An Optical Flow Approach to Photometric Stereo

José R. A. Torreão
Germana M. Mattos

Departamento de Informática, Universidade Federal de Pernambuco
50.732-970 Recife, PE, Brasil
{jrat, gmm}@di.ufpe.br

Abstract. We introduce a new approach to photometric stereo (PS) in which the optical flow resulting from the change of illumination of the imaged surface is estimated and used for depth reconstruction through a structure-from-motion scheme. The optical flow of the PS images is determined via a gradient-based algorithm, while a least-squares strategy is used for the structure-from-motion reconstruction, assuming that the irradiance pattern over the imaged surface undergoes a purely rotational rigid-body motion, when the illumination direction is changed. Our approach to PS has been successfully applied in the estimation of shape from both synthetic and real images of lambertian surfaces.

1 Introduction

Photometric stereo (PS) is a process which employs two or more images, obtained from a single camera under different illuminations, for the estimation of shape in computer vision [6]. In the standard approach to this process, a set of image irradiance equations of the form

$$I_i(s) = R_i(p,q), \quad i \geq 2 \tag{1}$$

are solved for the surface-gradient components, p and q, at each point $s = (x,y)$ in the image plane, where p and q are given in terms of the surface height function, $z(x,y)$, as

$$p = \frac{\partial z}{\partial x}, \quad q = \frac{\partial z}{\partial y} \tag{2}$$

In equation (1), the function $R_i(p,q) \equiv R(p,q,\hat{S}_i)$ is the reflectance map function for the i-th illumination direction, $\hat{S}_i$, and $I_i(s)$ is the corresponding image intensity function.

From the observation of the fact that pairs of photometric stereo images, when viewed under a stereoscope, produce an impression of depth which can be almost as striking as that resulting from stereoscopic pairs, we have been led to the study of PS as a geometric image-matching process. We have thus found, for instance, that by matching two PS images along a fixed direction, a disparity field can be obtained which carries both surface curvature and relative depth information [3,4]. Pursuing this line of research a bit further, we have lately considered the possibility of inferring shape from an analysis of the optical flow (apparent motion of the brightness pattern) which results from the change of illumination in PS. This can be viewed as a generalization of our previous approach where the matching process is not constrained to proceed along a fixed direction.

Here we report on the preliminary results of our optical-flow treatment of photometric stereo. We have applied a standard optical flow algorithm [2] to pairs of PS images, and used the resulting flow field as input in a least-squares estimation of structure from motion [1], assuming that the irradiance pattern on the imaged surface undergoes a purely rotational displacement, when the illumination direction is changed. This has been found to produce remarkably good shape reconstructions for lambertian-reflectance surfaces.

In the next section, we describe the flow-field estimation and structure-from-motion algorithms which have been employed in our approach; following that, we present results obtained from the application of such algorithms both to synthetic and to real PS images. In the final section, we give our concluding remarks and discuss possible extensions to the present work.

2 Photometric Stereo and Optical Flow

Let us consider a pair of photometric stereo images, $I_1(s)$ and $I_2(s)$, corresponding to the illuminations $\hat{S}_1$ and $\hat{S}_2$. If those illumination vectors are not far apart, and if the imaged surface is smooth, we can attempt to match the intensities in the two images, to obtain a disparity field $D(s) = (D_X(s), D_Y(s))$, for which $I_1(s) \approx I_2(s + D(s))$ at each point s in the image plane. Employing a Taylor-series expansion on the right-hand side of such equation, it is easy to see that the photometric disparity field satisfies the constraint

$$\Delta I(s) \approx D_X(s)\frac{\partial I_2}{\partial x} + D_Y(s)\frac{\partial I_2}{\partial y} \qquad (3)$$

which is the standard optical flow equation [1], with $\Delta I(s) \equiv I_1(s) - I_2(s)$ playing the role of the time derivative of the image intensity, and with the disparity vector playing the role of the flow velocity.

An optical flow algorithm, such as the one proposed by Horn and Schunk [2], can thus be employed for the estimation of the field $D(s)$, which will be henceforth referred to as the flow field. The algorithm in [2] introduces a functional $E \equiv E\{D(s)\}$ which is to be minimized to yield the flow estimates. Such functional is proposed in the form $E = E_S + \lambda E_C$, with λ as a parameter controlling the balance between the two terms, and with

$$E_S = \int\int \left[\left(\frac{\partial D_X}{\partial x}\right)^2 + \left(\frac{\partial D_X}{\partial y}\right)^2\right] + \\ + \left[\left(\frac{\partial D_Y}{\partial x}\right)^2 + \left(\frac{\partial D_Y}{\partial y}\right)^2\right] dxdy$$

and

$$E_C = \int\int \left[D_X\left(\frac{\partial I_2}{\partial x}\right) + D_Y\left(\frac{\partial I_2}{\partial y}\right) - \Delta I\right]^2 dxdy$$

measuring the departure of a given flow field from the smoothness and the optical-flow constraints, respectively. The measure $E\{D(s)\}$, discretized over the image grid, can be minimized through an iterative algorithm, as described in [1,2].

Once an estimate of the flow field has been obtained, a structure-from-motion scheme can be employed for reconstructing the shape of the imaged surface. For this purpose, an assumption has to be made as to the kind of 3-D motion that is associated with the change in illumination direction. We propose that, for surfaces with predominantly lambertian (i.e., diffuse) reflection, and for small changes in illumination, it is reasonable to assume that the irradiance pattern over the imaged surface undergoes a purely rotational rigid-body motion. Thus, the 3-D displacement of a given intensity element on the surface can be expressed as

$$\Delta R = \Theta \times R \qquad (4)$$

where $\Theta = (A, B, C)^T$ and $R = (x, y, z)^T$ are the rotation vector and the position vector, respectively, given in a coordinate system fixed with respect to the camera and with the $-z$ direction pointing along the optical axis. From (4), we obtain

$$\Delta x = Bz - Cy,\ \ \Delta y = Cx - Az,\ \ \Delta z = Ay - Bx \qquad (5)$$

Now, since we are assuming that the images are captured through orthographic projection, we would ideally expect to have $\Delta x = D_X$ and $\Delta y = D_Y$, i.e., the x and y components of the 3-D displacement vector would be given by the flow field. In this case, the first two equations in (5) would yield relations between the unknown z coordinate and the rotation-vector components, since x and y denote measured positions (pixel sites) in the image plane, and the flow (D_X, D_Y) has been estimated.

Considering that the estimation of the flow field is subject to errors, though, it is more convenient, instead of using such local approach, to try to obtain $z \equiv z(x, y)$ by a global least-squares strategy. We thus set out to minimize the functional

$$\int\int F(x, y)dxdy \qquad (6)$$

with $F(x, y) = [D_X - Bz(x, y) + Cy]^2 + [D_Y - Cx + Az(x, y)]^2$, which, by taking into account data available throughout the whole image, makes for a more robust estimation of $z(x, y)$.

Minimizing $F(x, y)$ with respect to $z(x, y)$, we obtain

$$z(x, y) = \frac{BD_X - AD_Y + C(Ax + By)}{A^2 + B^2} \qquad (7)$$

which gives the surface-height function at each point in the image plane in terms of the measured flow field and the unknown rotation parameters, A, B and C.

In order to estimate the rotation, we should go on to minimize (6) with respect to A, B and C. Before doing so, we can simplify the problem by taking advantage of the fact that in photometric stereo the illumination is controlled and we can thus choose illumination direction pairs for which certain components of the rotation vector are negligible. For instance, it is not difficult to verify that, for a change in illumination along the xz or yz planes, a rotation of the irradiance pattern about the z axis is most unlikely. We therefore expect the third component of the rotation vector to be $C = 0$, in such cases.

Thus restricting ourselves to rotations of the form $\Theta = (A, B, 0)$, we obtain, from the minimization of (6) with respect to A, the following relation between the rotation parameters:

$$A = \gamma B \qquad (8)$$

with

$$\gamma = \frac{\alpha \pm \sqrt{\alpha^2 + 4\beta^2}}{2\beta} \tag{9}$$

where

$$\alpha = \int\int [D_X^2 - D_Y^2] dxdy \text{ and } \beta = \int\int [D_X D_Y] dxdy$$

Equation (7) thus becomes

$$z(x, y) = \frac{1}{B}\left[\frac{D_X - \gamma D_Y}{\gamma^2 + 1}\right] \tag{10}$$

Since an independent estimate of B cannot be obtained via the structure-from-motion scheme, this parameter must be determined by some other means. By comparing equation (10) to results derived in our previous work with the geometric approach to PS [3,4,5], we have found it reasonable to employ for B, as a first approximation, the value of the rotation which the illumination vector undergoes when the PS pair is produced. With such approximation, equation (10) has been used for shape reconstruction from synthetic and real input images, and the obtained results have been found to corroborate the validity of our assumptions, at least for lambertian surfaces. In the following section, we present some of our results.

3 Experiments

Figures 1 to 3 illustrate shape reconstruction experiments with our optical flow approach to photometric stereo. Letters (a) and (b) in the figures show the PS input images, while letters (c) depict the estimated depth maps.

The first two experiments deal with synthetic images of lambertian surfaces: a sphere of radius 0.35 and an ellipsoid with x, y and z dimensions 0.25, 0.35 and 0.25, respectively. The third experiment deals with the real image of a vase, also of approximately lambertian reflectance. In all the experiments, the first image was obtained with central illumination ($\hat{S}_1 = (0, 0, 1)$), and the second one with an illumination vector rotated 10 degrees about the z axis ($\hat{S}_2 = (0.174, 0, 0.985)$). For the parameter B in equation (10), the value employed has thus been $B = 0.1745$, which correponds to the rotation of the illumination vector, in radians.

As can be observed from the figures, the reconstructions obtained are remarkably good, inspite of the many simplifying assumptions that have been made in the formulation of the problem. It is thus fair to surmise that such assumptions are not unreasonable, insofar as lambertian surfaces are considered. The main shortcoming of our approach seems to be a tendency to underestimate depth in regions where the gradient of the image intensities is small, as for instance on top of the sphere. This problem arises from the estimation of the flow field with Horn and Schunk's algorithm, since its iterative procedure is driven by the brightness gradient and does not always produce the correct results when such gradient is small [1].

4 Concluding Remarks

A new approach to photometric stereo has been proposed, whereby the flow of intensities resulting from the change of scene illumination is estimated and used in a structure-from-motion scheme for shape reconstruction. Our approach assumes that the irradiance pattern on the surface undergoes a purely rotational rigid-body motion, an approximation which, although very simple, has yielded good results for shape estimation from synthetic and real images of lambertian surfaces.

We are presently considering the extension of our framework to incorporate the possibility of deformable motion, which should allow the treatment of surfaces with more complex reflectance maps.

5 Acknowledgments

We thank Leopoldo França Neto and Mário Guimarães Neto for their expert help. JRAT also thanks Jonas de Miranda Gomes for a summer scholarship at IMPA, where part of the present work was done.

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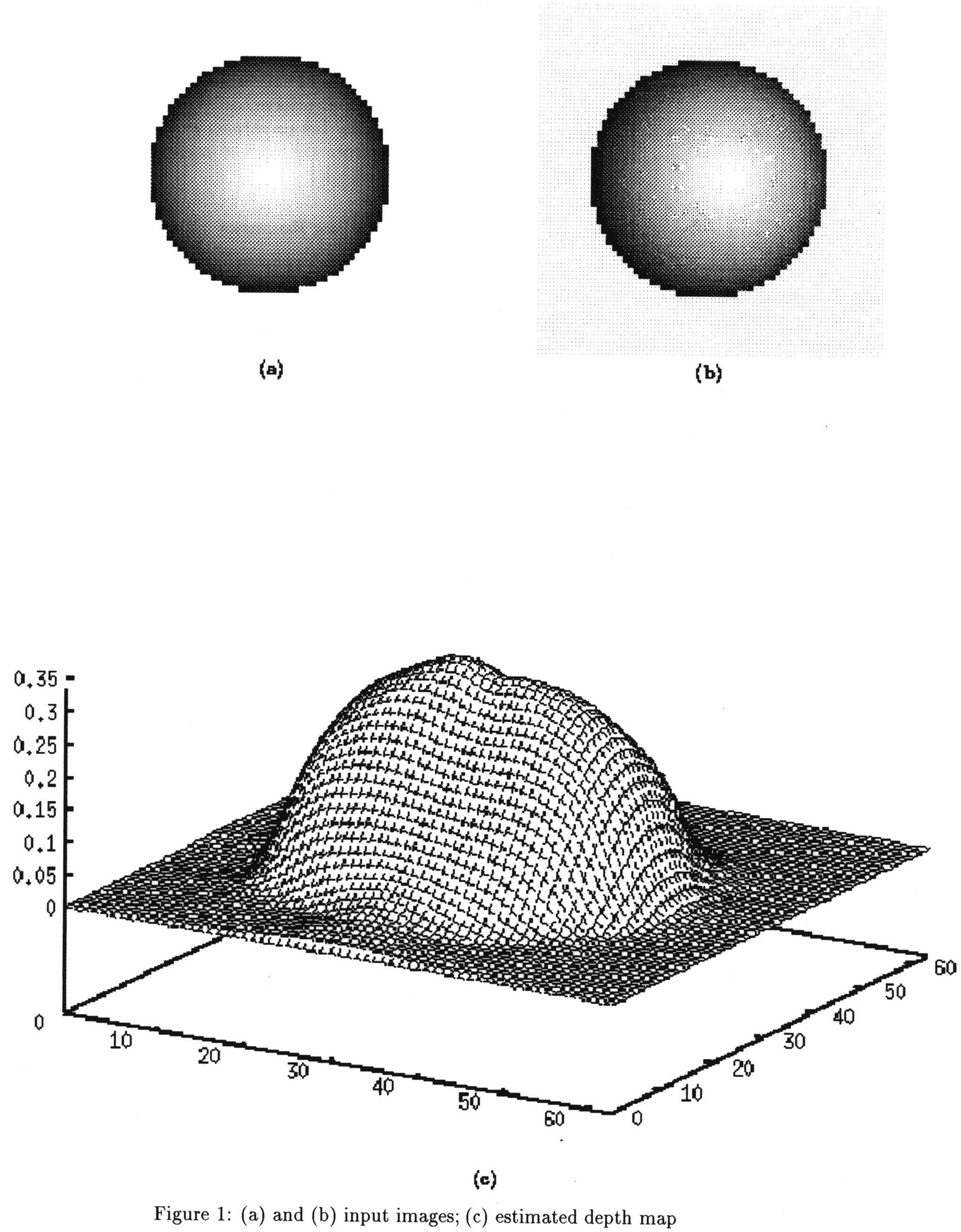


Figure 1: (a) and (b) input images; (c) estimated depth map

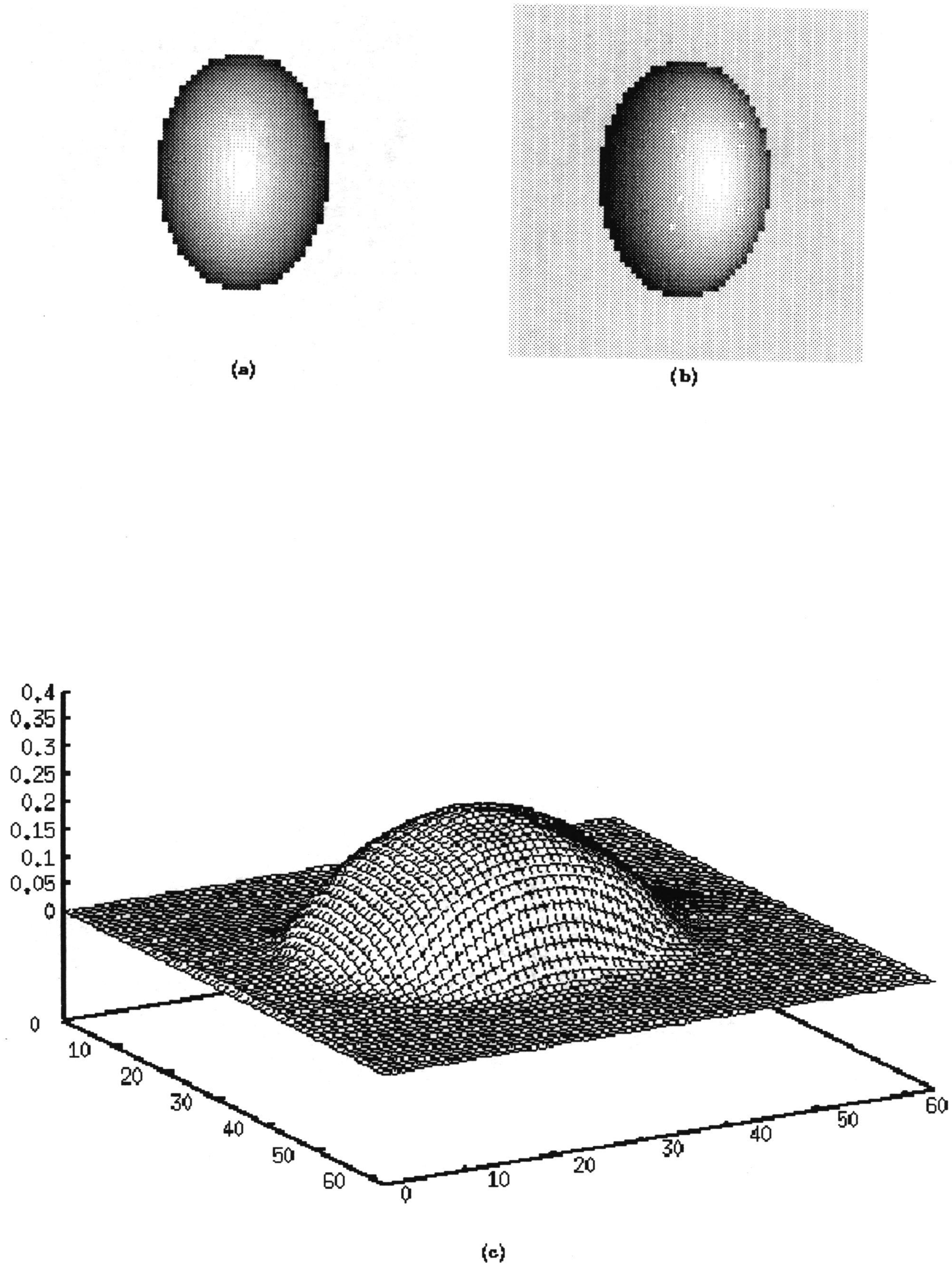
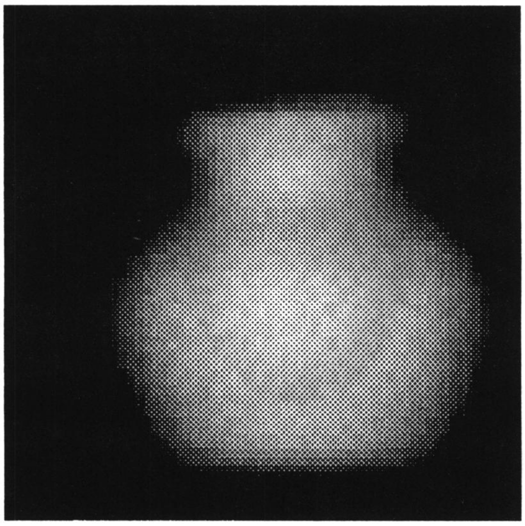
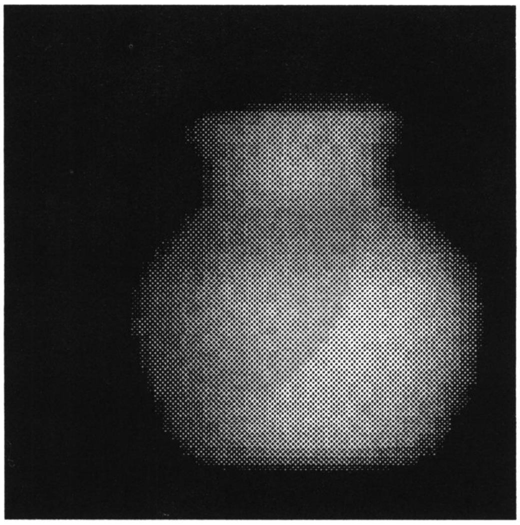


Figure 2: (a) and (b) input images; (c) estimated depth map



(a)



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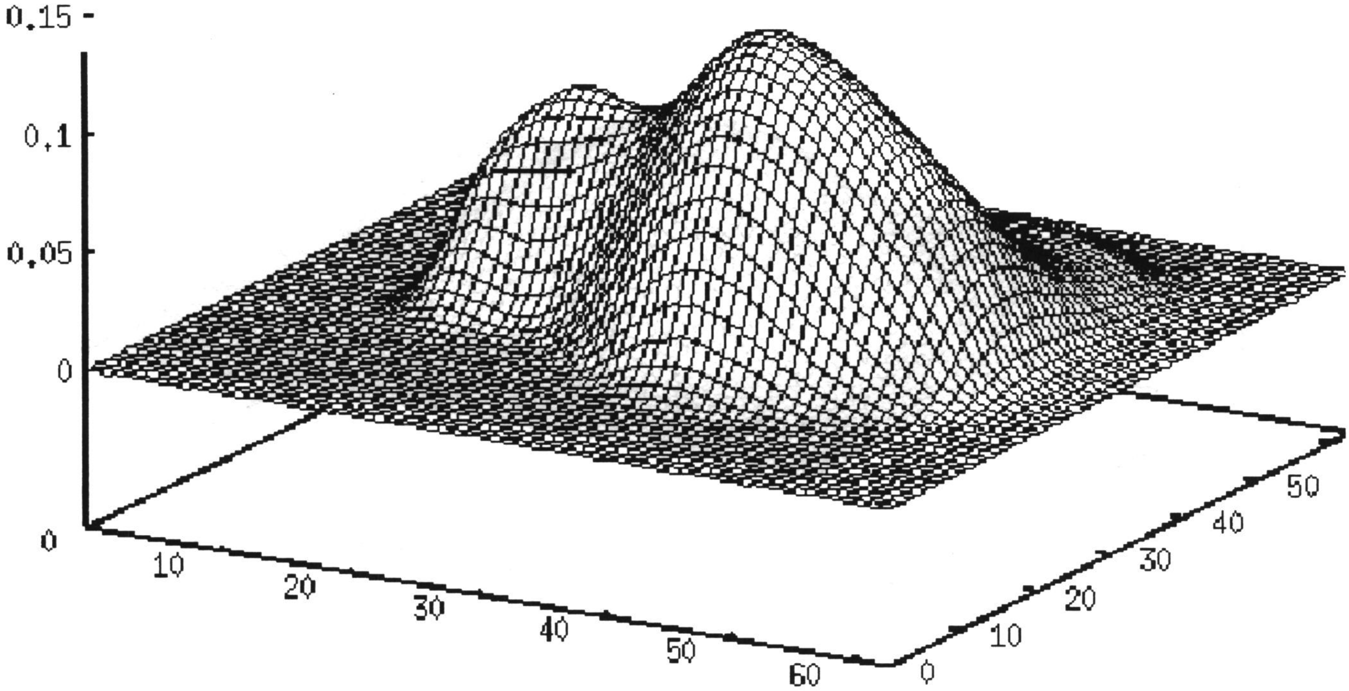

(c)

Figure 3: (a) and (b) input images; (c) estimated depth map