FINITE ELEMENTS APPLIED TO IMAGE SYNTHESIS AND THE SIMULATION OF NATURAL ENVIRONMENTS : SUMMARY OF A RESEARCH PROJECT

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Abstract : The present text describes briefly a long-term research project regarding the application of the finite element method to image synthesis and the simulation of natural environments. It reports new research and compatibly reviews emerging trends in the field. Various approaches are introduced and future works identified.

Keywords : Finite Element Method, Computer Graphics, Image Synthesis, Realism, Natural Scenes and Environments, Surface Modeling.

INTRODUCTION

This summary, following previous studies [Santo87a, Santo87b, Santo90, Santo91], reports briefly the first stages of a long-term research project which aims at developing finite element formulations for attacking specific problems of computer graphics (CG) and image synthesis (IS).

A new and promising line of research is therefore introduced in this text, while emerging trends in the field are implicitly reviewed and future developments identified. Various approaches, that represent an integrating framework, are briefly and compatibly described. This paper should be viewed in conjunction with a forthcoming, more elaborate survey [Santo92].

APPLICATIONS OF THE FINITE ELEMENT METHOD IN IMAGE SYNTHESIS

The finite element method (FEM) is undoubtedly the most versatile, powerful and widely used numerical method of analysis, which, much too inevitably, had to enter also the computer graphics field. This occurrence could only take place, however, after CG had grown to more elaborate, complex, advanced applications, revealing the need for the respective tools and techniques. It took therefore almost 30 years for CG to "discover" the FEM, after the method was formally christened [Clough60].

The first uses of the FEM within CG/IS can be traced from the works of A.H. Barr and others, in the development of physically-based models [e.g., Platt-Barr88, Barr88, Terzopoulos-Fleischer88], in the wake of finite difference studies [Terzopoulos-Platt-Barr-Fleischer87]. It was just natural that the FEM found its use in this area for engineering science has long-established theories and methods to solve problems of dynamics, elasticity, deformation, viscosity, etc, of interest to the corresponding CG applications.

In a similar, parallel endeavor, the FEM met at last its useful contribution to computer animation [Gourret88, Gourret-Magnenat.Thalmann-Thalmann89b], namely in applications where the method is particularly strong, e.g., the analysis of tension and deformation, suitable for solving hand-grasping problems [Gourret-Magnenat. Thalmann-Thalmann89a].

Many other areas of IS can benefit enormously from the FEM and other engineering tools. Techniques for photographic realism, simulation of texture, waves and natural scenes and phenomena in general, as well as more sophisticated approaches to surface and terrain modeling, are just a few of the innumerable possibilities. It is a fundamental object of the research project presented herein to deal properly with these and other CG aspects, as future works will report.

A PARADIGM : THE GENERATION OF SURFACES

The application of the finite element method (FEM) to the generation of curves and surfaces, has been considered since the sixties [e.g., Throsby69, Zienkiewicz-Phillips71]. In particular, the likenesses and differences between the discretization techniques of Steven A. Coons and the FEM, have been detected and discussed at length [Zienkiewicz71, Zienkiewicz77], with many efforts towards the concatenation of both lines of action being made. Indeed, a careful analysis allows us to recognize the analogy between the "Coons patches" and the isoparametric mapping concept of the FEM and, until today, this issue has deserved the attention of many researchers, although plainly satisfactory and practical results have not yet appeared.

The utilization of the FEM in the generation of surfaces follows two main approaches, both seen as interpolation techniques, according to the standard 'Computer Aided Geometric Design' classification [Santo87b] : the isoparametric mapping and the thin plate bending analogy. Recently, a more advanced, energy-based model has been developed which combines the more attractive features of the FEM with the modeling of deformable and free-form shapes, of interest in so many applications [Celnicker-Gossard91].

Isoparametric Forms

The isoparametric mapping concept constitutes the fundamental basis of finite element modeling. This classic formulation is found in the cited references and in FEM textbooks, and will not be elaborated on here.

3D surface elements generated by this technique should be viewed as macro-elements that can be generalized to other element types and irregular domains. The juxtaposition of various macro-elements forms the desired surface. This situation reveals the analogy with the Coons (or other parametric surface) patches : the initial subdivision of the domain in macro-elements, to which transformations are applied, generates what we can call a "multi-patch".

Plate Bending Analogy

The present case is fundamentally related to smooth surface fitting. In its simplest form, it can be described analytically in terms of the minimization of the integral, from the Lagrange Equation, plane stress state [Throsby69]:

$$\iint (w^2{}_{xx} + 2w^2{}_{xy} + w^2{}_{yy})dxdy,$$

i.e., the minimization of the total elastic strain energy of bending and torsion of a thin flat plate. Geometrically the problem is expressed as the minimization of the surface curvature, such as the first and second derivatives are continuous in the whole domain. The solution follows the standard FEM techniques.

Also in the study of plates and shells, recent works have appeared combining the Coons and the FEM techniques, such as the ones reported in Zhaobei-Zhiqiang87. In Lancaster-Salkauskas86 and Williams87 the concepts and methods related to the present case are summarized.

Representation of Natural Surfaces

The generation and computer representation of "natural" surfaces find useful application in several different areas, with special relevance to Civil Engineering, Architecture and Topography, in the particular context of Terrain Modeling, a sub-domain of Surface or Geometric Modeling. Until recently only classical methods have been applied, like the ones based on polygon discretization, triangulation, mesh generation, strings and contour lines. Other advanced approaches include smooth surface fitting, interpolation, reconstruction from contour lines and use of B-splines.

Fractal modeling seems to be best technique to date for handling the problem of simulating natural environments, terrains in particular, a well-known fact, sampled by countless works and applications. In Santo87b, a fractal-based scheme is derived and one application is developed in the context of Civil Engineering.

Many other approaches exist for the representation of natural surfaces but most of the works lean on some form of stochastic modeling for obvious reasons. Therefore, the so-called stochastic finite element method appears to be a natural, powerful and promising tool for tackling the problem of the generation, simulation, representation and visualization of natural surfaces, particularly competitive with fractal modeling and other analogous processes, as discussed below.

THE STOCHASTIC FINITE ELEMENT METHOD

The advent of the stochastic finite element method (SFEM), one of the more recent developments in the finite element domain, which is receiving lately increasing attention, has been motivated by the necessity that engineering has to deal with non-deterministic events or variables.

Since the FEM can be viewed as a numerical technique to solve differential equations, employing the method in combination with stochastic differential equations, results in the main SFEM formulation, which can be thus briefly described : partial differential equations, representing a certain system at a time *t*, are discretized and solved by the FEM yielding the initial state at instant *t0*. Integration in time, accomplished through the utilization of a stochastic differential equation solution method, gives the desired values of the respective unknowns. Typical problems include earthquake simulation, analysis of structures with random material properties, corrosion and wave propagation.

A simpler, more direct approach considers the stochastic fluctuation of the nodal coordinates in the FE displacement method. The basic steps for deriving a FE solution are [Zienkiewicz77] :

1 - Subdivision of the continuum into (finite) elements. This is performed "by hand" or, more appropriately, by compatible mesh generators.

2 - Evaluation of element stiffness and load terms. The element stiffness matrix is given by

$$K^e = \int_{V_e} B^T D B \, dV \text{ , with}$$

V_e - element volume

B - strain matrix, generally composed of derivatives of the shape functions

D - elasticity matrix, which contains the appropriate material properties. Relates the stresses to the strains : $s = D e$.

The load or nodal forces term can be expressed by

$F^e = \int_{V_e} N^T\, p\, dV + \int_{S_e} N^T\, q\, dS$, with

S_e - loaded element surface area
N - set of interpolation or shape functions
p - body forces per unit volume
q - applied surface tractions.

3 - Assembly of element stiffness and load terms into a global stiffness matrix and load vector. This is done mainly from the element connectivity or overall topology defined in step 1.
4 - Solution of the resulting system of linear simultaneous equations (unknowns = nodal variables). After accounting for the boundary conditions, a system of linear simultaneous equations is assembled. An adequate solution method is employed, e.g., Gauss elimination, frontal, profile, etc. The results are the nodal displacements, in the present synthesis.
5 - Post-processing. This refers to the calculation of quantities derived from the solution obtained in the former step, such as stresses and strains. Of course CG provides an invaluable help at this stage. Any serious FE system possesses powerful graphical capabilities, for obvious reasons.

Into the above standard steps, an "uncertainty" factor is included, consisting of a stochastic term to be added to the nodal coordinates. Since the displacements within an element are given in terms of the nodal points through the element shape functions, $d^e = \sum N_i d_i$, i=1,{no. of nodes of the element}, this can be expanded to account for a stochastic variation, i.e.

$d^e = d^o + \sum d_i a_i$, with d^o - deterministic part
a_i - stochastic part.

The same can be applied to the strain matrix B, the elasticity matrix D and the equivalent load term F^e, by adding stochastic terms as appropriate. Performing step 4 (solution), then, will yield a "random" pattern of nodal displacements.

APPROACHES TO SURFACE GENERATION

The techniques and theories of the SFEM, making it an ideal tool to attack problems involving "randomness", or that can be modeled by stochastic processes, can be directly extended to the applications of CG, Computer Aided Design and Computer Aided Geometric Design. The obvious case is the representation of natural environments or the simulation of natural phenomena.

Just as firstly reviewed in Santo87b, the SFEM can be particularly employed in the enhancement of graphical representations of terrain, and other Civil Engineering and Architectural applications.

Of course SFEM use is not restricted to surface modeling, but can encompass other situations that fractal methods deal with, as long as a compatible stochastic model is available, e.g., sea wave movements, texturing and, particularly, the simulation of earthquakes, fluid turbulence and wind action on structures, topics of extreme importance in the engineering field.

"Stochastic" Mesh Generation

One easy way to generate a natural-looking surface is by employing FE mesh generators. FE mesh generation is a very advanced field which dates back to the sixties, having reached a high level of sophistication.

The idea here is to add a "displacement factor" to the generated nodes, by use of a suitable stochastic distribution. The surface so obtained will exhibit a "random" characteristic, not unlike the fractal counterparts. Many other techniques can be utilized by applying "stochastic factors" within the own mesh generation algorithms.

"Stochastic" Triangulation

Another possible process is that of performing a FE based triangulation over a previously generated stochastic distribution of points. Of course the final result will not differ much from the one obtained by the previous approach. A stochastic factor can also be incorporated into the own specific triangulation algorithm and, here, certainly a wealth of possibilities exists, which deserves a thorough investigation.

Flexible Thin Plate Analysis

A true application of the SFEM can be achieved in the analysis of the bending of a flexible thin plate. Several cases can be considered, each with its own advantage and disadvantage, difficulty, appropriateness and characteristic.

Stochastic shape functions

As mentioned above, it is possible to include a stochastic parameter in various ways in the FE problem formulation, one of them being some kind of "stochastic" shape functions, which will certainly result in a final random-shaped form of the thin plate analyzed.

Stochastic load distribution

Another possibility is to apply a stochastically generated load distribution to the plate, performing the FE analysis afterwards. It is necessary to operate in the nodal forces term, adding a stochastic factor. There are many variations of this process, which also will need further experiments.

Stochastically imposed element thickness

Leaving the strict thin plate case aside, we can impose stochastically determined element thicknesses, which will also lead to "randomly" distributed nodal displacements.

Free and random vibration

This seems to be the more promising line of action. The flexural vibration of plates (and shells) is a very well studied and documented subject. The SFEM formulation can logically be extended to this case, although the question is obviously more complex. It is perhaps the best path to pursue, despite the clear less efficiency. The combination of the various "tricks" above with the vibration analysis of a plate, provides powerful tools, that nevertheless require a lot more research. Specifically, the proper selection of vibration modes, to suit particular situations, is a whole demanding object of trials and research.

Other Possibilities

Of course the simultaneous combination of the above approaches, in several different ways, gives room to many other possibilities that have to be investigated. Also, the direct application of stochastic differential equations and the stochastic fluctuation of the nodal coordinates, are especially suitable for thin plate analysis.

FURTHER RESEARCH

The first steps of a long-term research project regarding the development of new techniques for the generation and representation of natural environments, with special emphasis on surface generation, has been reported. From the above discussions, several lines of deeper investigation can be pinpointed, with special attention to the thin plate vibration case. But all the possible approaches described, namely the "stochastic" mesh generation and triangulation, will be subject of future research.

The present project is not limited to surface and terrain generation, nor to the SFEM. Many other areas of CG/IS will be dealt with by appropriate FEM models and techniques. Realistic image generation, simulation of texture, waves and natural scenes and phenomena in general, will be object of intensive research and publications.

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