deviation of the aircraft altitude at the i^{th} test point (ft);

The determination of the exact aircraft altitude from a single image picture requires a flight path over a known fixed vertical plane, inside the camera field-of-view and within the reference points. For each test point the Test Pilot should maintain the aircraft trajectory aligned with the tarmac centreline.

As an example, a successful test point execution of an ADS calibration FTC with HELIBRAS H-55 (Esquilo) Helicopter (Figure 2) was obtained while the aircraft flies inside the Valid Area (ARV) indicated by the red rectangle.

The reference points can be seen in the lower corners of ARV. These points are static metallic plates of $1m^2$ known fixed positions.

The ADS calibration flight consists of several Test Points (TP), carried out with evenly spaced speeds from the minimal (i.e. 1.2 times the Stall Speed) up to the maximum allowable speed with stabilized speed, altitude and attitude.



Figure 2. Valid Test Point with H-55 Helicopter

For each TP the aircraft flies over the airport tarmac centerline (Figure 4) and inside ARV. The camera is located in a fixed known place. The reference points are located close to the tarmac in an also known location to define a fixed baseline, in this application with $112.6\text{m} \pm 0.4\text{m} @ 1\sigma$, from which all measurements will be performed.

The TP validation is obtained with the conformance of all acquired data with the applicable requirements. The occurrence of any invalid TP requires its repetition. Therefore the use of real-time tools allows the acquisition of all necessary data while the aircraft is flying and prevents re-flying, which is very expensive and should be avoided.

The development of this tool required customized solutions to overcome the following main challenges:

- Although the reference points and center point are well defined (Figure 3a), the location of this point in the resulting image (Figure 3b) could be inaccurate due to image resolution. In this example the reference point captured by the current camera is just a 6x6 pixels square.
- The meteorological conditions could change very rapidly. Therefore the image background from which the target is recognized could change significantly (Figure 5).
- Determine a fixed reference point on the aircraft for measuring altitude and airspeed.

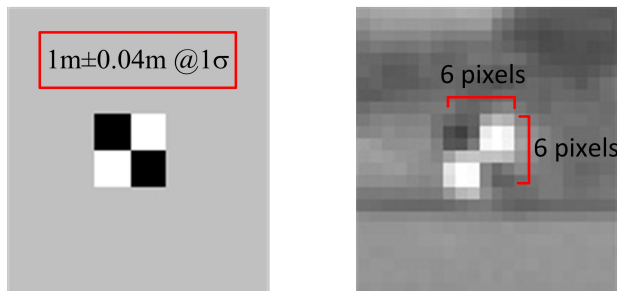


Figure 3. Reference Point (a) Original (b) Camera Image

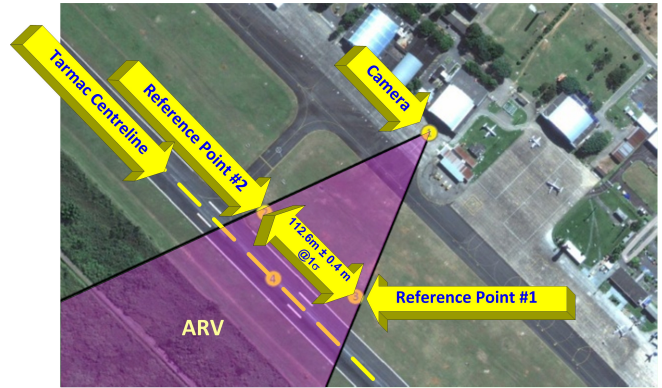


Figure 4. Test Site



Figure 5. Valid TP with diverse contrast and aircraft (XAT-26)

III. TOOL DEVELOPMENT

For this development all required data is provided by a fixed high speed camera that generates target image frames in grayscale.

In our test site there is a camera calibration board where camera and lens distortion errors are measured. With such information it is possible to compute the model to minimize such errors using [9]. The error compensation algorithm is included into the application. The camera optical centre axis is determined during calibration procedures. Then, during the flight, the camera position is determined, as well the position of two Reference Points (i.e. centre of the reference plate), located close and in parallel to the tarmac centreline. The measurement of these three points with the distance between the reference plates and the tarmac centreline allows the determination of local image reference plane coordinates where the aircraft flies.

This tool is designed to be used with experimental flight test in a restricted airspace. In this case, due to flight safety issues, the test flight will always be executed with a direct line of sight to the test bed. Therefore rain or cloud fading is

out of the operational scenario. Also due to the Test Site singularities, it will not be necessary to detect obstacles in the flight path, as in traditional optical tracking systems.

For the operational side, when starting the tool, the user should input:

- The size of the wingspan of the aircraft to be used to draw a straight line to indicate the minimum safe altitude for this FTC; and
- The frequency for data acquisition of camera image frames used to compute the true-reference test bed speed.

At each test point all data are recorded into the camera's buffer. At the end of the test point, the recorded video is downloaded to the computer through its network interface. Therefore the gathered information would be processed by the developed tool. After this, the camera's buffer should be cleared for the next test point.

The application architecture consists of (Figure 6):

- Video data acquisition over the TCP-IP Gigabit Interface;
- Target coordinate extraction from each video image frames;
- Target coordinate correction to minimize the lens distortion errors;
- TSPI computation algorithm; and
- Results displaying, storage and distribution over the TCP-IP LAN.

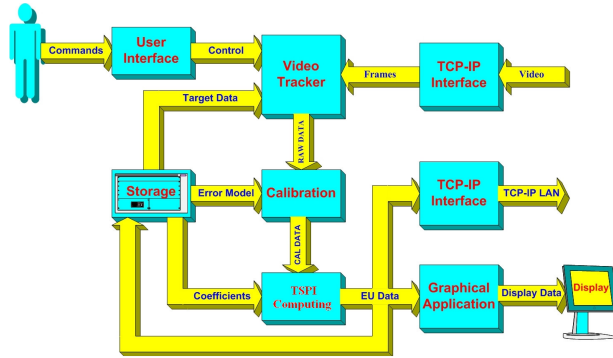


Figure 6. System Architecture, adopted from [1]

Data processing algorithm is executed into five steps and uses the following well known techniques:

- Reference point detection;
- Tarmac centreline detection;
- Aircraft detection;
- TSPI Computing; and
- Test Point Validation

A. Reference Point Detection

Initially an area is selected by the user around the reference point. Then the original image is cropped into the selected area to allow image sharpening [10], contrast adjustment, using linear mapping and binarization.

After that, the search for the central point location into the image will be performed. This location will be used as primary reference for data processing.

Once found, the central reference point will be highlighted in the original image; otherwise, a new area should be selected by the user.

B. Tarmac Centreline Detection

For the tarmac centerline detection, the airport runway location should be detected, so as to provide the reference baseline to compute the aircraft altitude. For this particular application, this baseline is located 4 pixels below the baseline defined by the central points of the two reference points.

C. Aircraft Detection

A segmentation process is used to detect the aircraft [11], in which an image frame is divided into mutually exclusive subsets, called regions or segments. Each region is homogeneous and uniform according to some properties such as texture or tone, whose values differ in some aspects and meanings of the properties of each neighboring region.

Sequential Image Frames ($I(t)$) acquired by the camera are processed in grayscale (Figure 7). Considering that camera is fixed throughout the whole sequence, the background remains static with minimal illumination changes.

Therefore the first TP image frame is used as a reference Background Image (IB) for the entire TP (Figure 8).



Figure 7. TP Image



Figure 8. Example of Background

The target probable location was defined by using classical methods [12, 13, 14, 15], and the best results were obtained with the Sobel Method [15], which computes the residue image (Irt) as [16] and:

$$Irt_{x_i y_j} = It_{x_i y_j} - IB_{x_i y_j} \quad (4)$$

The next step is to detect the edges of the resulting frame that corresponds to regions where there is a change of intensity in certain spatial area and direction.

This corresponds to regions of higher spatial derivatives of the $f(x,y)$ function that expresses the image pixel intensity. Since the light intensity of the aircraft is lower than the background, the residue image (Figure 9) is computed as:

$$Irt_{x,y} = \begin{cases} 255, & \text{if } (IB_{x_i y_j} - It_{x_i y_j}) > \mu \\ 0, & \text{if } (IB_{x_i y_j} - It_{x_i y_j}) \leq \mu \end{cases} \quad (5)$$

Where:

- $x_i y_j$ is the coordinate of the image;
- μ is the detection threshold.

After this, Connected Component Labeling (CCL) [17] is performed to detect the test bed. Initially, pepper noises (i.e. unwanted targets) should be removed by using morphological operators [18] (first erosion, then dilation) on the residue image. Then the CCL could be used. After that, the resulting components should be sorted in ascending size order.



Figure 9. Residue image with Sobel method

In normal operation, the largest components contain the searched target. The application typically finds only one component, which is large enough to describe the sought target (Figure 10). However, interfering objects (e.g. passing truck) could produce more than one component, which fits the target volume. In this case the frame is discarded.

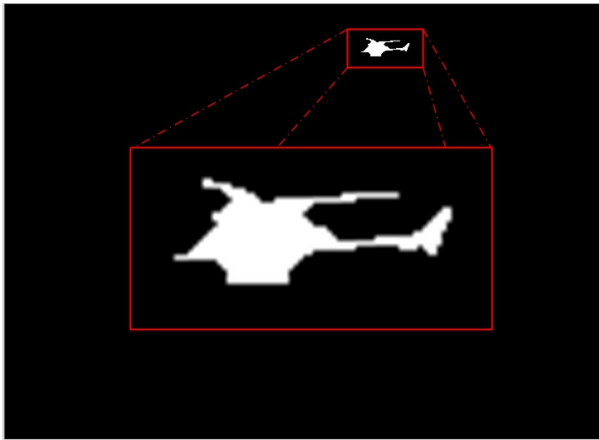


Figure 10. CCL Image

After this, only the perimeter pixels of aircraft inside the component are used (Figure 11). For the selected component perimeter (Ca) pixels, it is denoted:

$$Ca = V_{x_k y_l} \quad (6)$$

Where:

- V is the detected Component;
- x_k is x-axis pixel perimeter location; and
- y_l is y-axis pixel perimeter location.

Finally target position is considered to be the mean value of Ca pixels locations, estimated as follows:

$$\begin{cases} x_T = \frac{1}{N} \sum_{k=1}^N x_k \\ y_T = \frac{1}{N} \sum_{l=1}^N y_l \end{cases} \quad (7)$$

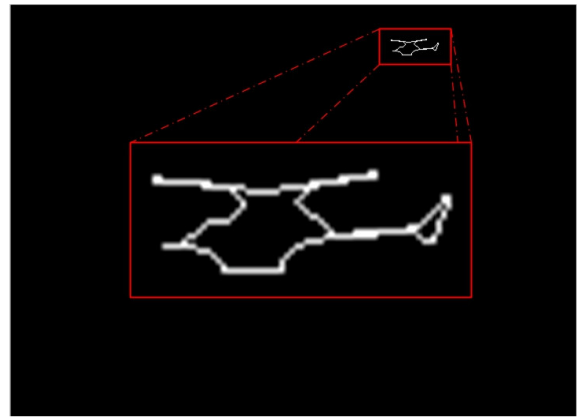


Figure 11. Perimeter Pixels of Aircraft

D. TSPI Computing

Now the requirement is to find a fixed reference point on the aircraft for measuring altitude and airspeed (i.e. TSPI, Time Speed and Positioning Information). Several tests were performed with the following techniques:

- Centroid; and
- Front, Rear and Bottom Edge detection;

The Front Edge method did not perform well with rotor wings (i.e. H-55 Helicopter), because of the constant area change in the frames caused by the motion of the main blades. The Centroid also suffers from this motion effect; however, it yielded the best results to compute altitude for rotor-wings.

The performance of bottom edge detection was not satisfactory either, due to the pepper noise. In this case, the count for image drops was very high.

Finally, the detection technique that performed better to compute airspeed for rotor wings was the rear edge detection. Moreover, the rear edge detection yielded good results for fixed-wings aircraft (i.e. XAT-26) and is used to compute altitude and airspeed.

Then, once found the target reference point, its altitude related to the Tarmac Centerline should be computed. In order to do so, boundary conditions are to consider that the aircraft path is aligned with the tarmac centerline, and the

camera lens distortion error has been previously modeled and minimized.

So, with the knowledge of the exact position of two fixed visual reference points, it is possible to compute the altitude of a third reference point, which is located on the aircraft, (e.g. The FTI Air Data Boom installed ahead the Test Bed) using the video tracking algorithm.

The True Airspeed (V_t) is computed by:

$$V_t = \frac{k\Delta S}{\Delta t} \text{ (m/s)} \quad (8)$$

Where:

- k is the image calibration factor, which in this case is 16.6×10^{-2} (m/pixel);
- ΔS is the pixel displacement between two consecutive frames (pixels); and
- Δt is the frame rate (frames/s).

The image calibration factor is computed from the real distance between references points (i.e. $112.6\text{m} \pm 0.4\text{m} @ 1\sigma$) divided by the number of pixels between them.

Optionally, this information could be determined from the real size of the reference plate ($1\text{m} \pm 0.04\text{m} @ 1\sigma$) divided by its pixels count.

E. Test Point Validation

Target altitude and True Airspeed are computed at every valid frame and stored.

Therefore, at the end each TP, the application checks the conformance of these parameters with the specific requirements (see Section II.A) to validate or reject the current TP.

Thus, it is possible to validate the TP in Near Real-Time increasing the efficiency of FTC.

Notes:

- Since the camera is in a fixed position, Steps III.A and III.B are executed automatically on the first video frame for each TP;
- The remaining steps (i.e. III.C to III.E) are executed for sub sequential video frames of current TP.

IV. TOOL EVALUATION

This tool was evaluated with a fully instrumented EMBRAER Jet XAT-26 and a HELIBRAS H-55 Helicopter during the CEV (Brazilian Flight Test School) ADS calibration FTC, carried out by 2010 and 2011 class students.

Several TP were executed in these FTCs, more than 300. The application performance evaluation employed video sequences from 1 to 5 seconds, totaling 430.918 different frames. All TPs were successfully executed in Real Time. So it is possible to include the gain (time and accuracy) provided by the application that improves the Flight Test Campaign efficiency.

The main application was developed under MatLab® environment and tested with Intel® Pentium IV Core™ 2 Duo CPU T5800 2.00 GHz notebook with Ethernet Gigabit Local

Area Network (LAN) interface, 4 Gb RAM and Microsoft Windows 7 Professional.

The camera used in this evaluation is a high-speed MacroVis V1.7.35 Gigabit Ethernet. Several tests were carried out with different camera configuration. As a result, it was noticed that higher resolution produces clearer aircraft silhouettes (i.e. better defined). In addition, a higher frame rate reduces motion blur effect that jeopardizes target detection and measurement accuracy.

The best optimal configuration for this application was obtained when the camera was configured to generate images in grayscale at 400 fps and 720i resolution.

The prototype application runs at $52 \text{ fps} \pm 1.6 \text{ fps} @ 1\sigma$ on average. Therefore the test results are produced in Near Real-Time. Considering the ADS application this performance was considered satisfactory.

The observation of a single frame of a TP (Figure 12), it can be viewed:

- The location of the aircraft centroid (blue cross);
- The rear edge reference point location (red cross);
- The minimal safety altitude (red line);
- The aircraft computed true airspeed (V) and its altitude Zpb (H);
- The ground reference points (green cross); and
- The tarmac centerline (white dotted line).



Figure 12. TP Snapshot and annotation text in image.

After tuning the tool performance was considered satisfactory and all TP produced adequate results. An example for a single test point it is presented the computed altitude (Figure 13) and associated uncertainty (Figure 14), which is $\pm 0.1331\text{m} @ 1\sigma$.

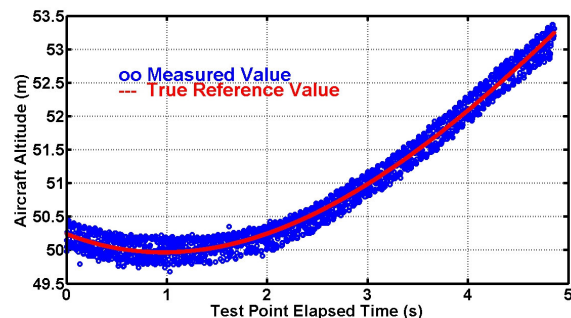


Figure 13. True Reference and Computed Altitude

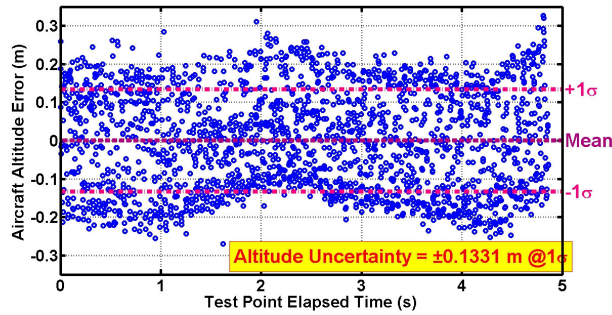


Figure 14. Altitude Uncertainty

Figure 15 shows the altitude computed by the application compared to the altitude received by the telemetry. The telemetry frame rate is 32 samples per second and the uncertainty of telemetry to altitude is $\pm 6.11\text{ft} @ 1\sigma$.

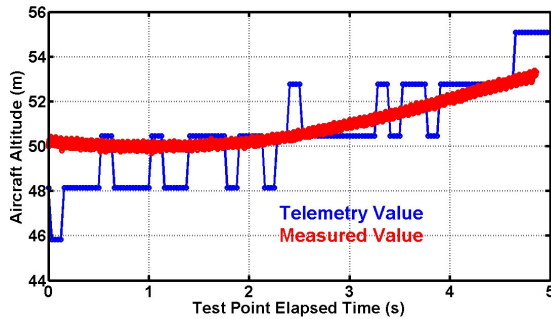


Figure 15. Telemetry and Computed Altitude

Due to the pixel resolution (i.e. $16 \times 10^{-02}\text{m}$) and the camera frame rate (i.e. 400 fps) the displacement of a single pixel/frame is equivalent to 230.4 km/h speed step.

Therefore considering that the helicopter is moving with lower speed (i.e. less than 80 km/h) this application fails to compute its exact speed. Figure 16 shows the speed computed by the Telemetry compared with the speed computed by the application. Even with the variation in speed measured by the application, note that the average speed can be considered satisfactory.

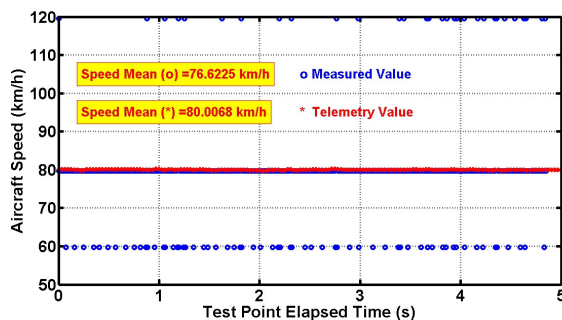


Figure 16. Telemetry and Computed Speed

It should be noticed that a single setup change (e.g. image resolution, image calibration factor, camera frame rate and target speed range) enhances the computed speed resolution and accuracy. For the used setup, it was verified

that the test bed position remained at the same pixel for several sequential samples.

V. CONCLUSIONS

The development and evaluation of a Tool for the Air Data System Calibration Flight Tests Campaign using Image Processing was successfully executed. This application integrates several simple yet efficient vision tools, which are easy to implement. The main challenge here is to compute in Real Time the aircraft altitude and airspeed using 400 fps High resolution images (i.e. 720i Format) without using special hardware to process the information. Therefore the processing application should be very efficient and specially tailored to satisfy this requirement. This tradeoff impairs the use of more complex image processing techniques.

The application can be customized for several aircrafts. As a result the system is very flexible and reliable and it can be used in wide range applications. The tests of this application showed satisfactory results compared to the Ground Telemetry System (GTS).

The next steps are:

- Evaluate the tool with other aircrafts;
- Improve system performance using:
 - Parallel processing techniques; and
 - Graphics Processor Unit (GPU) cards;
- Retrieve images directly from the buffer of high-speed camera; and
- Develop a tool to integrate this application with GPS and GTS.

REFERENCES

- [1] N. P. O. Leite, L. M. F. Lopes; W. O. Lima, and L. Roberto, "The Development of a Quasi-Real Time Data Processing Tool for the Calibration Flight Test Campaign of an Air Data System". In: European Test & Telemetry Conference, 2009, Toulouse. European Test and Telemetry Conference Proceedings. Paris : The Association Aéronautique and Astronautique de France, p. 130-135, 2009.
- [2] J. Roskam; "Airplane Flight Dynamics and Automatic Flight Controls, Part 1", Roskam Aviation Engineering, 1979.
- [3] N. P. O. Leite; F. Walter; "Multiple Time Base Synchronization Process Applied to the Flight Tests Campaign of a GPS Attitude Determination Algorithm", Proceedings of the International Telemetry Conference, ITC 2007, Las Vegas, 2007, pp. 1048 ~ 1057.
- [4] A. L. C. Forni, "Manual de Aerodinâmica, Documento nº 20-R-AH", Chapter 3, Divisão de Ensaios em Voo, 1995.
- [5] A. F. B. Edwards, "Flight Test Engineering Handbook – Air Force Technical Report nº 6273", Chapter 1, Edwards Air Force Base, 1966.
- [6] R. M. Arantes, "Introdução a Aerodinâmica Técnicas de Ensaios em Voo - Calibração, Documento nº E-B11", Chapter 2.3, Grupo Especial de Ensaios em Voo, 2003.
- [7] N. P. O. Leite, L. M. F. Lopes, and F. Walter, "The Integration of a GPS Receiver to a Quasi-Real Time Decision Aid Tool Used for the Air Data System Calibration Flight Tests Campaign". In: International Telemetry Conference, 2009, Las Vegas. ITC/USA 2009 Conference Proceedings. Las Vegas : The International Foundation for Telemetry, 2009
- [8] F. H. L. Pinto and F. W. Rodrigues, "Ordem de Ensaio - Calibração Anemométrica Document nº B11(A)/EFEV", Grupo Especial de Ensaios em Voo, 2007.

- [9] K. S. Fu; R. C. Gonzales; C. S. G. Lee. "Robotics: Control, Sensing, Vision, and Intelligence", McGraw-Hill, New York, 1987.
- [10] L. S. G. Kovaszny; H. M. Joseph. "Image Processing". Proc. IRE, vol. 43, pp. 560-570, 1955.
- [11] K. S. Fu, and J. K. Maui. "A Survey of Image Segmentation". Pattern Recog., vol. 13, no. 1, pp. 3-16.
- [12] L. G. Roberts. "Machine Perception of Three-Dimensional Solids". In Optical and Electro-Optical Information Processing, Tippet, J. T., ed., MIT Press, Cambridge, Mass, 1965.
- [13] J. M. S. Prewitt. "Object Enhancement and Extraction", In Picture Processing and Psychopictorics, Lipkin, B. S. and Rosenfeld, A., eds., Academic Press, New York, 1970.
- [14] G. S. Robinson. "Detection and Coding of Edges Using Directional Masks". University of Southern California, Image Processing Institute, Report no. 660, 1976.
- [15] J. R. Fram; and E. S. Deutsch. "On the Quantitative Evaluation of Edge Detection Schemes and Their Comparison with Human Performance". IEEE Trans. Computers, vol. C-24, no. 6, pp. 616-628.
- [16] R. J. Schalkoff. "Digital Image Processing and Computer Vision". John Wiley & Sons, New York, 1989.
- [17] R. C. Gonzalez and R.E. Woods, Digital Image Processing, Prentice-Hall, Inc., 2002.
- [18] J. Serra. "Image Analysis and Mathematical Morphology", vol. 2, Academic Press, New York, 1988.