Segmentation into fuzzy regions using topographic distance

SYLVIE PHILIPP-FOLIGUET[1]
MARCELO BERNARDES VIEIRA[1 2]
ARNALDO DE ALBUQUERQUE ARAÚJO[2]
[1] Equipe Traitement des Images et du Signal, ENSEA/UCP, 6 av. du Ponceau, 95014 Cergy Cedex, France
philipp@ensea.fr
[2] Departamento de Ciência da Computação, Instituto de Ciências Exatas, UFMG, Av. Antônio Carlos, 6627, 31270-901, Belo Horizonte, MG, Brasil
bernard@dcc.ufmg.br, arnaldo@dcc.ufmg.br

Abstract. This paper exposes an algorithm which leads to a fuzzy segmentation. This algorithm performs, as in the watershed method, a progressive flood of the gradient image from pixels of lowest gradients. It uses a new distance, called topographic distance. Any local minimum of the gradient norm image constitutes a seed for the region growing, avoiding the use of a marker image. These seeds constitute the cores of the initial fuzzy regions. Then the sites are gradually agglomerated to the region, while their membership degrees to the region decrease, according to the distance to the core and to the gradient norms, by the way of the topographic distance. The numerous fuzzy regions are then merged and the membership degrees of pixels to final regions are computed. Applications concern crisp segmentation of colour or gray scale images and pattern recognition from fuzzy regions.

Keywords: Segmentation, Fuzzy region, Watershed, Colour image, Topographic distance

1 Introduction

Image segmentation is defined as an image partition into regions, which are crisp sets (in opposition to fuzzy sets). But a complete segmentation or accurate regions are useless in many applications. In image indexing or in object detection for example, accurate edges are not awarded.

In crisp segmentation, there is a one-to-one choice for every site to belong or not to each region. But a pixel in the "center" of an homogeneous region, certainly belongs much more to this region than pixels agglomerated afterward, during a region growing for example. And a pixel close to a contour is certainly not in the same region than a pixel situated on the other side of this contour. To model this kind of knowledge, we suggest to build regions which are no more crisp sets but fuzzy sets, with for every site of the image a membership degree to every region.

There are two main types of method to segment images [5], the first one extracts the homogeneous areas (the regions), whereas the second one looks for the zones of transition (contours). Both approaches are complementary and none of them has proved its superiority over the other one, each of them has its advantages and its domains of application. The solution seems to be in a cooperative approach, using both types of methods.

The contour extraction has the advantage to generally provide well-localised contours, and works well when zones on each part of the contour have different average colours. But if these zones have different textures with the same colour, these methods work rather badly. Furthermore the extracted contours are rarely closed, specially in case of slow transition between regions and do not provide a segmentation strictly speaking (partition into regions).

Methods that look for regions better differentiate the textured zones, even if they have close colours. On the other hand these methods often have a bad behaviour in transition zones, providing imprecise contours or an over-segmentation of these zones. The image of regions so provided is directly a segmentation, but it often requires post-treatments such as small region removal.

To lead a cooperative contours / regions approach, we will perform a growing region, initialised by an image of gradient norm. The zones where the gradient norms are weak will be used as seeds for the region growing : they will constitute the cores of the fuzzy regions. Points with a high gradient norm will constitute dams, that are hard to cross. They will be bypassed, which allows to overcome the impulse noise, for example. For textured zones, criteria of size of the regions and of similarity of the gradient norms will be added, thereby avoiding over- segmentation often inferred by texture.

Membership degrees of sites to regions are then computed, based on a double criterion of spatial nearness and of gradient magnitude. They are computed from a "topographic distance", which is defined in §3.

2 Fuzzy segmentation

Let $\Omega \subset Z^2$ be a finite referential (set of N sites). A fuzzy segmentation of Ω is a set of M fuzzy regions R_j whose supports are included in Ω and defined by the two following axioms :

if μ_{ij} is the membership degree of site s_i to region R_j, then :

(a) $\forall i, \forall j, \mu_{ij} \in [0, 255]$

(b) $\forall j \ \sum_i \mu_{ij} \in]0, 255 \times N[$.

Notice that membership degrees are between 0 and 255, so they will equal 255 for the sites of the core and 0 for the sites which do not belong to the fuzzy region. The second axiom means that a fuzzy region must not be empty and must not be complete (equal to Ω).

This definition is based on Ruspini's definition of a fuzzy partition [12], but without the third axiom which imposes that for every site, the sum of the membership degrees to all regions is equal to 255.

Of course the terminology "fuzzy segmentation" can be found in several papers, but it is never clearly defined. The main ways which lead to fuzzy segmentation, as mentioned by Bezdek [2] are fuzzy thresholding, fuzzy pixel classification, region growing, or fuzzy rules.

The fuzzy thresholding aims at defining membership functions to object regions, based on a set of thresholds [10] [4]. Most of the methods of fuzzy pixel classification are based on the fuzzy c-means (FCM) algorithm [1], which does not lead to a properly so called segmentation, since it only classifies the pixels into fuzzy classes and does not create fuzzy regions. It is used by Boujemaa *et al* [3] to achieve a crisp segmentation on tomographic cardiac images. The number of classes is often fixed in the beginning. In the region growing methods, the problem is to find the seeds of the regions and the function linking region homogeneity and membership grades. Moghaddamzadeh *et al.* [9] have developed a complex algorithm of region growing limited by edges. The algorithm consists of two steps, the first one performs a coarse segmentation, seeds are chosen following a double criterion: most frequent colors and farness from edges. In the second step, which leads to a fine segmentation, seeds are pixels far from any of the regions of the first step. The membership function takes into account the color homogeneity and the farness to seeds. In [14] region growing is carried out by fuzzy rules involving fuzzy criteria such as region homogeneity, region size or gradient sharpness.

In this paper, we propose an algorithm which leads to a fuzzy segmentation. This algorithm performs a region growing from seeds whose membership degrees to the region are maximal (255). These seeds constitute the cores of the fuzzy regions. Then the sites are gradually agglomerated to the region, while their membership degrees to the region decrease, according to the distance to the core and to the gradient norm.

The growing region is performed by a progressive flood of the image of the gradient norms, as in the algorithm of watershed [8] [15] . In crisp segmentation, the growing of a region stops when it encounters another region, while we will pursue this growing, authorizing the overlap of the supports of fuzzy regions.

3 Topographic distance

It is interesting to define a distance between pixels which takes into account both the distance between sites (in Ω) and the roughness of the surface, that is to say a measure of the shortest 3D path drawn on the surface. Such a distance can be defined by the mean of a cost function [8].

Let f be an image : $\Omega \rightarrow Z$.

The π–topographic distance between $p \in \Omega$ and $q \in \Omega$ along path $\pi = \{ p = p_1, p_2, \ldots, p_n = q\}$ is :

$$T_\pi(p,q) = \sum_{i=2}^{n} C(p_{i-1}, p_i)$$

The cost function is :

$$C(p_{i-1}, p_i) = \left| f(p_{i-1}) - f(p_i) \right| + \text{dist}\,(p_{i-1}, p_i)\} \quad (1)$$

where dist is a distance in Z^2, which in the simplest case of 4-connectivity equals 1.

The topographic distance between two sites p and q is the minimum of all π–topographic distance between p and q along any path from p to q.

$$T(p,q) = \min_\pi T_\pi(p,q)$$

Contrary to Meyer's topographic "distance" [8], this measure T is a distance in the mathematical sense of term.

proof :

i. for all sites p and q, $T(p,q) \geq 0$, since T is the sum of costs, which all are positive or equal to zero.
ii. $T(p,q)$ is equal to zero if and only if it exists a path π, where all costs are equal to zero; this is only possible if all $f(p_i)$ are equal and all distances $\text{dist}(p_{i-1}, p_i)$ are equal to zero, that is to say if p and q are equal.
iii. for all sites p and q , $T_\pi(p,q) = T_\pi(q,p)$, since the costs are symmetrical, so does T.
iv. for all sites p, q, r $T(p,r) \leq T(p,q) + T(q,r)$, since T is the minimum of all π–topographic distances.

4 Algorithm

As in watershed algorithm [8] [15], we use a region growing which simulates the flood of the image from seeds. The algorithm is performed on gradient norm images. Seeds are automatically determined during image flooding and constitute the fuzzy region cores. To overcome the problem of over-segmentation, adjacent regions are merged, at the moment they get in contact, if one of them has a small area or a small depth. Membership degrees are finally computed.

4.1. Region growing and catchment basin merging

At first pixels are sorted according to their gradient norm. Levels are processed from the lowest to the highest gradient norm. The pixels of level (gradient norm) h are added to the basins computed up to level $h-1$ or form a new basin.

Solving the problem of region initialization of the basins is tricky. Andrade [6] builds region seeds during the image flooding : every local minimum of the image constitutes a region core and is labeled with a new label.

At level h, the supports of the fuzzy regions are spread by conditional dilations of supports of level $h-1$, conditionally to level h. At each dilation at level h, every pixel having at least one neighbor already labeled is processed. If all labeled neighbors have the same label, the current pixel takes this label. If there are different labels in the neighborhood, the pixel simply takes one of the labels and the merging is processed (cf. § 4.2). A new basin label is assigned to each connected block of pixels having no labeled neighbors, these pixels take 255 as membership degree to this region.

As Vincent & Soille [15] did, we use the following data structures to perform these steps :

- a label image,
- an image of geodesic distances (in Ω) to the region's core. This distance i mage identifies pixels of the successive dilations.
- a FIFO queue (first-in-first-out) of pixels, which stores the successive dilations.

Andrade *et al* [6] used a catchment basin merging to suppress the non-significant local minima. In this method, when two basins touch each other, their areas (numbers of pixels) and depths (bottoms' levels) are used to decide their merging. If the area or the depth of one of them are below the respective thresholds, the oldest, and deepest, basin absorbs the other one.

This method assumes that all pixels of the current level that belong to such basins have already been computed. We propose to delay the merging until all dilations of the current level have been achieved. We save the two basin labels in a FIFO queue of regions, rather than inspect them at the moment they get in contact. After the end of the support extension in the current flooding level, the FIFO is tracked to process the merging. Then the FIFO is emptied for the next flooding level. This ensures that the total area of the basins is taken into account in the merging test. Note that an absorbed basin will never absorb another basin.

We have yet reached the classical watershed. The main thing comes now : overflowing the initial catchment basins and computation of the membesrhip degrees.

4.2. Membership degrees

Membership degrees of pixels are computed according to the topographic distance to initial basin's core .

If s is a site belonging to fuzzy region R and v a neighbour of s, the membership degree of v to region R is computed using the definition of the cost (Eq. 1):

$$\mu_R(v) = \mu_R(s) - C(s,v) \qquad (2)$$

It can be easily deduced from Eq. (2) that if p and q belong to fuzzy region R, then

$$\mu_R(q) = \max \{0, (\min_{\pi} (\mu_R(p) - |f(p) - f(q)| - d_1(p,q))\} \qquad (3)$$

where d_1 is distance L_1, and π is a 4-connected path included in R and linking p to q.

(a) image f

10	15
10	10
0	0

(b) membership degrees

243	238
244	244
255	255

Table 1 : example of membership degrees.

The computation is concretely done by successive dilations and assignment of a new value if it is lower than a former one.

After merging, a penalty is applied to the membership values of the absorbed regions in order to have a higher degree in the absorbing basin's bottom :

$\mu(p) \leftarrow \mu(p) - |h_p - h_a|$ for every absorbed pixel p

h_p is the bottom's level of the basin containing p,

h_a is the bottom's level of the absorbing basin.

If both cores have the same level, the penalty equals zero and the region's core is non -connected.

5 Computational complexity

Gradient norm computation, immersion simulation and basin merging have $O(n)$ time complexity, where n is the number of pixels. The membership computation has $O(qn)$ complexity, where q is the total number of basins used in the process. Consequently, the overall process has $O(O(n) + O(n) + O(n) + O(n) + O(qn)) = O(qn)$ time complexity.

The smallest possible basin is composed of one pixel, with value h, which has its 4-connected neighbours with values higher than h. Consider a binary image with a rows and b columns, having in the even rows the pattern {0, 1, 0, 1, ...} and in the odd rows the pattern {1, 0, 1, 0, ...} like in a chess table. Such image has $\lceil a/2\rceil\cdot\lceil b/2\rceil+\lfloor a/2\rfloor\cdot\lfloor b/2\rfloor\le ab/2$ basins. This is the maximum number of basins for an image with dimensions a and b. As a result, $1\le q\le ab/2$ which leads to the worst case for our algorithm in terms of time complexity $O((ab)^2/2) = O(n^2)$. The quadratic time complexity is very restrictive for image applications but the worst case pattern is rare. Real images may have thousands of original basins, and, depending on parameters, hundreds of merged ones. Therefore, the computation time may be very high in some cases.

Actually, the algorithms have been very fast in our experiments with a linear complexity behaviour.

6 Results

The fuzzy segmentation algorithm can be used for two purposes. If crisp segmentation is awarded, the fuzzy segmentation must be "defuzzified", which is done by affecting each pixel to the region for which it has the highest membership degree. The other, and more important interest of this algorithm is to keep the fuzzy regions to compute scalar features.

6.1 Monochromatic images

The gradient norms may be computed with any filter, we used Shen-Castan filter [13] because of its accuracy in edge localisation.

On the noisy image of Fig. 1, intensity means of both parts only differ by 3 levels. The algorithm creates 287 fuzzy regions, which, after merging with thresholds of area and depth respectively equal to 50 and 2, are reduced to 2 regions (Fig. 2).

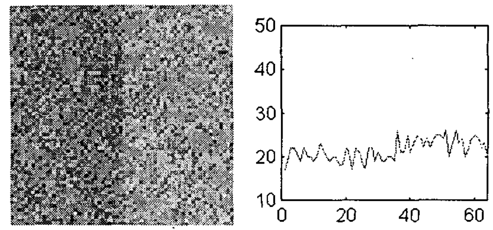


Fig. 1: Image (enhanced) and profile of a line

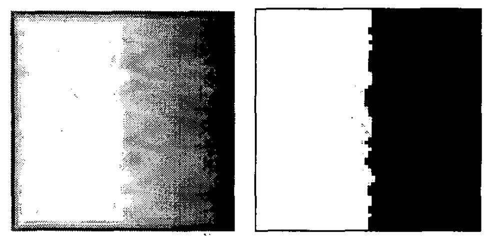


Fig. 2 : Membership degrees to left region and defuzzification

The image of Fig. 3, is a muscular slice which includes on one hand textured fibers and on the other hand uniform fibers, but so close to each other that they often are connected. We obtain 4434 fuzzy regions from the image of gradient norm, and 65 regions after merging with thresholds for area of 110 and for depth of 11 (Fig. 4). The threshold of area is tuned according to expected region size. Dark fibers are correctly segmented, the use of the gradient leads to an accurate boarder localization and almost all regions are obtained.

These results can be compared with two segmentation results from Cocquerez and Philipp [5] The first one is an edge detector, followed by a contour closing (Fig. 6) and an unsupervised segmentation method using Markov random fields (Fig. 7). The result obtained from fuzzy regions is better : region number is smaller, there is no oversegmentation due to texture, and borders between dark fibers, which are difficult to extract are often found (see zoom fig 5a).

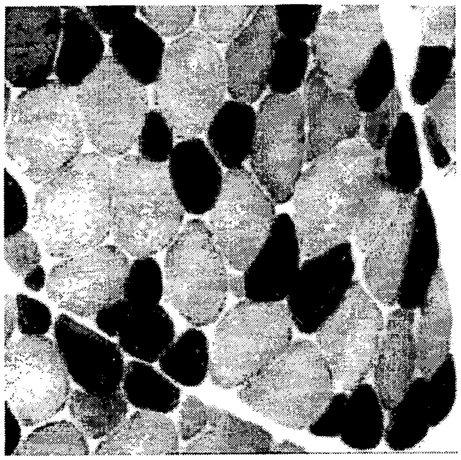
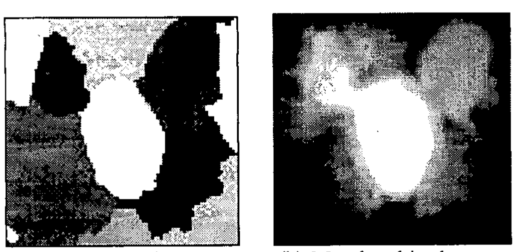


Fig. 3 : Original image (256 × 256) of a muscular slice

Fig. 4 : Crisp segmentation (65 regions)



(a) Region in the center (b) Membership degrees to this region

Fig. 5 : Zoom on a region of Fig. 3

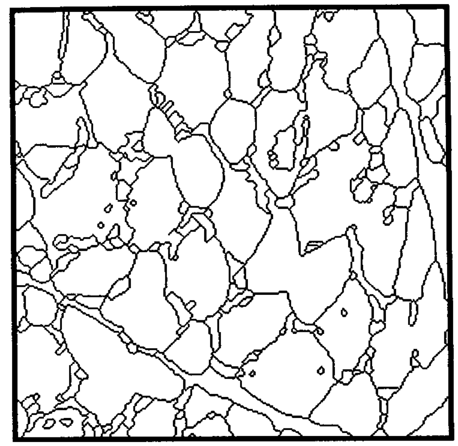


Fig. 6 : edge detection + contour closing

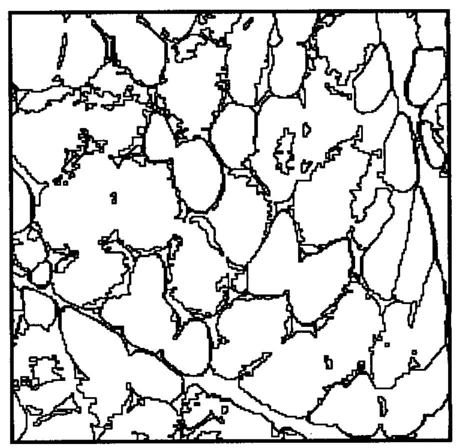


Fig. 7 : Markovian unsupervised relaxation

6.2 Fuzzy regions

The main result is the set of fuzzy regions. Fig. 5b displays one fuzzy region, with membership degrees of its elements. The core is multimodal, membership degrees are almost constant in the central fiber. They slowly decrease after crossing small edges, such as those with the two other black fibers, and they decrease a lot when encountering a sharp edge.

Rosenfeld [11] extended the classic features of area, perimeter, compactness, etc. to fuzzy regions. These attributes can then be used for pattern recognition, indexing, etc.

6.3 Colour images

This scheme is a general one, which can also be applied to colour images, as soon as a gradient norm is available. The Di Zenzo algorithm [7] is well adapted to compute the color gradient norm. Fig. 8 displays some results with a gradient norm computed by Di Zenzo algorithm with marginal gradients computed on each color band by Shen-Castan filter. For these two images, we show the maximum membership degrees (over all fuzzy regions) and crsip segmentation after defuzzification.

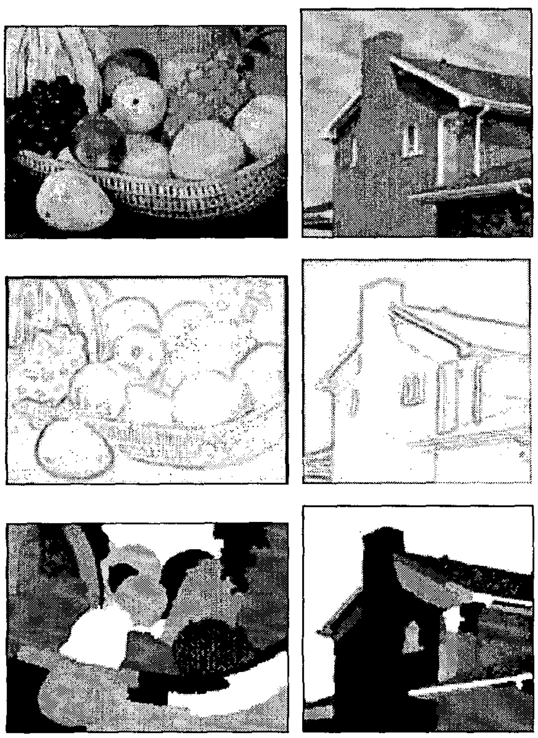


Fig. 8 : two images with maximum membership degrees (over all fuzzy regions) and crisp segmentation

7 Conclusion

We have proposed an original method of region extraction, consisting of a region growing constrained by edges. This method overcomes the problem of incomplete contours, since it provides fuzzy regions.

Membership degrees are high in the local minimums of the gradient norm (which well correspond to the homogeneous zones of the image) and which lessen when crossing a contour. For very spread regions, a weighting can be added between distance in Z^2 and difference of gradient norms, allowing to take differently into account distances to cores and gradient norms.

Contours are correctly positioned, because of the competition between adjacent regions (during defuzzification). The main advantage regarding segmentation is that it is completely automatic, since it does not use an image of region seeds. Regions are created dynamically during the process in every local minimum. Over-segmentation is reduced by merging regions on a double criterion of size and difference of depth. The first threshold is easy to tune, according to the size of the expected regions, the latter one is more delicate.

Shapes and membership degrees could be improved by using a distance more isotropic than L_1 distance. We are working on using a chamfer distance which is closer to Euclidean distance.

8 References

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