Image Data Fusion Using Projection over the Set of Solutions of an Underdetermined Least Squares Problem

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Abstract. This extended abstract presents a new image data fusion method that aims to combine images with different spatial and spectral resolutions. It is based on a projection onto the subspace of the set of solutions of an underdetermined least squares problem. The data fusion problem may also involve an interpolation process, when images with different spatial resolutions are used. In our case, this interpolation was performed by using a Bayesian approach, with or without spatial adaptation . As an example, the method is applied to SPOT images over São José dos Campos, SP, Brazil.

1 Introduction

The image data fusion problem has received much attention recently. This occurs because many images of various types are obtained for the same object. By using data fusion methods, it is possible to obtain products that incorporate the best characteristics of images from different sensors.

Many authors have published good results on image data fusion and three are discussed here, with special interest to this work. Brum [1] used the multispectral and panchromatic bands of SPOT to generate three synthetic bands with spatial resolution of approximately 10m and spectral resolution that is equivalent to the original multispectral bands. This work included a geometric interpolation of the multispectral bands. Mascarenhas et al. [3] proposed a new data fusion method using Bayesian statistical estimation theory, that uses the multispectral and panchromatic bands of the SPOT satellite to generate ideal synthetic multispectral bands, close to 10m spatial resolution. Zaniboni et al. [4] adapted the Bayesian method to work with locally adaptive correlation coefficients to generate synthetic multispectral images from the SPOT satellite.

In this work, an alternative to the Bayesian synthesis is proposed, that uses a projection of the interpolated images onto the subspace generated by the least squares solution of an underdetermined least squares problem that characterizes the synthesis problem.

2 The Bayesian data fusion

The Bayesian image data fusion was proposed by Mascarenhas et al. [2] and comprises two steps: (a) a new interpolator for the multispectral bands, obtained through the orthogonality principle, which will be used as a priori information for the second step; and (b) a new statistical synthesis formulation, using as observations the panchromatic and multispectral bands.

2.1 Spatial interpolation techniques

The non-homogeneous linear estimator proposed by Mascarenhas et al. [2] is in the form :

$$\hat{x} = Ay + b \qquad \text{(I)}$$

Under a Bayesian approach for estimation, the matrix A and the vector b will such that:

$$\hat{x} = E[x] + \sum_{xy} \sum_{yy}^{-1} (y - E[y]) \qquad \text{(II)}$$

where $E[x]$ is the statistical expectation, Σ_{xy} is the cross-covariance matrix of x and y and Σ_{yy} is the auto-covariance matrix of y.

2.2 Image synthesis techniques

The local, linear image synthesis process in [2] is also performed under a Bayesian framework. The *a priori* information is carried out by using the interpolated estimator $\hat{x}$ and its covariance matrix $\sum_{\hat{x}}$, as the *a priori* mean vector and covariance matrix, respectively.

The vector f of synthetic pixels is locally related to the observed vector z by using an observation matrix H, through a linear model, i.e.,

$$z = Hf + r \qquad \text{(III)}$$

where r represents an error vector on the linear model, with an assumed covariance matrix V.

This model (III) was implemented and tested by Brum [1], Mascarenhas et al. [2] and Zaniboni et al. [4].

The components of each row of the matrix H are defined by the fraction of the area under the ideal synthetic spectral sensitivity curves. The spectral relative response curves for each sensor will define the parameters of the matrix H [1,2,4].

Formulating the synthesis problem under a Bayesian framework, a linear minimum mean square error local estimation is performed, leading to the estimated synthetic bands.

$$\hat{f} = E[f] + CH^T(HCH^T + V)^{-1}(z - E[z]) \qquad \text{(IV)}$$

where $E[f] = \hat{x}$ and $C = \sum_{\hat{x}}$.

3 Image Synthesis Using a Least Squares Approach

The synthesis problem may be represented by the following equation, with f, z and H given in section 2.2:

$$z = Hf \qquad \text{(V)}$$

Note that equation (V) defines an underdetermined linear system, and the infinite number of least squares solutions, described by Rust and Burrus [3], lay over a subspace defined by:

$$f = H^+ z + (I - H^+ H)\, v \qquad \text{(VI)}$$

where H^+ is the *generalized inverse* of H, and v is any n-vector.

Let f' be the particular solution of (V) given by the orthogonal projection of the interpolated images $\hat{x}$ in the subspace $M = \{f \mid f = H^+ z + (I - H^+ H)v ,\ v \in R^n \}$.

Note that f' is the solution vector that is nearest of $\hat{x}$ (the interpolated images). See **Figure 1**. The rationale behind this procedure is the fact the synthetic bands should be spectrally similar to the interpolated multispectral bands, as was done in [2], under a different (Bayesian) approach.

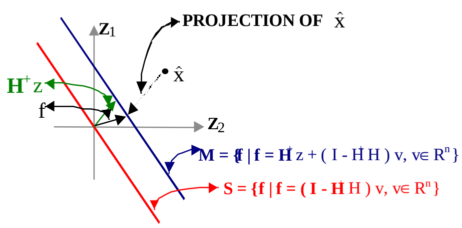

Figure 1 The vector f and subspace M (in R^2)

4 Experimental Results

Figure 2 shows the panchromatic and multispectral (band 2) images over São José dos Campos, SP, Brazil.

Note that both, the region denoted by A in panchromatic image and the region denoted by B in the multispectral image, are present in **Figure 3**, that shows the synthetic image that summarizes the information contained in both images, generated by using the least squares approach proposed in this work.

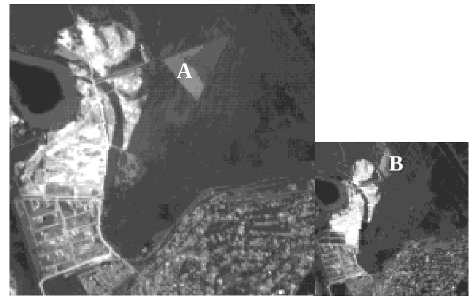

Figure 2 Original Images over SJ dos Campos

Figure 3 Synthetic image over SJ dos Campos

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