IFT–Watershed From Gray-Scale Marker

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Abstract. The watershed transform and the morphological reconstruction are two of the most important operators for image segmentation in the framework of mathematical morphology. In many situations, the segmentation requires the classical watershed transform of a reconstructed image. In this paper, we introduce the *IFT-watershed from gray scale marker* - a method to compute at same time, the reconstruction and the classical watershed transform of the reconstructed image, without explicit computation of any regional minima. The method is based on the *Image Foresting Transform* (IFT) - a unified and efficient approach to reduce image processing problems to a minimum-cost path forest problem in a graph. As additional contributions, we demonstrate that (i) the *cost map* of the IFT-watershed from markers is identical to the output of the superior gray scale reconstruction; (ii) other reconstruction algorithms are not watersheds; and (iii) the proposed method achieves competitive advantages as compared to the current classical watershed approach.

1 Introduction

We have developed a unified and efficient approach, called *image foresting transform* (IFT) [7], to reduce image processing problems to a minimum-cost path forest problem in a graph. The IFT has been successfully applied to boundary tracking, watershed transforms, morphological reconstructions, multi-scale skeletonization, fast binary morphological operations, multi-scale shape filtering, Euclidean distance transform, geodesic path computation, multi-scale fractal dimension, and shape representation by saliences [8, 6, 5, 11, 7]. In this paper, we propose a variant of the IFT-watershed from markers [8], called the *IFT-watershed from gray scale marker*.

The classical watershed transform of an input image I usually creates an over-segmented partition, where the catchment basins are influence zones of the regional minima of I [2]. In order to reduce the number of catchment basins in the segmentation, we usually apply to I a morphological reconstruction before the watershed transform. In [8], we proposed the IFT-watershed from markers which computes the watershed-from-markers transform without requiring minima imposition (see Figure 1). We presented in [6], the relation between watershed-from-markers transform and morphological reconstruction. In this paper, we show that the proposed method computes at same time the morphological reconstruction and the classical watershed transform of the reconstructed image, without explicit computation of regional minima (see Figure 2). There are competitive advantages of this approach as compared to the current classical watershed approach. It is conceptually solid with higher abstraction where the marker is specified by a gray scale image, it is faster and simpler by combining many steps in a single algorithm, and it does not require explicit computation of any regional minima.

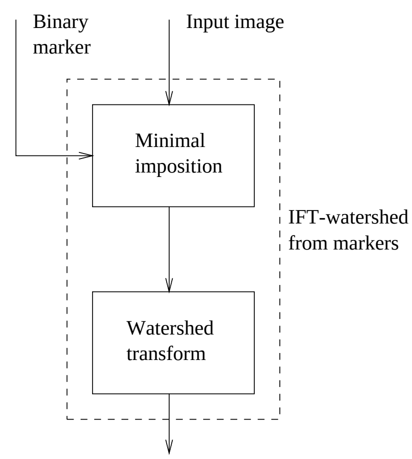

Figure 1: The IFT-watershed from markers does not require minima imposition.

We first review the IFT-watershed-from-markers algorithm (Section 2) and demonstrate that its *cost map* is identical to the output of the superior reconstruction algorithm

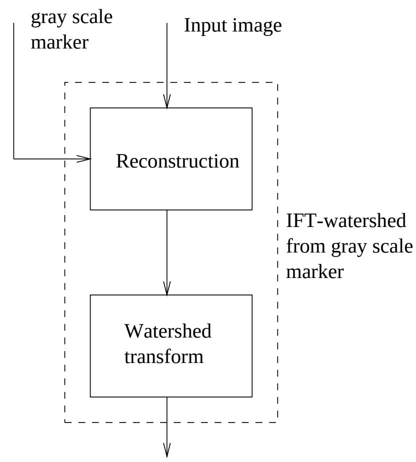

Figure 2: The IFT-watershed from gray scale marker computes the reconstruction and the watershed transform of the reconstructed image, simultaneously.

(Section 3). We then introduce the IFT-watershed from gray scale marker and its efficient algorithm in Section 4. In Section 5, we give an illustrative example of the method and, in Section 6, we demonstrate why other reconstruction algorithms are not watersheds. The conclusions are stated in Section 7.

2 The IFT-Watershed from Markers

Many problems in image processing can be interpreted as an image partition problem based on a given set of *root pixels*, where each root defines an *influence zone* consisting of the pixels that are "more closely connected" to that root than to any other, in some appropriate sense. The *image foresting transform* (IFT) reduces such problems to the computation of a *minimum-cost path forest* in a directed graph whose nodes are the image pixels and whose arcs are defined by an *adjacency relation* N between pixels. We denote by $N(p)$ the set of pixels adjacent to the pixel p (i.e. its neighbors in the graph).

The cost of a path in this graph is determined by a suitable *path-cost function* f, which usually depends on local image properties along the path — such as color, gradient, and pixel position. For suitable path-cost functions, one can choose the minimum-cost paths so that their union is an oriented forest, spanning the whole image. The nodes of a given tree in the forest are by definition the influence zone of the corresponding root.

The result of the IFT is an *annotated image*, that assigns to each pixel three attributes: its predecessor in the optimum path, the cost of that path, and the corresponding root. Its solution is usually obtained in linear time and requires a variant of the Dijkstra/Moore/Dial's shortest-path algorithm [4, 10, 3].

In this framework, the watershed-from-markers [2] can be computed by a single IFT [8], without minima imposition, where the labeled markers are root pixels. In this case, the root map can be replaced by the label map which corresponds to the catchment basins. The path-cost function f_m is given by:

$$f_m(< p_1, p_2, \ldots, p_n >) = \\ = \begin{cases} max\{I(p_1), I(p_2), \ldots, I(p_n)\} & \text{if } p_1 \text{ is a marker pixel} \\ +\infty & \text{otherwise,} \end{cases} \tag{1}$$

where $p_{i+1} \in N(p_i)$ and $I(p_i)$ is the value of the pixel p_i in the image I.

The watershed based on the IFT framework may be simplified to the following algorithm:

Algorithm 1 *Watershed from markers using hierarchical FIFO queue (HFQ).*

INPUT I: *input image,*
L: *labeled marker image;*
OUTPUT L: *watershed catchment basins;*
AUXILIARY C: *cost map, initialized to infinity;*

INITIALIZATION
for all pixels $L(p) \neq 0$
    $C(p) \leftarrow I(p)$, *insert* p *in* HFQ *with cost* $C(p)$*;*

PROPAGATION
while HFQ *not empty*
    $p \leftarrow$ *remove from* HFQ*;*
    for each $q \in N(p)$
        if $C(q) > \max\{C(p), I(q)\}$
            $C(q) \leftarrow \max\{C(p), I(q)\}$*;*
            Insert q *in* HFQ *with cost* $C(q)$*;*
            $L(p) \leftarrow L(q)$;

Two points are worth discussing in the above algorithm. Note that the pixels in the queue are never reevaluated as required by the original Dijkstra/Moore/Dial algorithm [4, 10, 3]. This was explained in [8] and the reason is that all the incident arcs to a node have the same weight. The other particularity of this algorithm is the use of a FIFO in the hierarchical queue. This is desirable if one wants to have the inclusion of a slight modification of the path-cost to make the catchment basin boundaries be in the medial line of plateaus regions in the image. The FIFO property of the hierarchical queue can be modeled by a lexicographic path-cost function as pointed out in [8].

3 Morphological Reconstruction

Morphological reconstruction is a powerful operator in the framework of mathematical morphology. It is the building block of connected operator design. In this paper, we will be interested in the sup-reconstruction, also called dual reconstruction, which is related to the watershed. The gray scale sup–reconstruction of image I from marker image J, $J \geq I$ is given by:

$$\phi_J(I) = \epsilon^{\infty}_{I,B}(J), \tag{2}$$

where $\epsilon_{I,B}(J) = \epsilon_B(J) \vee I$ is the geodesic erosion, $\epsilon_B(I)$ is the gray scale erosion of I by the structuring element B and $\vee$ is the pointwise maximum operator. The composition $\psi^n(I)$ is the successive application of the operator $n-1$ times: $\psi(\psi(\ldots(\psi(I))))$. The sup-reconstruction is a successive sequence of erosions of J by B, pointwise maximum with I until stability.

An important property, proved in [12], is that the sup-reconstruction of I from J is the same as sup-reconstruction of I from R_J, where R_J is given by:

$$R_J(p) = \begin{cases} J(p) & \text{if } p \text{ is regional minima of } J, \\ \infty & \text{otherwise.} \end{cases} \tag{3}$$

The regional minima M of a gray scale image is the largest connected component of pixels with same gray scale (plateau), such that every neighbor pixel of M has a strictly higher value.Fig 3 shows the relationship between R_J and J.

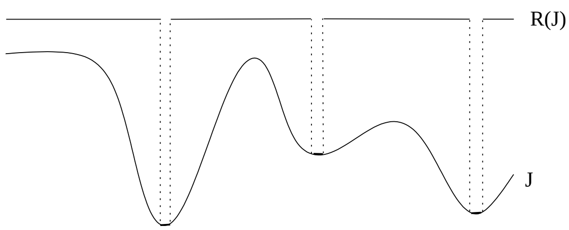

Figure 3: Functions R_J and J.

Vincent [12] has described several morphological reconstruction algorithms. The one based on a FIFO queue is shown below.

Algorithm 2 *Grayscale reconstruction using a queue.*

INPUT I: *gray scale mask image,*
J: *gray scale marker image,* $J \geq I$;
OUTPUT J: *sup-reconstruction of* I *from* J

INITIALIZATION
1. Compute $R_J(p)$ *by Eq. 3;*
2. Insert the pixels in the regional minima of J *in queue;*

PROPAGATION
3. **while** *queue not empty*
4. $p \leftarrow$ *remove from queue;*
5. **for each** $q \in N(p)$
6. **if** $J(q) > J(p)$ *and* $I(q) \neq J(q)$
7. $J(q) \leftarrow \max\{J(p), I(q)\}$;
8. Insert q *in queue;*

Note that algorithms 1 and 2 are very similar. We claim that the cost map C of algorithm 1 is the sup-reconstruction of I from any gray scale marker function J such that:

$$R_J(p) = \begin{cases} I(p) & \text{if } L(p) \neq 0, \\ \infty & \text{otherwise.} \end{cases} \tag{4}$$

We can state:

Proposition 1 *The cost map C in the algorithm 1 is the morphological reconstruction of I from any marker image J such that $J \geq I$ and R_J is given by Eq. 4.(See Fig. 4*

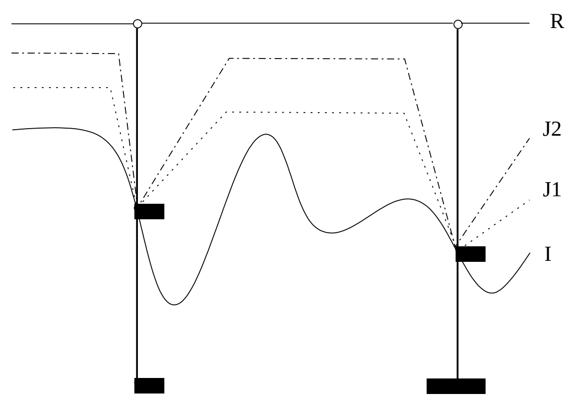

Figure 4: Functions I, R_J and two possibles J: $J1$, $J2$.

Prove 1 *Both initialization algorithms are equivalent, the pixels that are inserted in the queue are the regional minima of J. In the watershed, the inserted pixels have value $I(p)$ at the markers. The others pixels not inserted in the queue have cost infinity. So, both regional minima are equivalent and inserted in the queue. According to Vincent [12], the propagation order of the pixels does not matter in the reconstruction. Therefore, we can replace the FIFO in algorithm 2 by the HFQ of algorithm 1.*

Line 6 of algorithm 2 may be rewritten as: If $J(q) > J(p)$ and $I(q) < J(q)$, since $J \geq I$. So $J(q)$ must be greater than $J(p)$ and greater than $I(q)$, which is equivalent to line 6 of algorithm 1, where C plays the role of J. ∎

The consequence of this fact is that the morphological reconstruction, when implemented by the HFQ, can simultaneously output the result of the reconstruction and the catchment basins of I that were propagated from the regional minima of the marker J. In the IFT framework, the output of the reconstruction is given by the cost map and the watershed catchment basins by the root map. This result is not new, as we have presented in [8] that the maximum cost function of the IFT-watershed from markers intrinsically includes the minima imposition, and recently, and in more detail, with practical illustrative examples, we showed that the watershed-from-markers and the superior reconstruction are obtained at the same time by a single IFT [6]. Here we prove this result by reducing the algorithm 1 to algorithm 2.

A more important consequence of the above conclusion is that we can show that the morphological reconstruction of I from J, $I \leq J$, can be modeled as the cost map of an IFT, where the path-cost function f_r is given by:

$$f_r(< p_1, p_2, \dots, p_n >) = max\{J(p_1), I(p_2), \dots, I(p_n)\}. \quad (5)$$

Note that $J(p)$ is the initial cost of any path starting at p and all pixels p are root candidates (we usually call them *seed pixels*). Such a marker function can be seen as an initial restriction on the maximum path cost. However, since the cost of a path from a pixel p to a pixel q may be less than $J(q)$, only some candidate pixels will remain as root pixels in the minimal-cost path forest. Moreover, we wish to use this formulation to compute simultaneously the morphological reconstruction and the watershed transform, without explicit computation of any regional minima. This is addressed next.

4 The IFT-Watershed From Gray Scale Marker

Before presenting the IFT-watershed from gray scale marker we will introduce a further improvement of this algorithm by avoiding the explicit regional minima detection of the marker function, which is normally used to insert the pixels in the HFQ in the initialization phase of the IFT algorithm. This is also a contribution of the work that is not published elsewhere.

The IFT-watershed from gray scale marker has the same input as the morphological sup–reconstruction: I as the image and J as the marker, both gray scale images. When the regional minima of J is composed of more than one pixel, which is often the case, we would like to have only one root pixel for each regional minima. We propose an efficient variation of the IFT algorithm to make this happen. In the initialization step, we set the initial root map such that all pixels are candidate root pixels ($R(p) = p$). We insert all pixels in the HFQ with the initial cost given by $J+1$. The addition of one to the initial cost is the essential part of the technique. When a pixel is removed from the HFQ and is a root pixel, it means it is the first of its regional minima (called representative), and as such, its cost is corrected by subtracting one to it. Its neighbors belonging to the same regional minima will have their cost updated in the HFQ and will have their root map pointed to the representative of the regional minima. The technique of adding one makes the behavior of plateaus be dealt correctly in the FIFO property of the HFQ as all pixels in the HFQ will be updated as they were inserted with a higher initial cost. This removal and reinsertion in the HFQ for every pixel except the representative one of each regional minima is the price to pay for not computing the regional minima explicitly. When the HQF is implemented efficiently, it is well worth adopting the approach just described.

The IFT-watershed from gray scale marker algorithm is presented below.

Algorithm 3 *IFT-watershed from gray scale marker*

INPUT *I: gray scale input image,*
J: gray scale marker image, $J \geq I$;
OUTPUT *J: reconstruction of I from J,*
R: catchment basins of the watershed;

INITIALIZATION
for all pixels $R(p) \leftarrow p$
Insert all pixels p in HFQ with cost $J(p) \leftarrow J(p) + 1$;

PROPAGATION
while *queue not empty*
    $p \leftarrow$ *remove from HFQ;*
    if $R(p) = p$ *then* $J(p) \leftarrow J(p) - 1$;
    for each $q \in N(p)$
        if $J(q) > \max\{J(p), I(q)\}$
            $J(q) \leftarrow \max\{J(p), I(q)\}$;
            $R(q) \leftarrow R(p)$;
            Remove q from the HFQ;
            Insert q in the HFQ with cost $J(q)$;

5 Illustrative Example

We illustrate a simple example of using the IFT-watershed from gray scale marker.

Fig. 5 shows the original image K. The image is preprocessed by the gray scale area open to remove the white peaks in the middle of each cell. The resultant image I is shown in Fig 6. Fig. 7 shows the result of the watershed lines of IFT-watershed of image I from a gray scale marker obtained by adding 38 to the image I, superposed on the input image K. The classical way of doing this would be first computing image I_1 as a reconstruction of I from $I + 38$ (h-minima) and then computing the watershed transform of I_1. This filter h-minima filter removes any regional minima with depth smaller than $h = 38$. The classical approach took approximately 0.25 s to process a 256×256 image on a 1GHz Pentium and 0.15 s to compute the combined reconstruction and watershed using the IFT-watershed from gray scale marker.

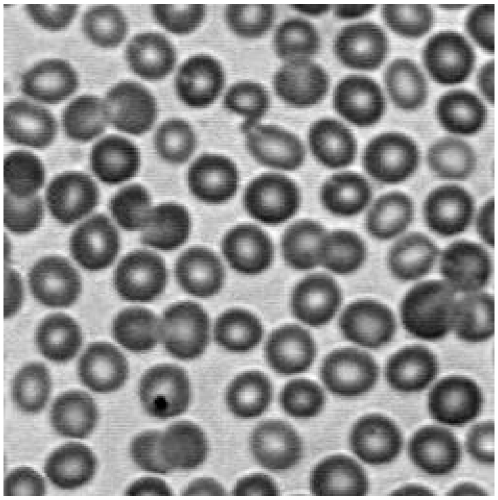

Figure 5: Image K: original image.

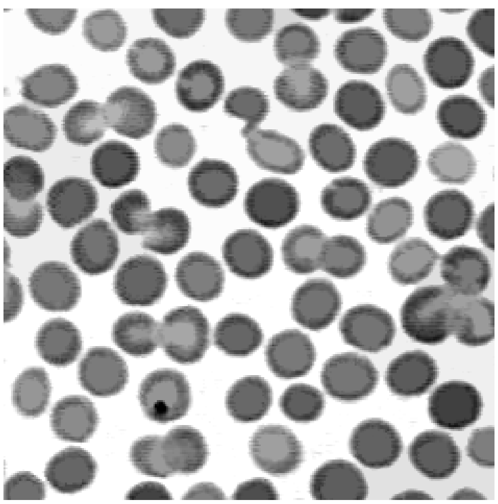

Figure 6: Image I: gray scale area open of K.

6 Why Other Reconstruction Algorithms Are Not Watersheds

Using the IFT framework, we have shown that the morphological reconstruction of I from J, with $I \leq J$, is the cost map of a minimal-cost path forest with the path-cost function f_r given by Eq. 5.

One of the simplest and general shortest path algorithms is the one introduced by Berge [1], which was designed for the classical distance path-cost function f_{sum}:

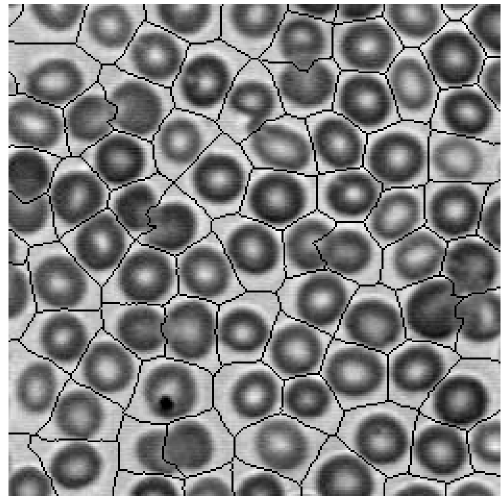

Figure 7: Watershed lines of the IFT-watershed of I from gray scale marker $I + 38$ superposed on the original image K.

$$f_{sum}(< p_1, p_2, \ldots, p_n >) = J(p_1) + I(p_2) + \ldots + I(p_n). \quad (6)$$

Berge's algorithm is based on a repetition until stability of the following situation that must be true in the shortest path solution. For any two neighboring nodes, $J(q) \leq J(p) + I(q)$. So this algorithm is a repetition in any scanning order of neighboring pixels of the above relation. This algorithm can also compute the root map of the forest partitioning.

Algorithm 4 *Berge shortest path.*

INITIALIZATION
  $R(p) \leftarrow p$;

repeat *until stability, using any scanning order*
  if $J(q) \geq J(p) + I(q), q \in N(p)$
    $J(q) \leftarrow J(p) + I(q)$;
    $R(q) \leftarrow R(p)$;

For the reconstruction algorithm, we have shown that the path-cost function is given by f_r which is based on the maximum pixel value in the path. Interestingly is the fact that the Berge algorithm in this case can compute the cost map correctly, but not the root map.

Rewriting the Berge algorithm using the path-cost function f_r and computing only the cost map, we arrived at the following algorithm.

Algorithm 5 *Berge minimal-cost path with f_r.*

INITIALIZATION
  $R(p) \leftarrow p$;

repeat *until stability, using any scanning order*
  if $J(q) \geq \max\{J(p), I(q)\}, q \in N(p)$
    $J(q) \leftarrow \max\{J(p), I(q)\}$;

which is the morphological reconstruction reported by Vincent [12] when the pixels can be updated in any scanning order.

Unfortunately, it is not possible to incorporate the root map updating in the Berge algorithm, as we demonstrate by a simple illustrative example. Suppose we have a small numeric image illustrated in Fig. 8a. The image has two valleys and each one has a non-infinity marker pixel with a small dot and a small square below the pixel value, in the figure. If we scan the pixels in raster order, for right and down neighbors using the Berge algorithm, the result of the first scan is presented in Fig. 8b. After the anti-raster scan order (left and up neighbors), note that the two top-right pixels with values 5 and 7 remained rooted at the squared dot marker, as their cost are correct already after first scan. Only the scanning order of the hierarchical queue propagates the root in the increasing path-cost order and as such can guarantee the root map in the path-cost function f_r.

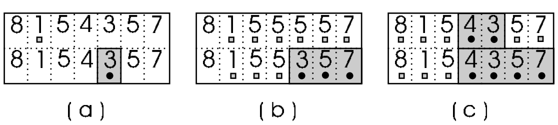

Figure 8: Berge algorithms are not watersheds: (a) initial image two markers; (b) raster scan right-down; (c) anti-raster left-up.

It is possible to compute the watershed transform algorithm using the Berge algorithm only if the path-cost function is in the form of a distance function of type f_{sum}. This is exactly the case presented by Meyer in [9] with the definition of the topographic distance.

In summary, the morphological reconstruction and the watershed transform are well explained by the minimal-cost path forest using the f_r path-cost function. The Dijkstra / Moore / Dial shortest path algorithm can compute the cost map (reconstruction) and the root map (watershed) of the minimal-cost path forest. The Berge algorithm using the f_r path-cost function can only compute the cost map (reconstruction). This latter algorithm belongs to the class of the parallel reconstruction algorithms. If it is desirable to compute the watershed using the Berge algorithm because of its suitability to hardware implementation, the topographic distance must be used.

7 Conclusions and Comments

In this paper, we have shown several results. Previously we have pointed that the morphological reconstruction algorithm based on the hierarchical queue is also a watershed transformation [6]. In this work, we have demonstrated this result by showing the equivalence of the morphological reconstruction algorithm using a FIFO queue and the cost map of the IFT-watershed-from-markers algorithm.

We have consolidate the importance of the IFT framework, where the image is modeled by a graph and the minimal-cost path forest is computed resulting in three maps: cost, root, and predecessor maps. In this context, the morphological reconstruction is the cost map of a minimal-cost path forest using the path-cost a function $f_r(p)$ of the maximum pixel values in the path and the pixel value of the gray scale marker function at p. We have shown that the Berge shortest path algorithm works for the computation of the minimal path-cost (reconstruction), but does not work for the computation of the root map (catchment basins), for this kind of path-cost. This is the reason why non ordered queue based reconstruction algorithms are not watersheds.

We have introduced the IFT-watershed from gray scale marker which is a new concept where the watershed is not computed from a set of labeled markers but from a gray scale function. This concept puts together several steps used in classical watershed based segmentation strategies in a single algorithm.

We have presented an efficient IFT-watershed from gray scale marker algorithm which computes simultaneously the watershed and the sup-reconstruction and avoids the explicit computation of the regional minima of the marker function.

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