Depth Estimation Through a Disparity-Based Photometric Stereo

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Abstract. Here we revisit a recently introduced process of relative depth estimation through the matching of photometric stereo images. By considering the general solution of the differential equation which relates surface depth to the disparity map produced by the matching process, we are able to obtain a more consistent formulation of such disparity-based approach to photometric stereo. Also, we introduce a multiscale matching procedure, based on a new stochastic metaheuristic for combinatorial optimization, which yields more reliable disparity maps in shorter processing times. Finally, we employ a simple least-squares regression in a calibration strategy for estimating the parameters required by our reconstruction approach. Shape-estimation experiments with real images are presented.

Keywords: Computer Vision, Photometric Stereo.

1 Disparity-Based Photometric Stereo

We consider two photometric stereo images, $I_1(s)$ and $I_2(s)$, where $s=(x,y)$ denotes a general point on the image plane. If such images correspond to two illumination directions which are not far apart, and if the underlying surface is smooth, we may attempt to match them to obtain a disparity field similar to the ones resulting in stereoscopy. Calling $D(s) = (D_x(s), D_y(s))$ the disparity field, we would have

$$I_1(x, y) \approx I_2(x + D_x(s), y + D_y(s)) \tag{1}$$

from which we obtain, through a Taylor-series expansion,

$$\begin{aligned} \Delta I(s) &\equiv I_1(x, y) - I_2(x, y) \approx \\ &\approx D_x(x, y)\frac{\partial I_2(x, y)}{\partial x} + D_y(x, y)\frac{\partial I_2(x, y)}{\partial x} \end{aligned} \tag{2}$$

Now, if we assume that a linear approximation of the reflectance map function is applicable, we may rewrite $\Delta I(s)$ as

$$\Delta I \equiv I_1(x, y) - I_2(x, y) = k_0 + k_1 p + k_2 q \tag{3}$$

where

$$k_0 = k_{01} - k_{02},\ k_1 = k_{11} - k_{12} \text{ and } k_2 = k_{21} - k_{22} \tag{4}$$

with

$$k_{0i} = \bar{k}_{0i} - k_{1i} p_0 - k_{2i} q_0, \quad \text{for } i = 1,2. \tag{5}$$

and

$$\bar{k}_{0i} = R_i(p_0, q_0), \quad k_{1i} = \frac{\partial R_i(p,q)}{\partial p}\Big|_{po,qo} \quad \text{and} \quad k_{2i} = \frac{\partial R_i(p,q)}{\partial q}\Big|_{po,qo} \tag{6}$$

where $R_i(p,q)$, for i =1,2, denote the reflectance maps associated with the two imagens, which are functions of the surface gradient components, $p = \partial z / \partial x$ and $q = \partial z / \partial y$, and (p_0, q_0) denotes the orientation around which the linear expansion is taken. From (3) and (2), we thus get

$$k_1 p + k_2 q = D_x(s)\frac{\partial I_2}{\partial x} + D_y(s)\frac{\partial I_2}{\partial y} - k_0 \tag{7}$$

which is the differential equation relating the disparity field to surface depth. Now, for two sufficiently close illumination directions, it is possible to obtain another relation between $D(s)$ and $z(s)$, by requiring that the displacement of a given irradiance patch over the imaged surface be perpendicular to the local normal vector, which is given by $\vec{n} = (-p, -q, 1)$. Since, for an orthographic projection geometry, any such displacement can be denoted by $(D_x(s), D_y(s), V(s))$, where $V(s)$ is the unobservable displacement component along the optical-axis direction (direction *z*), we thus have

$$D_x(s)p + D_y(s)q = V(s) \tag{8}$$

In order to obtain the depth map, *z*(*s*), in terms of the disparity field, *D*(*s*), we must therefore find a solution to (7) which is also consistent with equation (8). Employing Lagrange's method [Hildebrand (1962)], we thus find that the following relations must hold

$$k_2 dx - k_1 dy = 0 \quad \text{and} \quad D_y(s)dx - D_x(s)dy = 0 \tag{9}$$

From where we may take

$$\frac{D_y(s)}{D_x(s)} = \frac{k_2}{k_1} \tag{10}$$

Proceeding with the solution to (7) we must then solve the ordinary differential equation

$$\frac{dz}{D_x(s)\dfrac{\partial I_2}{\partial x} + D_y(s)\dfrac{\partial I_2}{\partial y} - k_0} = \frac{k_1 dx + k_2 dy}{k_1^2 + k_2^2} \tag{11}$$

which can be rewritten as

$$(\vec{k}.d\vec{s})(\vec{D}(s).\vec{\nabla} I_2) - k_0(\vec{k}.d\vec{s}) = (k_1^2 + k_2^2)dz \tag{12}$$

where $\vec{k} = k_1\hat{x} + k_2\hat{y}$, $\vec{D}(s) = D_x(s)\hat{x} + D_y(s)\hat{y}$, $\vec{\nabla} I_2 = \frac{\partial I_2}{\partial x}\hat{x} + \frac{\partial I_2}{\partial y}\hat{y}$ and $d\vec{s} = dx\,\hat{x} + dy\,\hat{y}$. Using the property of the double vector product, the factor $(\vec{k}.d\vec{s})(\vec{D}(s).\vec{\nabla} I_2)$ can be rewritten as $[(\vec{k}.\vec{D}(s))d\vec{s} + \vec{k}\times(\vec{D}(s)\times d\vec{s})].\vec{\nabla} I_2$, and the second term inside the brackets is easily found to vanish, due to (9). Thus, (12) becomes

$$(\vec{k}.\vec{D}(s))\vec{\nabla} I_2 .d\vec{s} - k_0(\vec{k}.d\vec{s}) = (k_1^2 + k_2^2)dz \tag{13}$$

The term on the left-hand side of the above equation becomes a complete differential, *df*, if we assume, as was done in the original formulation of DBPS [Torreão et al. (1995)], that $\vec{D}(s)$ varies slowly with position across the image plane, when compared to $I_2(s)$. In such case, we will have

$$df = \vec{\nabla}[(\vec{k}.\vec{D}(s))I_2(s) - k_0(k_1 x + k_2 y)].d\vec{s} = \\ = (k_1^2 + k_2^2)dz \tag{14}$$

where $f = (\vec{k}.\vec{D}(s))I_2(s) - k_0(k_1 x + k_2 y)$. The general solution to (7) can be written as

$$z(x,y) = \frac{f + F(k_2 x - k_1 y)}{k_1^2 + k_2^2} \tag{15}$$

where *F* is an arbitrary function of its argument, which comes from the first expression in (9) [Hildebrand (1962)]. Equation (15) can be easily related to the expression for *z*(*x*,*y*) obtained in the optical-flow approach to photometric stereo [Torreão et al. (1995)]. Moreover, if we employ (10), such equation becomes

$$z(x,y) = \frac{D_x(s)I_2(s)}{k_1} - \frac{k_0(k_1 x + k_2 y)}{k_1^2 + k_2^2} + \\ + \frac{F(k_2 x - k_1 y)}{k_1^2 + k_2^2} \tag{16}$$

which obviously includes, as a particular case, the expression given in [Torreão et al. (1995)], where it was assumed that $k_2 = 0$.

It is interesting to remark that, even starting from a disparity-vector along an arbitrary direction, we have been able to arrive at a depth map which depends only on the component $D_x(s)$, without having to consider any particular illumination directions, in contrast to the formulation of [Torreão et al. (1995)].

Apart from $I_2(s)$, which is an input image, and from $D_x(s)$, which can be reliably estimated through a multiscale version of the microcanonical optimization algorithm [Torreão-Roe (1995)], equation (16) depends only on the parameters of the linear approximation of the reflectance map, which we obtain through a calibration approach based on a simple least-squares regression [Fernandes (1997)].

2 Experiments

Here we present two 3-D reconstructions yielded by the DBPS strategy above described (Experiments 1 and 2). The calibration surfaces employed for the estimation of the reflectance map were spheres of approximately the same reflectance as the test objects. For each experiment (Figures 1, 2, 3 and 4) we show the input image pair, the resulting disparity map, and one view of the reconstructed surface, with the intensities of one of the corresponding input images mapped onto it. The illumination directions chosen, in all cases, were (-0.36,0,1) and (0.36,0,1), and equation (16) was employed for all reconstructions, with the function *F* taken equal to zero.

References

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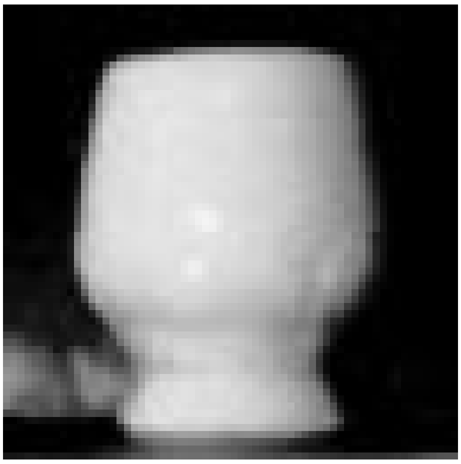
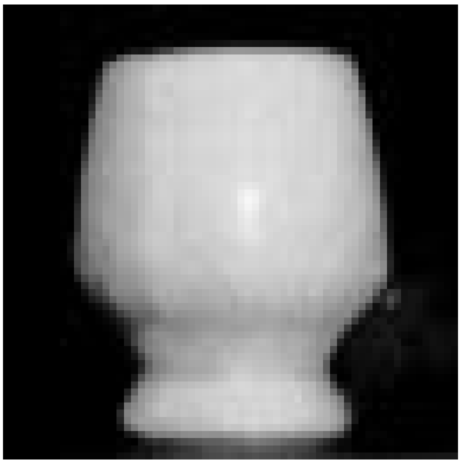
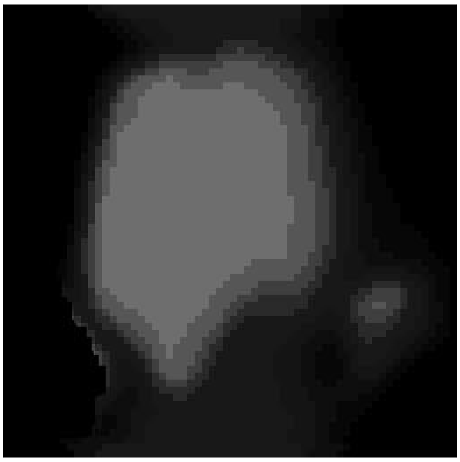


(a)



(b)



(c)

Figure 1. Experiment 1. Input images (a) and (b); disparity map (c).

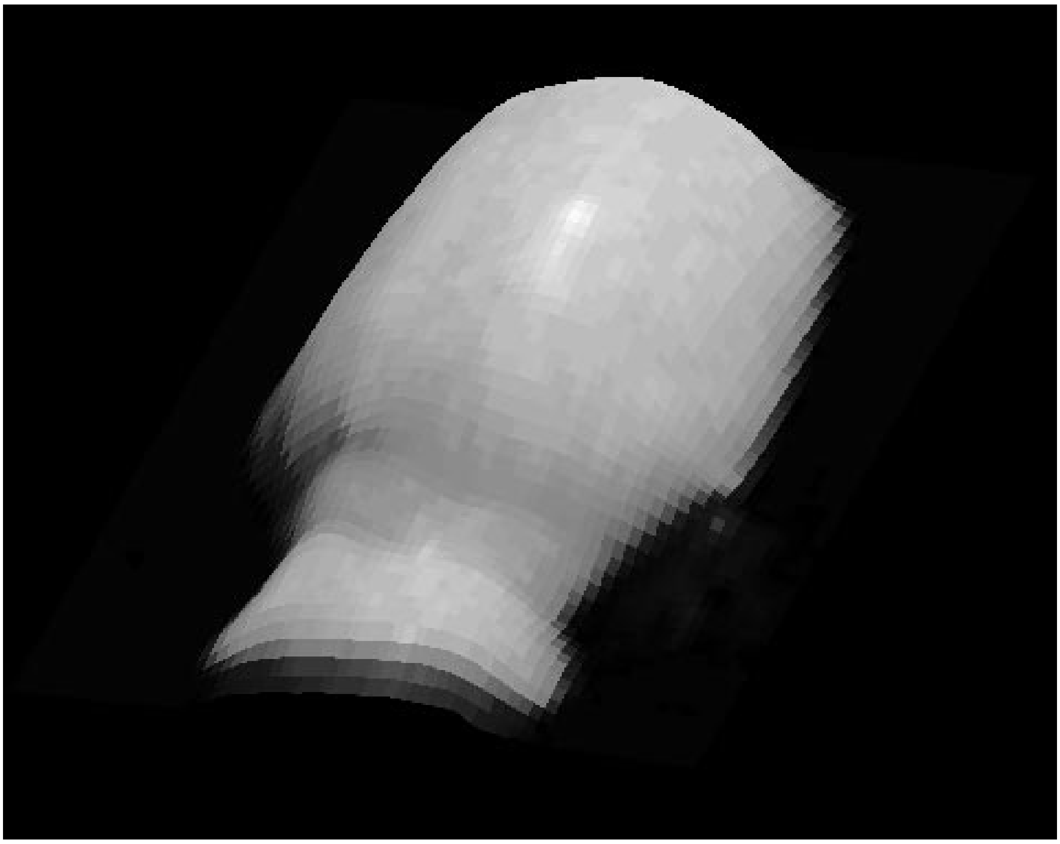
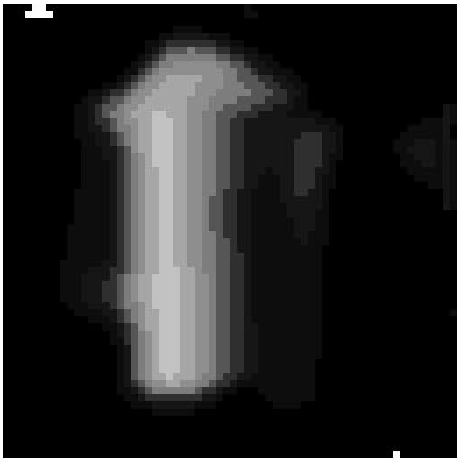


Figure 2. Experiment 1. Reconstructed surface.

(a)

(b)



(c)

Figure 3. Experiment 2. Input images (a) and (b); disparity map (c).

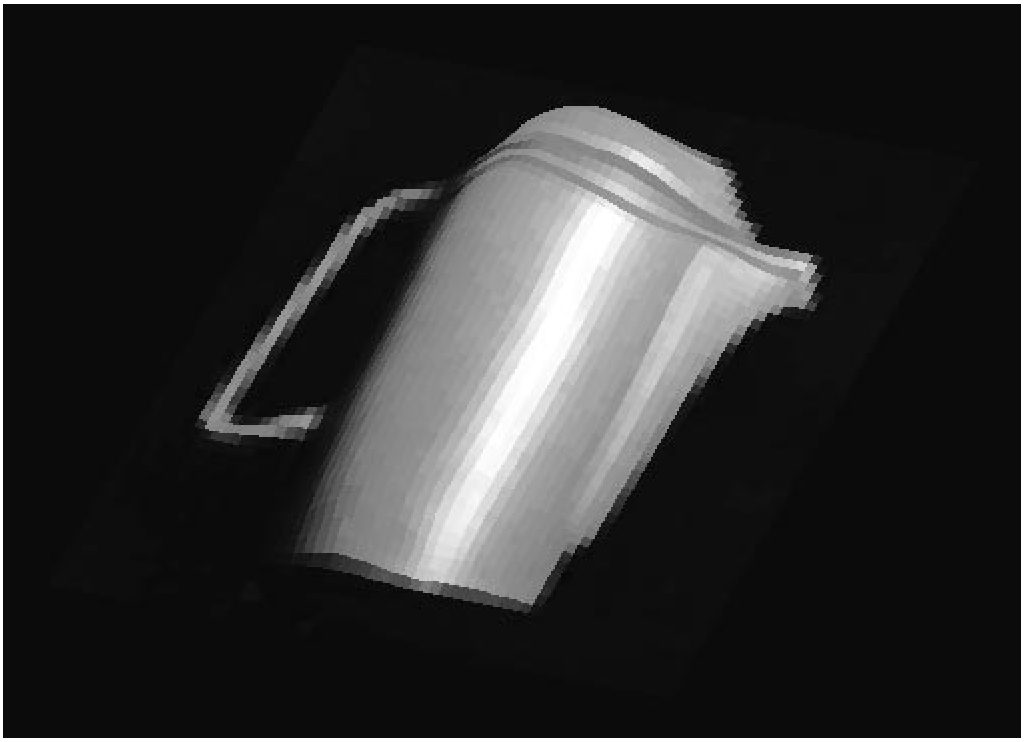


Figure 4. Experiment 2. Reconstructed surface.