Improvement and Invariance Analysis of Zernike Moments using as a Region-based Shape Descriptor

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Abstract

An improved Zernike moment using as a region-based shape descriptor is presented. The improved Zernike moment not only has rotation invariance, but also has scale invariance that the unimproved Zernike moment does not have. The experimental results show that the improved Zernike moment has better invariant properties than unimproved Zernike moment using as region-based shape descriptor.

Keywords: *Zernike moment; Invariance; Shape; Region-based; Content-based; Image retrieval; trademark;*

1. Introduction

In order to retrieve an image from a large database the descriptor should have enough discriminating power and immunity to noise. In addition, the descriptor should be invariant to scale and rotation, not to mention the computation efficiency. The Zernike moment descriptor has such desirable properties: rotation invariance, robustness to noise, expression efficiency, fast computation and multi-level representation for describing the various shapes of pattern [1]. But there is a drawback for the Zernike moment descriptor, it does not have scale invariance. At present, with a proper normalization method of images, scale invariance can also be achieved in some degree [2,3], but it is not the real scale invariance. The Zernike moments gained by using this method can not well reflect the feature of original shape, because scaling a shape will unavoidable lead to the loss of some information in the shape, especially when the shape is shrunk. For example, when a circle shape in a digital image is shrunk to some degree, it maybe becomes a square shape.

In this paper, a region-based shape descriptor is presented, which utilizes a set of the magnitudes of Zernike moments. Then we analysis the invariance properties of Zernike moments and present improved Zernike moments. Finally we show that the improved Zernike moment has better invariant properties than unimproved Zernike moment using as region-based shape descriptor, which can be used effectively as a global shape descriptor of an image, especially for a large image database.

2. Definition of Zernike moments

In (ρ,θ) polar coordinates, the Zernike radial polynomials $\left\{R_{nm}(\rho)\right\}$ are defined as [3]

$$R_{nm}(\rho)=\sum_{s=0}^{\frac{n-|m|}{2}}\frac{(-1)^s(n-s)!}{s!(\frac{n+|m|}{2}-s)!(\frac{n-|m|}{2}-s)!}\rho^{n-2s} \qquad (1)$$

where n is a non-negative integer, and m is a non-zero integer subject to the following constrains: $n-|m|$ is even and $|m| \le n$.

The (n,m) order of the Zernike basis function [2], $V_{nm}(\rho,\theta)$, defined over the unit disk is

$$V_{nm}(\rho,\theta) = R_{nm}(\rho)\exp(jm\theta), \quad \rho \le 1 \tag{2}$$

The Zernike moment [4] of an image is then defined as

$$Z_{nm} = \frac{n+1}{\pi} \iint_{\text{unit disk}} V_{nm}^{*}(\rho,\theta) f(\rho,\theta) \tag{3}$$

where V_{nm}^{*} is a complex conjugate of V_{nm}.

3. Analysis of invariance properties of Zernike moments

To calculate the Zernike moments of an image $f(x,y)$, the image (or region of interest) is first mapped to the unit disk using polar coordinates, where the centre of the image is the origin of the unit disk. Those pixels falling outside the unit disk are not used in the calculation. The coordinates are then described by ρ which is the length of the vector from the origin to the coordinate point and θ which is the angle from the x axis to the vector ρ, by convention measured from the positive x axis in a counter clockwise direction. The mapping from Cartesian to polar coordinates is:

$$x = \rho\cos\theta \qquad y = \rho\sin\theta \tag{4}$$

where

$$\rho = \sqrt{x^2 + y^2} \qquad \theta = \tan^{-1}\left(\frac{y}{x}\right) \tag{5}$$

However, $\tan^{-1} A$ in practice is often defined over the interval $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$, so care must be taken as to which quadrant the Cartesian coordinates appear in. Translation and scale invariance can be achieved by normalizing the image using the Cartesian moments prior to calculation of the Zernike moments [2]. Translation invariance is achieved by moving the origin to the centre of the image by using the centralized moments, causing. $m_{01} = m_{10} = 0$. Following this, scale invariance is produced by altering each object so that its area (or pixel count for a binary image) is $m_{00} = \beta$, where β is a predetermined value. Both invariance properties can be achieved using:

$$g(x,y) = f\left(\frac{x}{a} + \bar{x}, \frac{y}{a} + \bar{y}\right) \tag{6}$$

where $a = \sqrt{\frac{\beta}{m_{00}}}$ and $g(x,y)$ is the new translated and scaled function. The error involved in the discrete implementation can be reduced by interpolation. As we have stated that the scale invariance using this method is not the real scale invariance.

Further, the absolute value of a Zernike moment is rotation invariant as reflected in the mapping of the image to the unit disk. The rotation of the shape around the unit disk is expressed as a phase change, if ϕ is the angle of rotation, Z_{nm}^{R} is the Zernike moment of the rotated image and Z_{nm} is the Zernike moment of the original image then:

$$Z_{nm}^{R} = Z_{nm}\exp(-jm\phi) \tag{7}$$

$$|Z_{nm}^{R}| = |Z_{nm}\exp(-jm\phi)| = |Z_{nm}| \tag{8}$$

Fig. 1 shows the process of extracting the Zernike moment descriptor from an image. First, the input image is binarized. Since the Zernike moments are defined over a unit disk, the radius R of a circle is determined to enclose the shape completely from the centroid of the binarized shape in the image to the outer most pixel of the shape. The shape is then re-sampled to normalize to the predetermined size [5]. This normalization step helps to achieve the scale invariance for the descriptor. Thirty-six Zernike moments of order zero to ten in n and m are then extracted from the normalized image, and the magnitudes are used as the descriptor. The total number of moments used in the shape descriptor was determined experimentally.

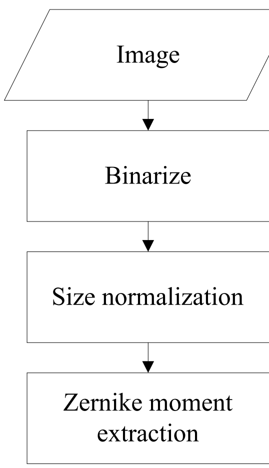

Figure 1: The block diagram of Zernike moment extraction.

4. Improvement of Zernike moment descriptor

(1). Get the order (0,0) geometric moment m_{00} of the image.

$$m_{00} = \iint f(x,y)dxdy \tag{9}$$

from physics viewpoint, m_{00} is the expression of mass or area of an image.

(2). Compute the various order Zernike moments

$$Z'_{nm} = \frac{n+1}{\pi} \iint_{D^2} f(x,y)[V_{nm}(x,y)]^* dxdy \tag{10}$$

where $D^2 : x^2 + y^2 \le 1$

(3). Normalize the Zernike moments

$$Z_{mn} = \frac{Z'_{nm}}{m_{00}} \tag{11}$$

where, Z_{mn} is the improved Zernike moments we presented.

(4). Because Z_{mn} is Complex, we often use the Zernike moments modules $|Z_{mn}|$ as the features of shape in the recognition of pattern. In the following parts of the paper, Zernike moments mean the Zernike moments modules.

Fig. 2 shows the process of extracting the improved Zernike moment descriptor from an image. What in the dashed rectangle is the improved Zernike moment extraction.

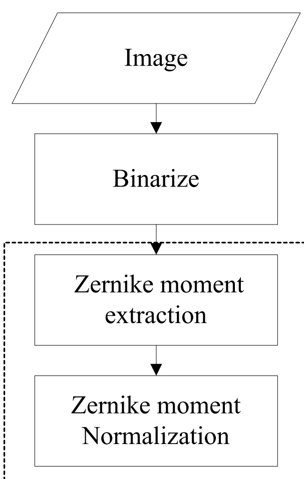

Figure 2: The block diagram of improved Zernike moment extraction

In the improved Zernike moments, there are three moments are constant, $Z_{00} = \frac{1}{\pi}$ and $Z_{11} = Z_{1-1} = 0$.

Before proof above proposition, we first give the definition of radial-polar coordinate system.

Definition 1

Let f be an image and (x_c, y_c) be its centroid. We use

$$\rho = \sqrt{(x-x_c)^2 + (y-y_c)^2} \tag{12}$$

$$\theta = \tan^{-1}((y-y_c)/(x-x_c)) \tag{13}$$

to define our radial-polar coordinate system.

Proof.

We have

$$R_{00}(\rho) = \frac{(-1)^0 (0-0)!}{0!\left(\frac{0+0}{2}-0\right)!\left(\frac{0-0}{2}-0\right)!}\rho^0 = 1 \tag{14}$$

so

$$V_{00}^*(\rho,\theta) = R_{00}(\rho)\exp(-i\cdot 0\cdot\theta) = 1 \tag{15}$$

Hence

$$\begin{aligned} Z_{00} &= \frac{\frac{0+1}{\pi}\iint_{D^2} f(x,y)V_{00}^*(\rho,\theta)dxdy}{m_{00}} \\ &= \frac{\frac{1}{\pi}\iint_{D^2} f(x,y)\cdot 1\cdot dxdy}{m_{00}} \end{aligned} \tag{16}$$

Now by Definition 1, the support of f is enclosed by the unit disk D^2. We may thus remove the domain of integration, yielding

$$\begin{aligned} Z_{00} &= \frac{\frac{1}{\pi}\iint f(x,y)dxdy}{m_{00}} \\ &= \frac{m_{00}}{\pi\cdot m_{00}} \\ &= \frac{1}{\pi} \end{aligned} \tag{17}$$

We also have

$$R_{11}(\rho) = \frac{(-1)^0 (1-0)!}{0!\left(\frac{1+1}{2}-0\right)!\left(\frac{1-1}{2}-0\right)!}\rho^1 = \rho \tag{18}$$

so

$$V_{11}^*(\rho,\theta) = R_{11}(\rho)\exp(-i\theta) = \rho(\cos\theta - i\sin\theta) \tag{19}$$

Taking real and imaginary components of $V_{11}^*(\rho,\theta)$, we have

$$\operatorname{Re} V_{11}^*(\rho,\theta) = \rho\cos\theta \tag{20}$$

$$\operatorname{Im} V_{11}^*(\rho,\theta) = -\rho\sin\theta \tag{21}$$

and using Definition 1, this gives

$$\operatorname{Re} V_{11}^*(\rho,\theta) = x - x_c \tag{22}$$

$$\operatorname{Im} V_{11}^*(\rho,\theta) = -(y - y_c) \tag{23}$$

Hence

$$\begin{aligned} \operatorname{Re} Z_{11} &= \operatorname{Re}\frac{\frac{1+1}{\pi}\iint_{D^2} f(x,y)V_{11}^*(\rho,\theta)dxdy}{m_{00}} \\ &= \frac{\frac{2}{\pi}\iint_{D^2} f(x,y)\operatorname{Re} V_{11}^*(\rho,\theta)dxdy}{m_{00}} \end{aligned} \tag{24}$$

Now by Definition 1, the support of f is bounded by the unit disk D^2. We may thus remove the domain of integration, yielding

$$\begin{aligned} \operatorname{Re} Z_{11} &= \frac{\frac{2}{\pi}\iint f(x,y)(x-x_c)dxdy}{m_{00}} \\ &= \frac{\frac{2}{\pi}\{m_{10}\}}{m_{00}} \\ &= 0 \end{aligned} \tag{25}$$

where m_{10} is order (1,0) geometric moment, we know $m_{10} = m_{01} = 0$ when the origin of the Cartesian coordinates locates the center of a shape.

Similarly $\operatorname{Im} Z_{11} = 0$, so $Z_{11} = 0$, Then $Z_{1-1} = 0$ by $Z_{nm} = \left(Z_{n-m}\right)^{*}$.

5. Experimental Results

To evaluate the performance of the proposed improved Zernike moment descriptor, experiments have been conducted. The experiments mainly address the improved Zernike moments possess the better properties of rotation invariance and scale invariance than unimproved Zernike moments. The test image database consists of about 3,000 device-mark type trademark images (marks that contain graphical or figurative elements only) in MPEG-7 test material ITEM S8. The first 10 trademark images are shown in Figure 3.

3(a) 3(b) 3(c) 3(d) 3(e)

3(f) 3(g) 3(h) 3(i) 3(j)

Figure 3: the first 10 trademark images of 3,000 device-mark type trademark images

The experiments are made according to the following steps:

Step 1. Image Transforming: Transformations have been applied to the test images as described below.

1. Rotation – The image is rotated by the following angles: 5°, 30°, 50°, 90°, 120° and 150°.
2. Scale - The image is scaled by the following factors: 300%, 230%, 120% and 90%.
3. Rotation/scale - The image is printed on a laser printer. Ten prints of each image were digitized using both video and digital cameras, with rotation and scale changes except for perspective effects.

Step 2. Improved Zernike moments Computing: The improved Zernike moments of the images transformed are computed.

Step 3. Analysis of Improved Zernike moments: Plot it to show the improved Zernike moments possesses the properties of rotation invariance and scale invariance.

Figure 4 shows the experiment result of Figure 3(a) for improved Zernike moments and unimproved Zernike moments. Figure 5 shows the experiment result of Figure 3(b) for improved Zernike moments and unimproved Zernike moments.

Because the differences of the improved corresponding Zernike moments of the transformed images are very tiny, it is high difficult to show all of their discrepancy, we only show the improved Zernike moments of original image, the image rotated by 50° angle and the image scaled by 120% factor. Their improved Zernike moments are plotted in the blue dot, green plus sign and red asterisk respectively in Figure 4(a) and 5(a). We also plot the unimproved Zernike moments of corresponding images in Figure 4(b) and 5(b). From above figures, we can see that improved Zernike moments possess better rotation invariance and scale invariance properties than unimproved Zernike moments.

Figure 6 shows the mean error values of improved and unimproved Zernike moments respectively for the first 10 trademark images. From this figure we can see the mean error values of improved Zernike moments are smaller than unimproved.

6. Conclusion

In this paper, we showed that the improved Zernike moment descriptor not only has better rotation invariance, but also has better scale invariance than unimproved Zernike moment descriptor. It was very effective in retrieving an image from a large image database.

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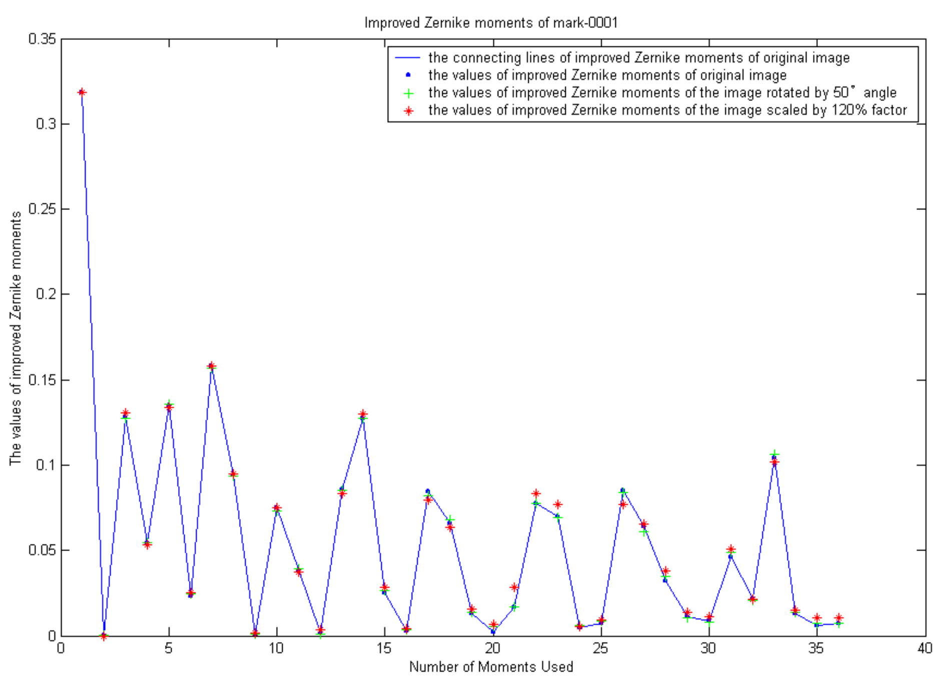
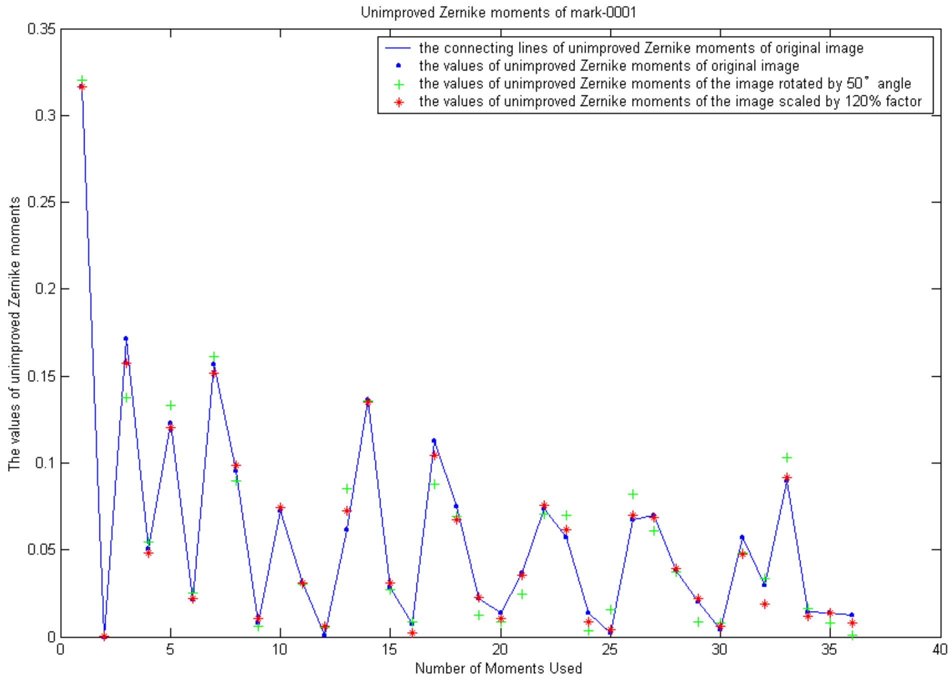
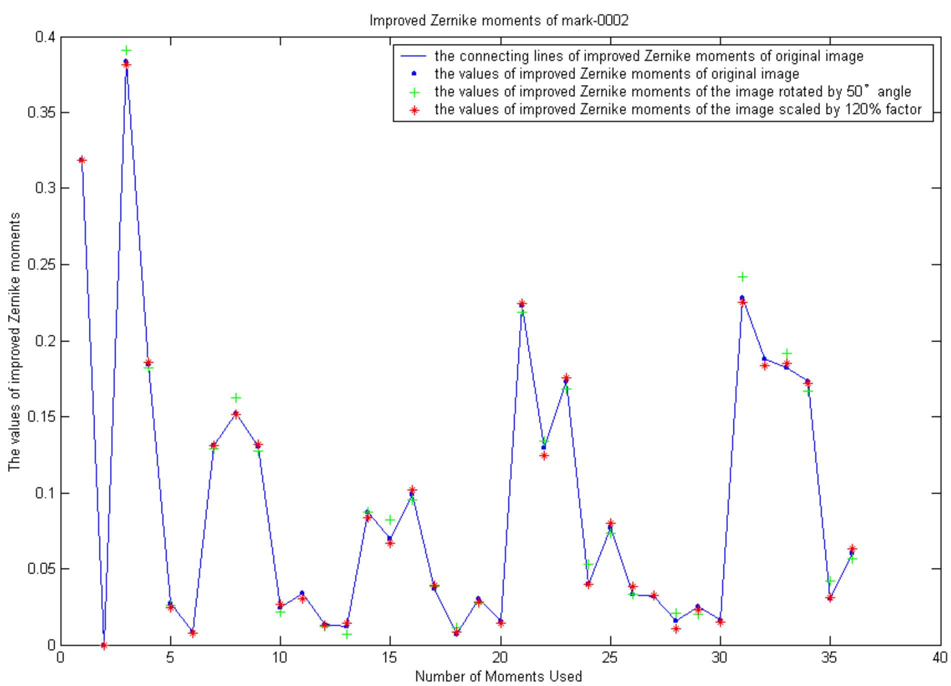


4(a) improved Zernike moments

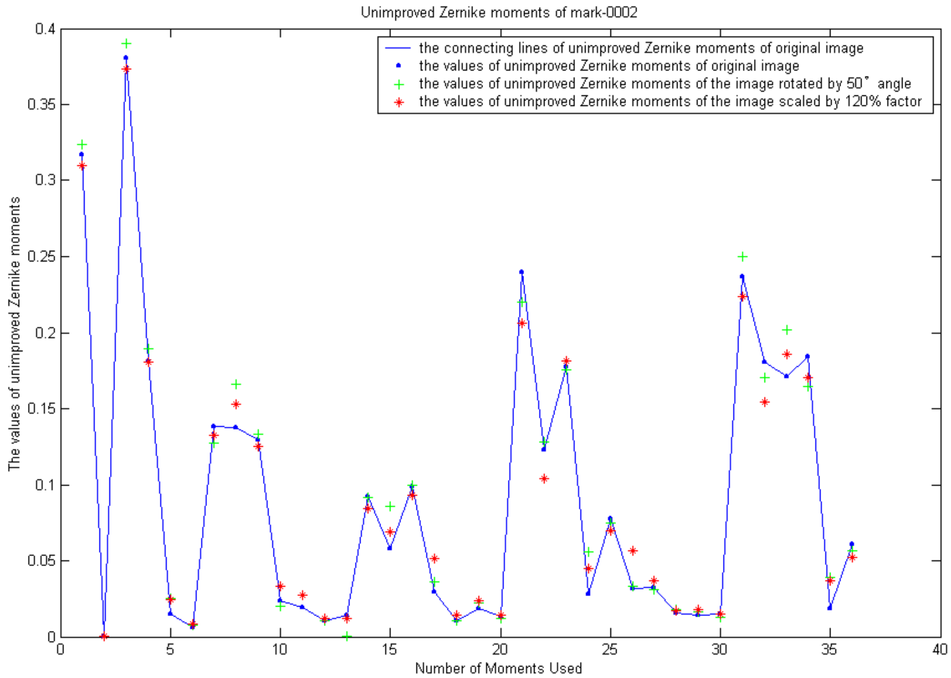


4(b): unimproved Zernike moments

Figure 4: Invariance of Zernike moments of 3(a)



5(a) improved Zernike moments



5(b): unimproved Zernike moments

Figure 5: Invariance of Zernike moments of 3(b)

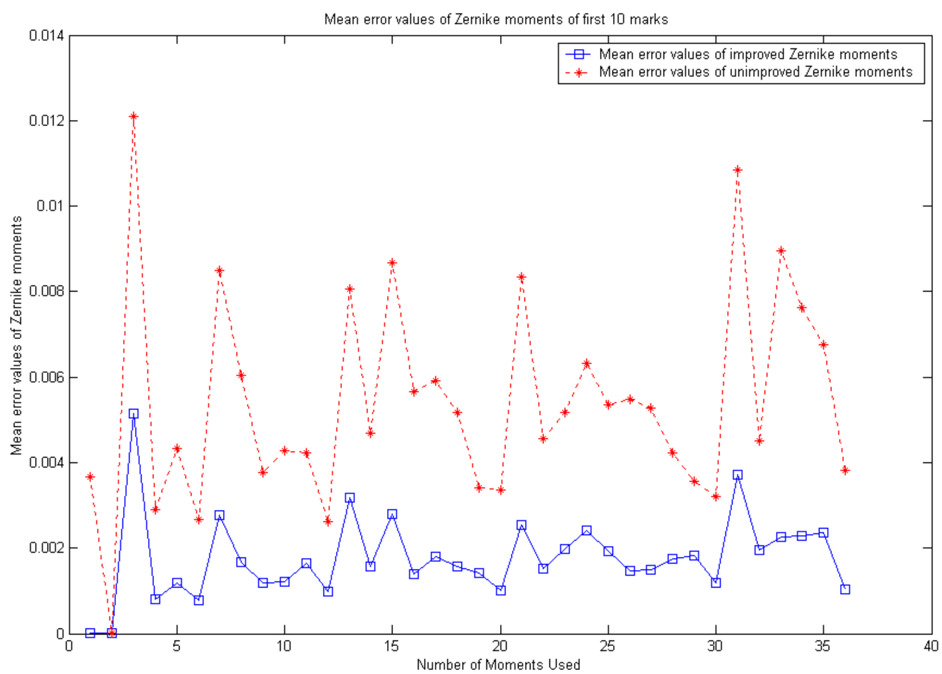


Figure 6: Mean error values of improved and unimproved Zernike moments