Analyzing Polarimetric Imagery with $\mathcal{G}_p^0$ Mixture Models and SEM Algorithm

Michelle M. Horta
Instituto de Física de São Carlos
Universidade de São Paulo
São Carlos, Brazil
michellemh@if.sc.usp.br

Nelson D. A. Mascarenhas
Departamento de Computação
Universidade Federal de São Carlos
São Carlos, Brazil
nelson@dc.ufscar.br

Alejandro C. Frery
Instituto de Computação
Universidade Federal de Alagoas
Maceió, Brazil
acfrery@pesquisador.cnpq.br

Abstract

This paper presents the use of a finite mixture model for multi-look polarimetric SAR *image analysis. The pixels are complex covariance matrices set as a $\mathcal{G}_p^0$ mixture distribution. The parameters are estimated with the* SEM *algorithm Experimental results on real* SAR *data are reported, showing that a careful statistical model is important.*

1. Introduction

In multi-look polarimetric imagery (PolSAR), the statistical modeling process turns out to be able to design effective techniques for processing and analysis. This type of data has been classically described using the complex Wishart and Polarimetric K distributions [1, 2].

The $\mathcal{G}_p^0$ distribution was proposed to model extremely heterogeneous areas, but it can also be used to describe heterogeneous and homogeneous clutter [1]. In this paper, PolSAR data are modeled as a finite mixture of $\mathcal{G}_p^0$ distributions, where each component corresponds to a specific statistical land-cover. The parameters are estimated with the stochastic expectation-maximization (SEM) algorithm [3]. The $\mathcal{G}_p^0$ mixture model was compared with the Wishart mixture model.

2. Model for PolSAR Image Data

In the multi-look PolSAR imagery the observed value at each pixels is formed a complex covariance matrix [1] related to the dielectric properties of the scene. In this paper, each pixel information, denoted Z, obeys a mixture model $\mathcal{M}(\boldsymbol{\theta},\boldsymbol{\rho})$ given by $f(z)=\sum_{i=1}^{g}\rho_i f_i(z,\boldsymbol{\theta}_i)$, where $\sum_{i=1}^{g}\rho_i = 1$ are non-negative proportions and $f_i(z,\boldsymbol{\theta}_i)$ are densities that characterize the $\mathcal{G}_p^0(\alpha_i, C_i, n)$ law, given by [1]:

$$f_i(z,\boldsymbol{\theta}_i) = \frac{n^{mn}|z|^{n-m}\Gamma(mn-\alpha_i)}{h(n,m)|C_i|^n\Gamma(-\alpha_i)(-\alpha_i-1)^{\alpha_i}} \cdot (nTr(C_i^{-1}z)+(-\alpha_i-1))^{\alpha_i-mn}, \tag{1}$$

where $h(n,m)=\pi^{m(m-1)/2}\Gamma(n)\cdots\Gamma(n-m+1)$, m is the number of polarimetric components, n the number of looks, Tr and $|.|$ are the trace and determinant and $\alpha<0$ is the roughness parameter. When $\alpha\to-\infty$, under certain conditions, the pixels obey the complex Wishart law $\mathcal{W}(C_i,n)$, given by [2]

$$f_i(z,\boldsymbol{\theta}_i) = \frac{n^{mn}|z|^{n-m}\exp(-nTr(C_i^{-1}z))}{h(n,m)|C_i|^n}. \tag{2}$$

In [1], the $\mathcal{G}_p^0(\alpha_i, C_i, n)$ parameters were estimated by the moment method. These estimators are integrated in the SEM algorithm.

Given $\widehat{m_i}(\boldsymbol{Z})$ the ith-order sample moment of the N sample $\boldsymbol{Z}=(Z_1,\ldots,Z_N)$ from an area, the covariance matrix is computed by

$$\hat{C}=\widehat{m_1}(\boldsymbol{Z}). \tag{3}$$

The roughness is calculated with the mean of the estimates $\widehat{\alpha}_i$ of each intensity channel $j\in\{hh,hv,vv\}$ by

$$\frac{\Gamma^2(-\hat{\alpha}_i-1/4)\Gamma^2(\hat{n}+1/4)}{\Gamma(-\hat{\alpha}_i-1/2)\Gamma(\hat{n}+1/2)\Gamma(-\hat{\alpha}_i)\Gamma(\hat{n})}-\frac{\widehat{m_{1/4}}^2(\boldsymbol{Z}_j)}{\widehat{m_{1/2}}(\boldsymbol{Z}_j)}=0. \tag{4}$$

	Law	Kappa	Accuracy
Image 1 (558×491)	$\mathcal{G}_p^0(\alpha, C, n = 3)$	0.44	68%
	$\mathcal{W}(C, n = 3)$	0.22	56%
Image 2 (890×491)	$\mathcal{G}_p^0(\alpha, C, n = 3)$	0.70	85%
	$\mathcal{W}(C, n = 3)$	0.52	72%

Table 1. Classification with SEM Algorithm.

3. SEM Algorithm

The SEM is an iterative stochastic algorithm which formalizes the problem of the parameters estimation of a mixture distribution as an incomplete data problem [3]. It avoids the analytical maximization of the expected log-likelihood function and the moment estimators are used instead. Given $\boldsymbol{\theta}_i^k$ the ith component parameters in the kth iteration, where $\boldsymbol{\theta}_i^k = (\alpha_i^k, C_i^k)$ for $\mathcal{G}_p^0$ law or $\boldsymbol{\theta}_i^k = C_i^k$ for Wishart law, the algorithm has three steps and alternates between these three steps until convergence. The image can be grouped using, for instance, a maximum a posterior decision rule.

The *E-Step* updates the posterior probabilities for each pixel and each mixture component by $\tau_{ij}^k = \rho_i^k f_i(z_j, \boldsymbol{\theta}_i^k)/\sum_{\ell=1}^g \rho_\ell^k f_\ell(z_j, \boldsymbol{\theta}_\ell^k)$.

The *S-Step* randomly samples a label for each pixel according to the current estimated τ_{ij}^k of the jth pixel partitioning, thus, the image in g groups $\{Q_1^k, \dots, Q_g^k\}$.

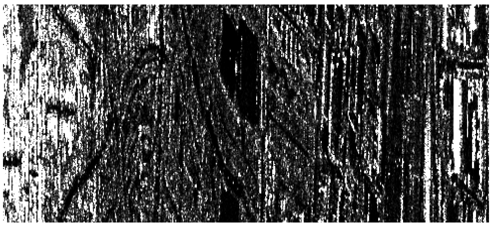
The *M-Step* updates the parameters estimates by $\rho_i^{k+1} = \#\{Q_i^k\}/N$ and $\boldsymbol{\theta}_i^{k+1}$ are updated by equations (4) and (3) with the pixels of group Q_i^k.

4. Results

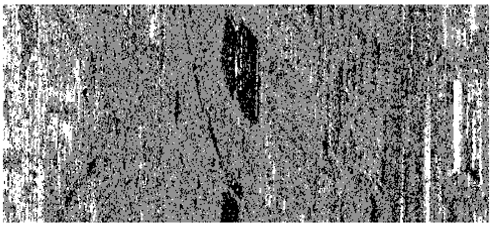
We composed two 3-looks images using real images with 558×491 and 890×491 pixels, and averaging pixels by columns. The images are parts of a single-look complex L band obtained on May 10, 1999 over Wessling, Germany. A visual inspection suggests that the scene consists of three main regions: pasture (dark areas), forest (gray areas) and urban spots (light areas).

The $\mathcal{G}_p^0$ and Wishart mixture models were applied in the classification problem where the number of groups and looks are known ($g = 3$, $n = 3$). The Table 1 and Figure 1 show the classification results. The table shows the Kappa coefficient of agreement and the overall accuracy.

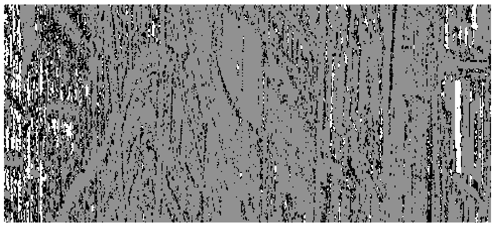
The comparison cleary shows that the SEM method using $\mathcal{G}_p^0$ mixture model leads to better results. The $\mathcal{G}_p^0$ law has the roughness parameter in order to achieve more flexibility in describing different types of areas: extremely heterogeneous, heterogeneous and homogeneous; while the Wishart distribution is a particular case of the $\mathcal{G}_p^0$ law, describing homogeneous areas.



(a) Intensity image - HH



(b) $\mathcal{G}_p^0$ mixture model



(c) Wishart mixture model

Figure 1. Classification with SEM Algorithm.

5. Acknowledgments

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References

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