Image based simulation methods for depositional systems modeling

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Abstract—In this work, we present two geostatistical methods to model geological structures that exhibit directional features in a tree structure, like fan deltas and turbidite channels. The first method is a multiple point geostatistical algorithm called directional field-based simulation (DIR-SIM). The directional feature of the training image is used to create a new object that we call training directional field (TDF), which contains the direction in each point of the image. We propose to apply this object as a fundamental tool in the simulation. The second method is an object- based simulation called SKE-SIM which uses a training image to extract the distribution of selected parameters to build the turbidite channel system. The idea is that the training image can be well represented by a one-dimensional object that we call, skeleton. 1

I. INTRODUCTION

An important problem in geology is the modeling of certain type of reservoirs whose image representation cannot be assumed as the realization of a stationary point process, however it has a well-defined geometric structure, like fan deltas and turbidites deposits.

The purpose of this work is the modeling of geologic structures, which spatial continuity can be assumed to be reflected by training images like the red channel system in the figure 1(b); these images present a particular geometry that we called **tree-like**.



Fig. 1. (a) The Wax Lake Delta. (b) The channel system highlighted in red is the training image.

In the thesis two simulation methods are proposed. The first method uses techniques from multi-point

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geostatistics. It is based on the fact that our training images can be well represented by a vector field. The second method is object-based and it originates from the idea of interpreting tree-like training images as the thickening of a graph.

Multi-point geostatistics was developed to overcome difficulties that appeared in traditional methods, for the lack of capability of representing curvilinear and continuous structures. They depend on the concept of training image as source of spatial continuity; it is a conceptual image that is assumed to contain all possible structures believed to appear in the geological body. The training image has been used to obtain statistics and conditional probabilities with the aim to reproduce them. A different perspective is the pattern reproduction approach, where the image is used as a source of patterns, that are located in the simulation area trying to provide consistency between them.

The second simulation proposed, SKE-SIM, is object-based. This type of geostatistical method is applied to geological structures that can be described by parametric geometries. The geological structure is represented by a collection of well-defined geometric objects. Such geometric objects are defined by parameters deduced from the available information. Then, the simulation proceeds sequentially creating the objects and placing them in the simulation grid until a criteria is attained.

The objective of SKE-SIM is to build a 3D model of the channel system using information extracted from a tree-like image. The 3D channel system is assumed to be located in the half-space $\{z \leq 0\}$ and the training image is thought as its projections to the plane xy. The channel system is interpreted as a thickening and deepening of a uni-dimensional object, called skeleton. The training image is used to define probability distributions from which the parameter values of the skeleton are sampled.

Problem

Turbidite deposits are generated by turbidity currents and related gravity flows. Turbidite reservoirs are distinguished by a complex structure of sand bodies arranged in channels and lobes. This kind of reservoirs still represent an important source of oil exploration in Brazil. The cost of drilling a single well can easily exceed 100 million dollars and the success rates are around 15 to 30 percent, then the risk involved in the exploitation must be determined. It is necessary an adequate representation of these reservoirs, so the main objective of this work is to present methodologies for modeling the depositional architecture of turbidite channels, creating different scenarios that are equally likely and realistic.

The turbidite channel system that will be modeled is represented by binary 2D images. Because of the particular structure of the modeled object, this work is restricted to images with a special geometry, that we called tree-like. We have not formal definition, but tree-like images should have the following features: (1) At each black point, one direction in which the channel appears to be developing can be perceived. (2) The tree-like image has a directional interval. If a point p is taken in a tree-like image, tree directions stand out, see figure 2. In the direction u the point gets away from the object in a short distance, because in this direction the channel is crossed through its width. In directions w and v the point p tends to continue for a longer distance within the object. In fact these two directions give the idea of the tendency of the channel to develop. But one of them goes against the flow of the channel. To decide which one, the small region where the point p is analyzed, has to be seen as part of to the entire channel system, see figure 2. Observing the general structure of the image, v can be chosen like the appropriate direction in the point p. (3) Like a tree, the body has a starting region O from which branches are born, ramify and continues.

The *directional interval* is defined as an interval containing the directions that are presented in the tree-like image. It can be established visually, for example in figure 2 the directions in the tree-like are inside the red cone.



II. DIRECTIONAL FIELD-BASED SIMULATION OF NON-STATIONARY GEOSTATISTICAL MODELS

In this section it is proposed a multipoint-geostatistics method using tree-like training images. This method is divided in two parts. First, due to the directional characteristics of the training images used, a directional field is obtained defining a direction at each point within the body in the training image. In the second part, this field is employed to guide the simulation algorithm using directions in a context of similarity between patterns.

A. Training directional field construction

The training images **TI** used are binary, i.e. one point is inside the channel system or not. They can be described by the function ti:

$$ti(p) = \begin{cases} 1, & \text{if } \mathbf{TI} \text{ contains sand in p} \\ 0, & \text{if } \mathbf{TI} \text{ contains no sand in p} \end{cases}$$

Motivated by the directional property of the tree-like TI, a new object called **training directional field** (**TDF**), with the directions over the reservoir, is determined. This is represented by the function tdf:

$$tdf(p) = \begin{cases} \alpha_p, & \text{if } ti(p) = 1\\ \text{ND}, & \text{if } ti(p) = 0 \end{cases}$$

 α_p is the direction of the development of the channel at point p. **ND** means there is not a defined direction. The construction of the TDF starts with a tree-like training image T_0 (figure 3(a)). To this image it is applied an erosion (operation in morphological image processing). The resulting image is given by T_1 (figure 3(c)). In the figure 3(b) the contour C_0 of T_0 is shown.

Then the erosion is applied to the image T_1 . This process, is repeated successively generating two sequences, one with the contours C_0, C_1, \ldots, C_k and another with T_0, T_1, \ldots, T_k , see the figure 3. Then, T_0 can be described as $T_0 = C_0 \cup C_1 \cup \ldots \cup C_{k-1} \cup T_k$.



Fig. 3. Sequences T_n and C_n .

Fig. 2. Three directions are perceived, v and w in the channel development direction and u in the width direction of the channel. Visual definition of the directional interval in a tree-like image.

We start defining the TDF on the contour. The direction at a point is defined using the contour to which it belongs, taking the direction that the point must follow to continue within the contour. The idea is to give at a point $p \in T_0$ a value for the slope of the tangent line to the contour curve that passes through that point.

The tangent in p will be approximated using two secants passing through p and nearby points, which are selected taking steps from p on the contour. Then, to construct the TDF, two step sizes n and m are fixed. By taking these steps from a point p, two final positions are reached, points q_n and q_m . The direction α_i of the secant passing through p and q_i is defined as the angle between the line $\overline{pq_i}$ and x axis, for i = n, m. The TDF is defined as the average of these two directions. In the figure 4(a) it can be observed the two points q_1 and q_3 taking steps of size 1 and 3 from the point p. In the point r no steps from them can be taken, so the definition of the TDF is reserved for later.



Fig. 4. (a)Steps of size 1 and 3 from point $p_{\cdot}(a)$ The TDF defined in the contour C_0 (b) TDF defined in the contours C_0, C_1, C_2 , the yellow points do not have a assigned direction yet.

The TDF is defined in the points of the successive contours C_n . The direction in the interior points, T_k , and those that belong to some C_n but do not have a direction yet, is defined by the average of the known directions around it (see figure 4(b,c)).

B. Directional field synthesis

The algorithm presented, called **DIR-SIM**, is a MPS method but instead of a training image it uses a training directional field. It can be described in two steps: (1) The TDF is preprocessed to extract and collect the patterns. (2) The patterns are used in the sequential simulation to simulate one point each time.

Pattern database

A template T with L-form is used to scan the TDF, the template is centered at each point p in the TDF, and the patterns Pat_p^T are stored. The figure 5 illustrates a template, represented by the red points. The patterns are not independent of their location, because inclination and thickness of the channels differ between regions, then together with the pattern Pat_p^T , the position of the point p(x, y) is stored.

Comparison criterion

To compare two patterns centered at p and at q, a measure of similarity is defined by:

$$d_{tdf} = \sum_{x \in T} (Pat_p^T(x) - Pat_q^T(x))^2$$

| -2 | -1 | | 0 | 1 | 2 | |
|----|----|---|----|---|---|--|
| | | | | | | |
| • | • | | 1 | • | • | |
| • | • | 1 | -2 | • | • | |

Fig. 5. Illustration of the template concept.

 d_{tdf} is the summation of the angles differences between the two patterns. In the tdf is not define ND in both positions then the difference is 0. In the case when a position in one pattern has a defined direction but the corresponding position in the other pattern does not, i.e. $Pat_p^T(x) = \text{ND}$ and $Pat_q^T(x) \neq \text{ND}$, the assigned value is $\frac{\pi}{b}$. The value of the parameter b depends on the directional interval adopted. For example, it can be chosen b such that $\frac{\pi}{b}$ corresponds to the length of the directional interval interval, this makes sense because it is the maximum value that the difference between two directions can take.

The local distance d_{loc} between the patterns centered at p and at q is the squared Euclidean distance of the points p and q. The comparison between two patterns is given by the total distance d_{total} , that is the convex combination of these two distances $d_{total} = \beta d_{tdf} + (1 - \beta) d_{loc}$ where $\beta \in [0, 1]$.

Simulation Algorithm

The algorithm starts with an empty simulation region R. The model simulates one point each time, following a unilateral path, from bottom to top and left to right starting on the left bottom corner. Some values are copy to initialize the simulation. To simulate one position q, the pattern with center q in R with template T is compared against the pattern base, looking for a similar one. The value in the center of the selected pattern is assigned to the position q. The same process is repeated for each position in R. The goal of this simulation is to ensure that the newly assigned value will maintain as much local similarity between the two patterns as possible.

C. Simulations

In this example the training image comes from a photograph of an alluvial fan in Taklimakan Desert in China, given by figure 6(a). The blue left region is the active part of the fan, it was highlighted in red and used as training image (first image in the figure 7). The size of the training image is 183 X 183. In figure 7, it can be observed the two sequences T_n and C_n , erosion was applied 2 times. In the image 9 it is shown 6 realizations using the parameter $\beta = 0.5$

III. Skeleton based simulation

In this section it is presented another approach to model turbidite channels. The method is called **skeleton based simulation**, SKE-SIM and it is an object based model and starts with a training image which is represented by a one-dimensional object called training skeleton. From this



Fig. 6. (a) Alluvial Fan, China. (b) The channel system highlighted in red.



Fig. 7. Erosion applied two times.



Fig. 8. The training directional field of the image.

new object, information about the direction and length of the channels is extracted and it is used to simulate others skeletons. These new skeletons are used to create a 3D model of channels inside a turbidite lobe.

Training Skeleton

A channel system can be approximated by a one dimensional structure, that will be called *skeleton*. It reflects the global behavior of the channel system. The skeleton is a graph in the plane formed by edges that are straight lines. The nodes represent the channel bifurcations and all skeleton has a special node corresponding to the root of the graph.



Fig. 9. Realizations obtained from the simulation using $\beta = 0.5$.

SKE-SIM starts with a 2D binary training image, as presented in figure 10(a). It is obtained one-dimensional representation of the channels by erosion, preserving the connectivity of the branches, using the image processing program called FIJI.It can be found in https://imagej.net/Fiji. The figure 10(b) is the result obtained.



Fig. 10. (a) Training image. (b) One-dimensional representation obtained by using FIJI. (c)Training skeleton

Now the skeleton is built connecting the nodes by straight lines, see figure 10(c). It will be called **training skeleton**.

Extracting information from the training skeleton

Now, two information will be extracted of the training skeleton to guide the generation of others skeletons, the bifurcation angles α and β and the length of the edges a and b (see figure 11). The information is analyzed to obtain probability distribution functions for the angles and lengths.



Fig. 11. Bifurcation angles and lengths.

Skeleton definition

A skeleton Sk is defined by two sets: the edges E (channels) and the nodes N (bifurcation or convergence

points of channels). A node $n \in N$ is an object formed: a point $p_n \in \mathbb{R}^2$, a vector $\alpha_n \in \mathbb{R}^2$ and an integer $M_n \in \mathbb{N}$. α_n contains the directions of the edges that arrive at the point p_n . M_n is the number of edges that arrive or left the point p_n , it is called the *mark* of the node. M_n is always a value in the set $\{1, 2, 3\}$.

Skeleton synthesis

The construction of the skeleton aims to mimic the channel system development. The idea is to generate a sequence of skeletons $Sk_0, Sk_1, Sk_2, \ldots, Sk_n, \ldots$

For the skeleton Sk_m , each node $n \in N_m$ with mark different to 3 is bifurcated. If the node has mark 1 then it is bifurcated in two channels, on the contrary if it has mark 2, only one channel is generated. The angle and the length of the new edge is chosen using the probability distributions extracted from the training skeleton.



Fig. 12. Skeleton bifurcation.

Given a skeleton Sk_m , its set of nodes N_m is traversed and the nodes with mark different to 3 are bifurcated generating new edges. All the resulting edges are stored in a set denoted by E_{pos} of the possible edges to be included in the new skeleton Sk_{m+1} . In the figure 12(b) the green edges form the set E_{pos} from the skeleton in the figure 12(a). This set is traversed in a random order, that in the figure 12 is given by the number at the end of each edge. Every time one edge in E_{pos} is analyzed, another skeleton is created by adding the new edge. $Sk_{m,n}$ is the skeleton obtained by inserting the edge in the position n in E_{pos} to the previous skeleton $Sk_{m,n-1}$.

The insertion process to generate $Sk_{m,n}$ from $Sk_{m,n-1}$ is not just adding the *n*-th edge, since this edge could intersect the already generated skeleton. If it has not a intersection then it is included. In the figure 12, the skeleton synthesis can be observed, in (c), (d) it is presented the case when the edge have no intersection. In the others, it is shown what happen when the edge intersect the previous skeleton.

A. 3D turbidite channels simulation

This work was mainly motivated by turbidite channels, those channels formed inside turbidite lobes. The first step in the 3D modeling will be to create the lobes, inside which the channels will be placed. It is adopted the turbidity lobe modeling proposed in the work [1]. Once the lobe has been created the skeleton synthesis is done inside it. This means that the skeleton development is constrained by the lobe boundary.

Lobe Modeling

The turbidite lobe is constructed using basically three parameters: depth, width and length l. In the figure 13(a) the general structure given to the lobe modeled and the parameters used can be observed.



Fig. 13. Lobe parameters.

The algorithm creates the lobe by first creating two regions, one in the plane xy and another in the plane xz(see figure 13(b)), so these regions should coincide with the projections of the lobe to those planes. These curve will determine the volume of the lobe, they are connected by quarters of ellipses. When all this ellipses are joined a volume is obtained. The lobe is represented in a 3D grid. To determine whether a cube in the grid is inside a lobe or not, its center is evaluated in the equations that determine the lobe.

Now, the skeleton will be used to construct a 3D model of a turbidite channels. The skeleton must be fitted into the lobe. Then the skeleton is thickened and also it is given a depth according to the lobe depth parameter. Each edge is used as the center line of a channel. The cross section orthogonal to the edge is a half-ellipse.

B. Simulations

The training image used is given by the figure 14(a). From this image, using FIJI, the training skeleton is obtained and it is illustrated in the figure 14(b).



Fig. 14. (a) Training image. (b) Training skeleton.

From the training skeleton, the distribution of the bifurcation angles and channel lengths are obtained and used to generate others skeletons. In the figure 15 a simulation of one lobe and one channel system is presented. Figures 15(a, b) show only the channel system, in (a) an upper view is shown. In the figure 15(c), the same channel system is illustrated inside the lobe. The figure 15(d) shows a cross-section, perpendicular to the x axis, of the total system, here the ellipsoidal form of the channels is appreciated.



Fig. 15. One channel system simulation.

IV. CONCLUSIONS

It was presented a non-stationary multiple point method using training images with a particular geometry, that was called tree-like. The main contribution is the introduction of a new object called training directional field TDF, defined from the training image. This field can be interpreted as the directions followed by the tree-like image. The idea is that the image being simulated should follow the TDF. This is done by including the directions in the calculation of the distance between patterns. One of the advantages of this method is that the realizations obtained by the simulation, present the same geometrical characteristics that the training image and the variability between realizations is good.

It was also present a 3D object-based modeling for turbiditic channels inside a lobe was presented. To build the channel system, information is required to determine the parameters values. Our idea is to use a tree-like image that represents the projection to the plane xyof the channel system. This is the training image and contains the basic geometry desired in the object. A linear approximation of this image is obtained, this is called training skeleton. From it, two probability distributions for the parameters values (bifurcation angles and length) are obtained. This is employed to generate new skeletons.

The skeleton is constructed sequentially. In each step, a couple of new edges emerge from each of the final nodes. The directions and length of those edges are determined using the probability distributions. Some rules are applied for especial edges or when an intersection occurs. The boundary of the projection of the lobe to the plane xy was used as constraint. Once the skeleton is simulated, the 3D channel system is created inside the lobe.

SKE-SIM is not a complex method, it does not try to mimic the detailed physical behavior of the phenomena. Nevertheless, the results are visually appealing. The small time of simulation is one of the advantage of this method. The construction of the skeleton is very fast. The main factor increasing the simulation time is the number of cells in the grid.

Another advantage of SKE-SIM is that the simulation has a well-defined geometry. This helps in computing relations between the different parts of the channel system. This can be used to insert internal properties of the rocks in the channel system, as porosity and permeability.

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