Corner detection within a multiscale framework

Iális C. Paula Jr, Fátima N. S. Medeiros
Depto. de Eng. de Teleinformática
Universidade Federal do Ceará (UFC)
Fortaleza, CE, Brazil
Emails: {ialis,fsombra}@ufc.br

Francisco N. Bezerra
Inst. Fed. de Educação, Ciência e Tecnologia
Maracanaú, CE, Brazil
Email: nivando@ifce.edu.br

Daniela M. Ushizima
Math and Visualization Groups
Lawrence Berkeley National Lab.
Berkeley, CA, USA
Email: dushizima@lbl.gov

***Abstract*—We present a new multiscale method for corner detection. The proposed algorithm embodies an undecimated wavelet decomposition of the angulation signal of a shape contour to identify significant points on it. It detects peaks that persist through several scales from the correlation signal between scales of its non-orthogonal sub-band decompositions. These peaks correspond to high curvature points (HCPs). Furthermore, we compare the proposed method with others available in the literature, including the well-known curvature scale-space (CSS) method. The quantitative assessment of the algorithms is provided by some figures of merit (FOM) measures that indicate which method better detects the relevant points in terms of compaction and shape reconstruction.**

***Keywords*-corner detection; high curvature points (HCP); shape reconstruction; curvature space-scale (CSS).**

I. Introduction

In the context of shape analysis and recognition, the topic of shape representation presents several works about corner and dominant point detectors by contour analysis. Corner detection is an important task in computer vision and image processing purposes [1]–[4]. Applications that rely on corners include scene analysis, feature matching, robot navigation, shape similarity and object tracking, among many others. One important class of edge detection models often suppose that corners correspond to the high curvature points (HCP) of an object profile. Basically, high curvature points in images represent useful information as they enable the identification of representative features in many shapes and can be detected by contour analysis. A large number of corner detectors have been proposed in the literature [5]–[8]. The current techniques can be broadly categorized into two groups named as intensity-based [6], [7], [9], [10] and contour-based detection [6], [11]–[14] methods. The former includes the algorithms that estimate a measure to indicate the presence of a corner directly from the image gray values. The latter consists of the methods that prior recover image contours and then search for curvature maxima or inflection points along these contours [15]. The proposed method belongs to the latter group since our database consists of shapes that we first extract the respective contours. Corner detectors can also be classified into two approaches: single-scale detectors [16] and multiscale detectors [4], [7], [11]. Single-scale detectors work well only if the image has similar size features; otherwise either fine or coarse scale features are poorly detected. Multiscale corner detectors based on the classical scale-space theory [17] have been proposed to improve the effectiveness in the more general situation of relevant features with various sizes.

This paper introduces a new method for shape corner detection and it can be classified as a contour-based multi-scale category. The method starts decomposing the contour orientation function which is obtained via the original image chain into scales of different details. Thus, local maxima and minima of consecutive scales of this function are obtained through a multiscale wavelet decomposition [5], [6], [11] to identify the shape dominant points. In the category of contour based multiscale methods, there are several corner detectors [7], [8], [15], [18]–[21] available with specific design for different applications. The proposed method is assessed by using the reconstruction error and compacting ratio measures and compared with three methods [8], [11], [12], [22] including the well known curvature scale-space (CSS).

This paper makes two main contributions, first in the idea underlying inter-scale correlation to detect dominant points in planar curves, and secondly presenting new criteria to evaluate the performance of corner detectors. The methods directly related to our work are reviewed in Section II. Section III describes the proposed corner detection technique. The evaluation metrics described in Section IV are used to compare the results of the methods in Section V. Section VI concludes this paper.

II. Related Methods

In this section, we present three methods available in the literature and named as Lee *et al.* [11], Marji & Siy [12] and CSS [17]. The main reason for choosing the classical well-known CSS and Lee *et al.* methods in this study is the multiscale nature of them both, and therefore are suitable to be compared with the proposed corner detector. Regarding the multiscale method introduced by Lee *et al.* [11] it uses information of local extrema and modulus of a threescale wavelet decomposition of the angulation signal to detect corners. In fact, to evaluate dominant points detectors, we have also included the Marji & Siy technique. This last method also proposes an evaluation measure which is applied in Section IV. The performance assessment of the methods relies on four measures such as the number of detected corners, integral square error (ISE), compacting ratio (CR) and figures of merit.

A. The Curvature Space-Scale Technique

One of the most popular multiscale curvature representation of 2D curves is the curvature scale-space [17], which has been improved and applied in different works [7], [8], [14], [15], [18]–[21], [23].

The CSS technique is suitable for recovering invariant geometric features, curvature zero-crossing points [17] and/or extrema [19], of a planar curve at multiple scales. To compute it, the curve Γ is first parameterized by the contour point t:

$$\Gamma(t) = (x(t), y(t)). \quad (1)$$

An evolved version Γ_σ of Γ is defined by [18] as:

$$\Gamma_\sigma = (X(t,\sigma), Y(t,\sigma)), \quad (2)$$

where

$$X(t,\sigma) = x(t) \otimes g(t,\sigma) \; ; Y(t,\sigma) = y(t) \otimes g(t,\sigma). \quad (3)$$

The symbol $\otimes$ refers to the convolution operator and $g(t,\sigma)$ denotes a Gaussian of width σ. Note that σ describes the *scale* parameter. The process of generating evolved versions of Γ as σ increases from zero to infinity (∞) denotes the evolution of Γ. This technique is suitable for removing noise and smoothing a planar curve as well as gradual simplification of its shape. In order to find curvature zero-crossings or extrema from evolved versions of the input curve, it is necessary to accurately compute curvature, k, on an evolved version Γ_σ. Curvature k on Γ_σ is given by [18]:

$$k(t,\sigma) = \frac{X_t(t,\sigma)Y_{tt}(t,\sigma) - X_{tt}(t,\sigma)Y_t(t,\sigma)}{\sqrt[3]{(X_t(t,\sigma)^2 + Y_t(t,\sigma)^2)}}, \quad (4)$$

where

$$X_t(t,\sigma) = \frac{\partial}{\partial t}(x(t) \otimes g(t,\sigma)) = x(t) \otimes g_t(t,\sigma), \quad (5)$$

$$X_{tt}(t,\sigma) = \frac{\partial^2}{\partial t^2}(x(t) \otimes g_t(t,\sigma)) = x(t) \otimes g_{tt}(t,\sigma), \quad (6)$$

Similarly, $Y_t(t,\sigma)$ and $Y_{tt}(t,\sigma)$ can be defined by following the expressions in Equation (5) and Equation (6). The literature reports that this technique is applied to corner detection and to other several applications [20], [21]. This algorithm designed by Mokhtarian and Suomela [8] requires the Canny edge operator applied to the gray level image to obtain a binary edge image. In the sequence, the algorithm fills the gaps and finds the T-junctions of the operator output. Thus, it computes the curvature at a high scale σ_{high} for each edge contour. The local maxima of curvature are chosen to be the initial corners whose absolute curvature are above a threshold th and twice as much as one of the neighboring local minima. The CSS method identifies a corner on the evaluated contour and the edge extraction algorithm marks a T-corner at the end of the other contour. In the sequence, the algorithm searches for the points marked by the edge-extraction step. Thus, the T-junction corners are removed when compared to the corners from curvature and if they are very close to each other. The Canny detector step is unnecessary when applying this method in segmented (binary) shapes. In addition to other inherent parameters of the CSS technique, σ_{high} defines the scale for which corners are chosen. In [8] it is suggested $\sigma_{high} = 4.0$.

B. Lee et al. Technique

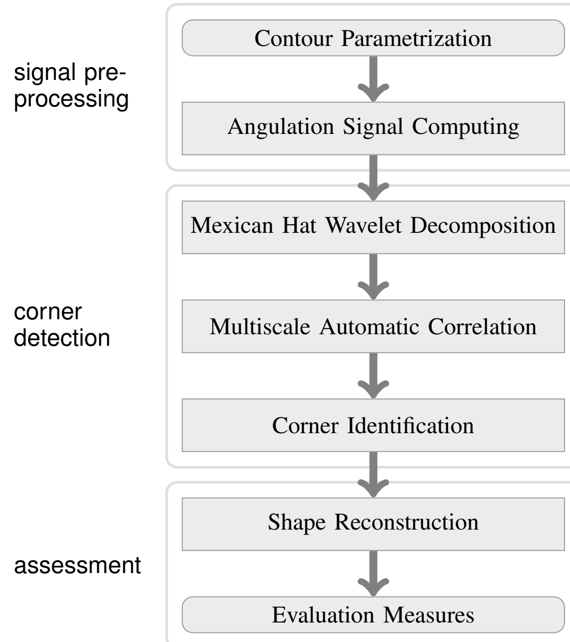


Fig. 1. The methodology for dominant points detection.

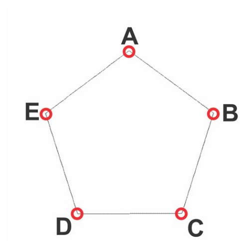
Lee *et al.* presented in [11] an analysis of the behavior of the wavelet transform modulus maxima with different corner and arcs models. These authors also used the second derivative of the Gaussian as the wavelet function to develop a multi-scale corner detection algorithm based on wavelet decomposition of contour orientation.

Corners and arcs are relative terms and largely depend upon the shape of the object under consideration. If there are sharp corners at the boundary, smooth curvature changes will not be recognized as corners. On the other hand, if it consists only of smooth curvature changes then these curvature changes will be recognized as corner points. This approach exposes the parameterized corner models as disadvantages in order to identify the adequate set of corners for any shape.

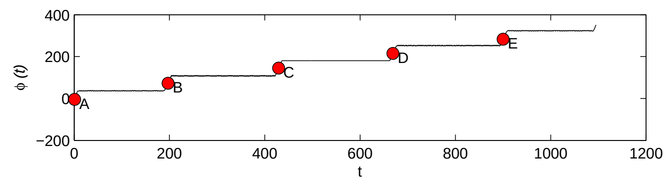
C. Marji & Siy Technique

Polygonal approximation is a mechanism to achieve feature extraction due to the simplicity of the shape representation that in turn allows for simple and efficient algorithms to recognize patterns and features. Most existing algorithms achieve shape representation by finding corner points and connecting them with line segments [24].

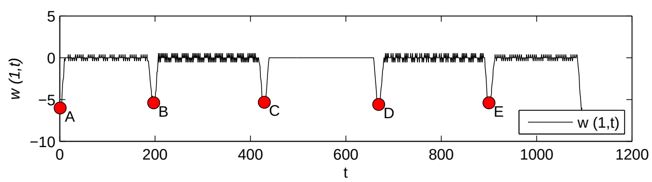
The corner points are also known as dominant points because they dominate the shape representation. This was originally discussed by Attneave [25] who concluded that these high curvature points dominate human perception of shapes and can therefore be used as a good characterization of a contour.



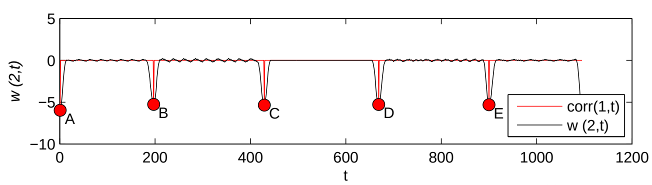
(a) Original shape and its ground-truth corners (red circles).



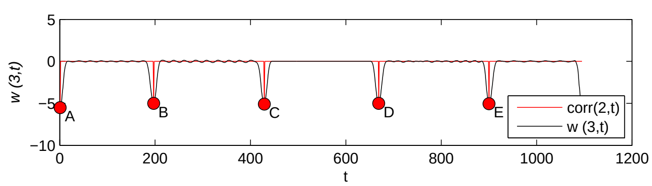
(b) Angulation signal with its highlighted corners (red marks).



(c) Highlighted corners into the first level of detail coefficients.



(d) Highlighted corners into the second level of detail coefficients.



(e) Highlighted corners into the third level of detail coefficients.

Fig. 2. Correlation analysis between wavelet coefficients of different scales of a shape contour.

Marji & Siy implemented an algorithm [12] that determines the region of support automatically for the detection of dominant points. The approach has to be competitive to return results close to the ground truth to each shape and it finds a small number of false positives. However, their results return higher error reconstruction values which, in spite of containing dominant points, do not conform to human perceptions.

III. Methodology

Corners are local features and wavelets are suitable to evaluate significant features that persist over several scales. Attempting to identify corner candidates, inter-scale correlation by using Mexican hat wavelet decomposition searches for candidates which occur where there are maxima in the wavelet decomposition at several adjacent scales. We have chosen the Mexican hat for identifying changes in non-stationary angulation signals due to the fact that it is a real function and it is convenient to identify local maxima and minima. Fig. 1 summarizes the proposed methodology for multi-scale corner detection. The signal acquisition module from the methodology consists of the contour extraction of a segmented shape. As there are various ways of representing a shape contour by signals, we adopt to generate the contour signal which follows the 4-directional chain code [3]: such method starts from an initial point of contour and follows it in a clockwise manner. For each point, the chain-code retrieves the location of the actual point and describes the direction to be followed in the contour in order to find the next point. Its output must be parameterized and the result is executed in a corner detector like an 1-D angulation [11] or curvature [8] signal.

The angulation signal is described by Equation (7) where the parameter q is called smoothing level. According to Lee *et al.* [11] the determination of the parameter q depends on the orientation resolution and the corner discrimination ability. Lee *et al.* also observed the orientation profile of a corner becomes a ramp-like profile instead of a step. Thus, the parameter q provides smoothed orientation profiles and the angulation, $\phi(t)$, is given by:

$$\phi(t) = \tan^{-1}\left(\frac{y_{t+q} - y_{t-q}}{x_{t+q} - x_{t-q}}\right). \qquad (7)$$

Our multiscale corner detector searches for local features at different scales by using a non-orthogonal wavelet decomposition namely Mexican hat. The angulation signal is convolved with a scaled version of the mother wavelet at scales

$$c(s,t) = \phi(t) \otimes \psi_s(t), \qquad (8)$$

where the symbol $\otimes$ denotes the convolution operation, $c(s,t)$ corresponds to the smoothed version of the angulation signal at scale s and $\psi_s(t)$ is a scaled version of the mother wavelet. The function $\psi_s(t)$ is such that

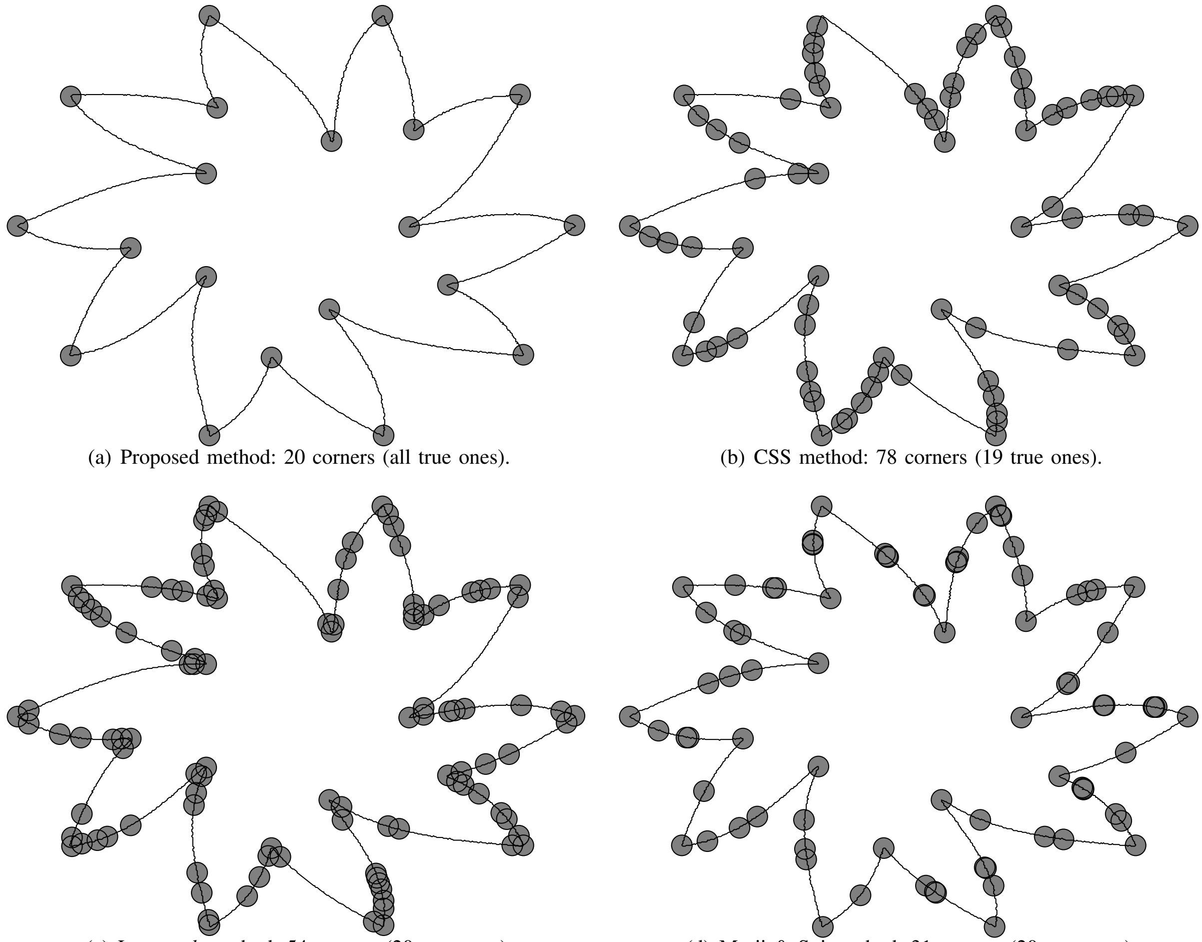
$$\psi_s(t) = \frac{1}{\sqrt{s}}\psi\left(\frac{t}{s}\right) \qquad (9)$$

and the mother wavelet $\psi(t)$ is the second derivative of a Gaussian, i.e. a normalized Mexican hat function [26] given by

$$\psi(t) = \frac{2}{\pi^{1/4}\sqrt{3\sigma}}\left(\frac{t^2}{\sigma^2} - 1\right)\exp\left(\frac{-t^2}{2\sigma^2}\right). \qquad (10)$$

Thus, by applying Equation (8) to the angulation signal it yields the approximation wavelet coefficients (smoothed signal) in each scale s. Hence, the difference between two successive smoothed signals generates the detail wavelet coefficients, $w(s,t)$, as Equation (11) summarizes

$$w(s,t) = c(s,t) - c(s-1,t), \quad s = 1,2,3 \qquad (11)$$



(a) Proposed method: 20 corners (all true ones).
(b) CSS method: 78 corners (19 true ones).
(c) Lee *et al.* method: 54 corners (20 true ones).
(d) Marji & Syi method: 31 corners (20 true ones).

Fig. 3. Corners detected (circle marks) from a sample shape with 2708 contour points and 20 true corners.

where $c(0,t)$ corresponds to the angulation signal.

The proposed methodology states that corners occur where there are extreme values, i.e. maxima and minima values in the non-orthogonal wavelet coefficients $w(s,t)$ at two or more adjacent scales. The novelty of this multiscale framework consists in inspecting these peak values by using correlation signals obtained from the redundant scales of the wavelet decomposed signal without the usage of a threshold. Thus, different redundant scales are generated and later correlated to detect persistent peaks which are likely to be candidate corners in the decomposed signals. Our experimental findings indicate that the multiscale correlation analysis at the first three scales ($s = 1, 2, 3$) is able to reveal redundant information that remains over scales and it is likely to be a candidate corner. The spatial correlation $corr(s,\, t)$ between adjacent scales, s and $s+1$, observed for a contour point location, t, for all N points, is described by the element-by-element product below:

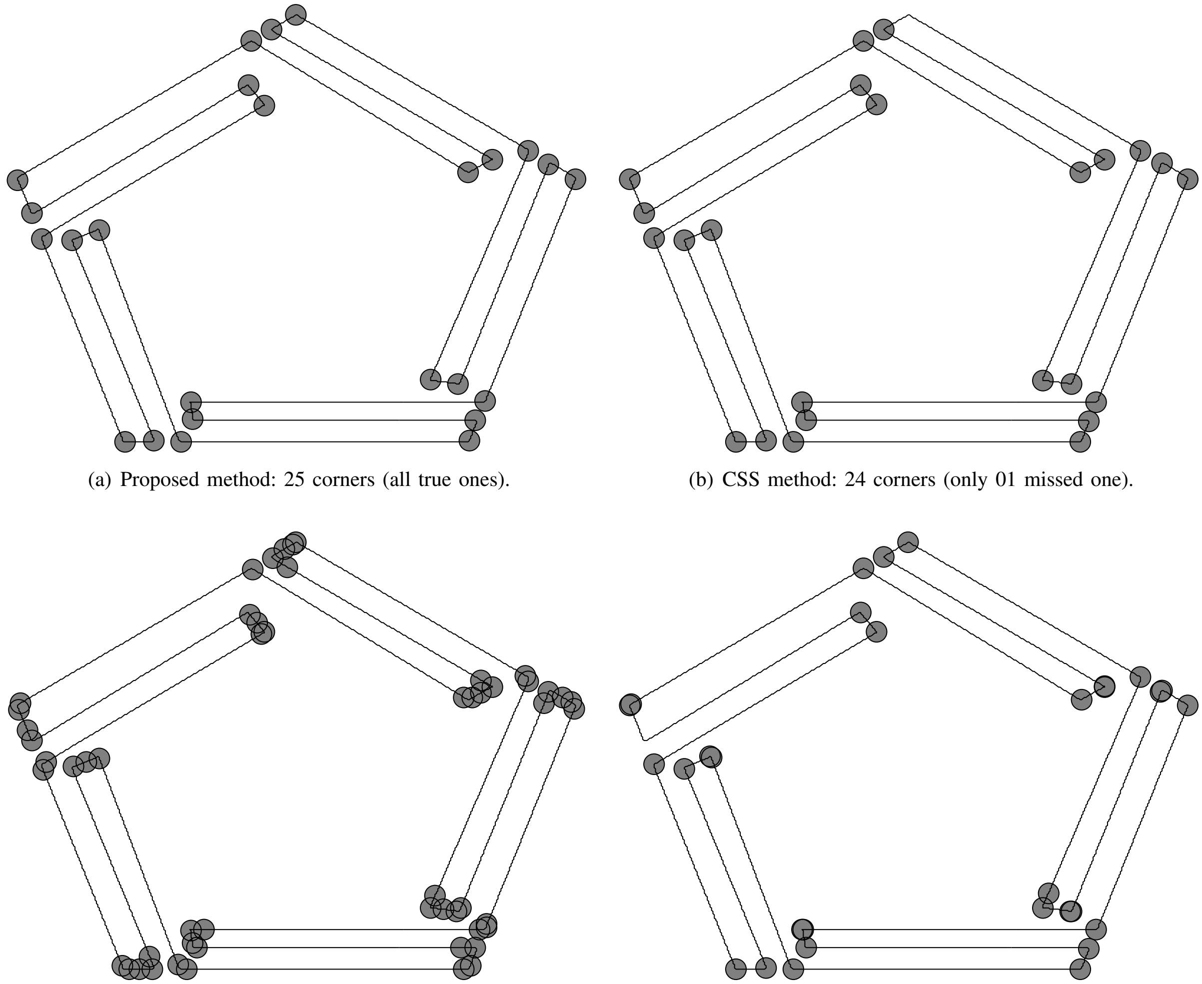
$$corr(s,\, t) = w(s,\, t) \cdot w(s+1,\, t), \quad t = 1, 2, \ldots, N. \quad (12)$$

Our approach is synthesized in Fig. 2 and therefore it depicts that real corners have large amplitude over many wavelet scales, and false corners dies out swiftly with increasing scale. The small red marks in Fig. 2 illustrate the selected candidate corners in the array. The angulation signal is decomposed in several scales: a detail coefficient (black line) is compared to the one in the next level. This result shows that corners (red marks) correspond to the largest amplitude values which persist on the correlation signal (red line) over many wavelet scales.

Following the statement that false corners dies out swiftly with increasing scale, the algorithm identifies on the correlation signal whether a candidate corner is true or false by testing the inequality:

$$|corr(s,\, t)| > |w(s,\, t)|. \quad (13)$$

When Equation (13) is confirmed (for scale s and all N points) it means that large spatial correlation values between two consecutive scales point to real corners. Afterwards, they constitute an array of probable corners. The algorithm investigates whether the candidate corner presents the largest value in the neighborhood where the correlation is high in order to differ a high curvature point from a sharp correlated false



(a) Proposed method: 25 corners (all true ones). (b) CSS method: 24 corners (only 01 missed one).

(c) Lee *et al* method: 54 corners (18 true ones). (d) Marji & Syi method: 31 corners (24 true ones).

Fig. 4. Detected corners (circle marks) from a sample shape with 2876 contour points and 25 true corners.

corner. The absence of corners or other significant features in the neighborhood of a contour region allows the false corners to be removed from the array. After selecting the corners, they are used to reconstruct the original shape and assess the methodology.

IV. Evaluation measures

The proposed assessment methodology employs the spline reconstruction algorithm [27] to rebuild the set of shapes from the detected HCPs in order to evaluate and compare the methods.

Into the Fig. 1, there is the shape reconstruction module. This one involves the curve fitting which is used to minimize the ISE and it evaluates the best set of endpoints which describes the smallest ISE on the shape contour. A new contour $\Gamma^*(t)$ is spline reconstructed by using the set of identified corners. The error is calculated from the difference between the original and reconstructed contours, e.g., $e(t) = \Gamma(t) - \Gamma^*(t)$, for all N points. The final ISE value [27] is attained by:

$$ISE = \sum_{t=1}^{N} e^2(t). \tag{14}$$

To evaluate shape reconstruction by using dominant points extracted from its contour, these works [12], [28] present different quantitative measures to report the performance of their algorithms. Among them, there are the ISE and CR measures. The latter is defined by:

$$CR = \frac{N}{N_D}, \tag{15}$$

where N is the total number of boundary points and N_D is the number of detected HCPs. The compaction ratio expresses the ability to compact the shape. Sarkar [28] combined ISE and CR measures to define a figure of merit (FOM), given by:

$$FOM = \frac{CR}{ISE}. \tag{16}$$

A modified version FOM_{CR_n} of this figure of merit which favors the CR measure was introduced in [12]:

$$FOM_{CR_n} = \frac{(CR)^n}{ISE}, \text{ for } n > 1. \tag{17}$$

We propose a new figure of merit FOM_{ISE_n} that favors the ISE measure in order to identify the algorithm that minimizes the shape reconstruction error and occasionally achieves the best performance with a low CR value, simultaneously. We define FOM_{ISE_n} as:

$$FOM_{ISE_n} = \frac{CR}{(ISE)^n}, \text{ for } n > 1. \tag{18}$$

The parameter n is introduced to control the contribution of the numerator in FOM_{CR} and of the denominator in FOM_{ISE} to the overall result to reduce the imbalance between the two terms. In order to improve the performance of the figures of merit we set $n = 3$, experimentally.

V. Simulation Results

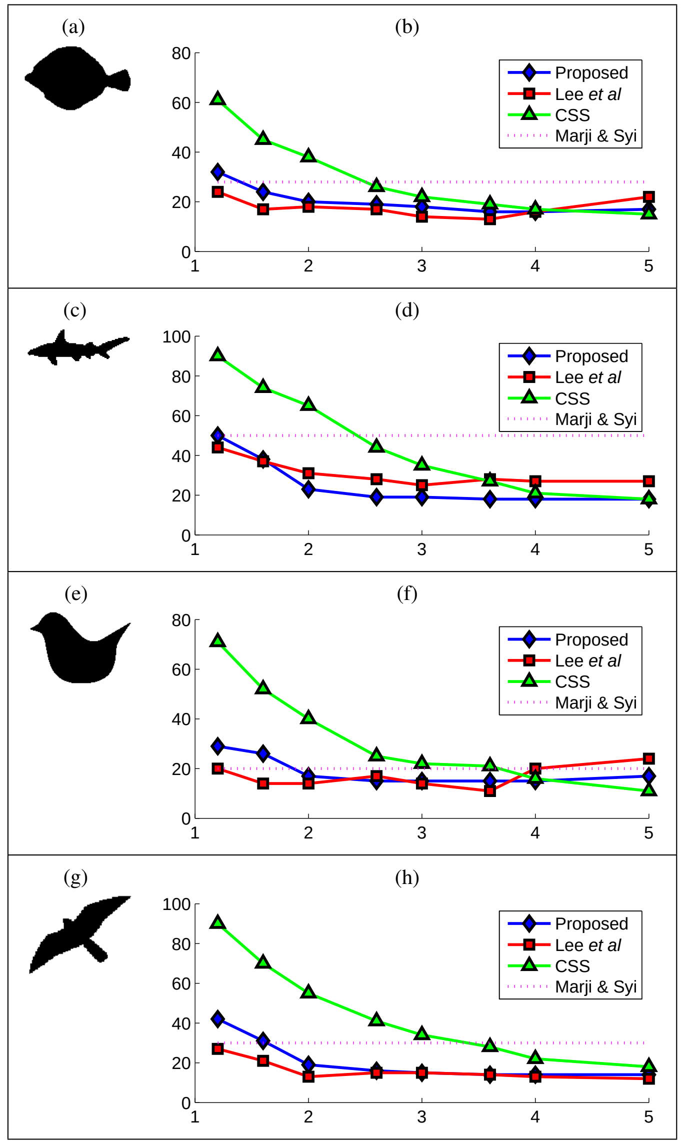
This section presents the experiments and evaluates the effectiveness of the proposed technique for dominant point detection. The results were obtained by applying the methods to a set of shapes of different sizes and formats that belong to a dataset of 104 binary images from MPEG7 Part B [29]. It is noteworthy that 10% of the images from the dataset present noisy contours. We also compared the results of the proposed method with those obtained by other contour-based methods (wavelet transform [11], dominant point detector [12] and curvature space-scale [8] approaches).

Moreover, the proposed method achieved the best results for $q = 7$ (Equation (7)) and $\sigma = 5$ (Equation (10)). These parameters were defined experimentally in order to accomplish the minimum reconstruction error and the maximum number of detected corners that match the ground-truth ones for all shapes of this set. Fig. 3 and Fig. 4 show two samples of the shape database that the ground-truth corners are known and its results for all the studied methods. We can observe that the proposed method succeeded in detecting the true corners when comparing to the others. The number of false corners produced by the CSS was higher than the other methods, and thus it performed inaccurately for the sample in Fig. 3. On the other hand, it is closer to a better result than the others methods in Fig. 4. Although the Marji & Siy and Lee *et al.* methods have detected the correct number of true corners in Fig. 3, and also return an imprecise quantitative results in Fig. 4, false corners were also detected.

Fig. 5 displays some examples and their identified corners. Fig. 5(a) shows a comparison about HCPs detection between CSS and proposed methods with a reference image from [15], [17]. In this case, the CSS approach returns a more representative array of corners than the proposed one, even with matching points. The presented sample in Fig. 5(b) exhibits the emphasized points outside the boundary from the gray level image.

Table I exhibits the number of detected corners by all the discussed methods. These data are computed to sample

TABLE I
Comparative analysis of methods for different standard deviation values and HCPs.



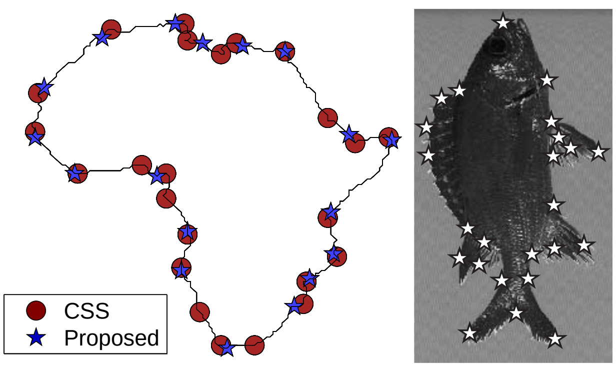
shapes from the database and using different values for the standard deviation parameter σ. We note a decreasing number of corners with the growth of σ for all methods. After a certain σ value, further increasing it has minimal influence over the number of corners, which remains stabilized. The proposed approach prevails in the least amount of identified corners when σ is equal to 4. Its computational complexity is $O(n^2)$, where n is the number of points on the shape contour. The Marji & Syi method requires no standard deviation in its definition and therefore does not present itself as a multiscale approach. So there is no variation of this parameter to be displayed in Table I(b), I(d), I(f) and I(h).

Table II shows the results for the evaluated methods. The CSS corner detector was simulated for $\sigma = 4.0$ which is the value suggested in [8]. We observe that a higher FOM_{CR_3}

TABLE II
COMPARATIVE EVALUATION OF CORNER DETECTORS.

Shape	Method	HCP	FOM_{CR_3}	FOM_{ISE_3}	Time (s)
 (a) S1 - 368 contour points	Proposed	11	2178.30	0.05	0.11
	CSS	14	3554.80	0.16	0.63
	Lee *et al.*	36	451.86	0.61	0.15
	Marji & Siy	30	853.80	0.96	3.24
 (b) S2 - 1825 contour points	Proposed	108	3071.90	5.13	0.57
	CSS	168	2382.90	5.01	2.42
	Lee *et al.*	204	628.62	4.30	0.61
	Marji & Siy	131	1619.40	2.15	15.00
 (c) S3 - 2353 contour points	Proposed	24	234440.00	1.41	0.75
	CSS	23	9975.90	0.02	2.90
	Lee *et al.*	42	46957.00	0.86	0.79
	Marji & Siy	32	116080.00	1.27	18.29
 (d) S4 - 370 contour points	Proposed	18	974.21	0.03	0.11
	CSS	24	761.71	0.14	0.57
	Lee *et al.*	42	118.76	0.05	0.15
	Marji & Siy	34	7.95	0.01	1.94
 (e) S5 - 1763 contour points	Proposed	109	1667.60	1.99	0.47
	CSS	154	889.72	1.37	2.34
	Lee *et al.*	182	339.05	0.50	0.50
	Marji & Siy	159	606.99	0.96	10.98

value indicates higher shape compaction. Furthermore, a higher FOM_{ISE_3} implies a better shape reconstruction.



(a) Corners in Shoreline of Africa image [17]. (b) Corners in Fish image [14].

Fig. 5. Other results: (a) plot of identified corners by CSS (circle marks) and proposed methods (pentagrams marks); (b) detected corners in a non-synthetic image.

The shape S1 has only a minimal amount of points identified as corners: only 3% of the contour points (11 dominant points). As a consequence, the proposed method reached a greater CR than others, as we see in the FOM_{CR_3} measures. Marji & Siy obtained the best reconstruction of S1 with higher FOM_{ISE_3}. The proposed method also shows the best reconstruction for the shapes S2, S3 and S5. As a limitation of the method, there is no strong impact on the reconstruction for all the shapes. For each level of wavelet decomposition, the algorithm discards irrelevant information of the contour representation signal and retains dominant points, which are candidates to be corners. Thus, it illustrates the trade-off between a shape highly compressed and well reconstructed.

The shape S5 presents a certain amount of noise in its contour. The proposed method detected a minimum amount of 6% of corners (109 points) from the total number of contour points. For this shape, our approach obtained both the best compaction and the best shape reconstruction among all the methods.

The method Marji & Siy produces good results and achieves a good number of true corners, but its execution time is affected in some algorithm steps. As described in [12], this corner detector has four basic steps for finding the corners: support region, node strength, sorting nodes and shape covering. In the region of support, we seek to identify for each boundary point which neighbor provides the largest line so that does not generate a high error in the reconstruction in this connection. This occurs both clockwise and counterclockwise. This approach provides successive visits to all neighbors from a point with the aim to find the one that is the closest and achieves small reconstruction error. Regarding the sorting nodes task, one can conclude that it increases the computational cost with the amount of contour points. Its computational complexity is $O(m * k * n^2)$, where n is the number of points on a curve, k is the length of a region of support and m corresponds to the number of different regions of support.

The CSS approach also presents a computational time higher than the proposed method. This is due to the Gaussian function is applied to the shape outline several times and whose standard deviation changes in all iterations, as defined in [8], [17]. This process is performed to generate the space-scale map from the original method and then run the corner detector with the σ_{high} [8]. This method achieves a computational complexity $O(n^3)$, where n is the number of points on a contour.

The experiments were performed on a 1.8 GHz Core 2 Duo with Matlab© v2.6 source code. Data in the rightmost column of Table II indicate that the proposed approach performed quite similar to the Lee *et al.* technique regarding the execution time and thus they both spent the least time to process each shape from the data set. Moreover, the CSS and Marji & Siy methods spent much more time than those aforementioned.

VI. Conclusions

We have presented in this paper an innovative approach for corner detection that uses correlation between scales of the normalized Mexican hat wavelet detail coefficients. The method may be viewed as a multiscale detector and furthermore the amount of information changes when the scale is modified. Therefore, relevant information or real corners persist over many scales while false corners dies out swiftly. Moreover, many points are suppressed and the most relevant shape corners are emphasized. This new approach has overcome other methods discussed in this paper in terms of compacting ratio for this set of binary shapes. Despite it achieved shape compaction while retrieved ground-truth corners, the method did not perform quite well when applied to gray level images. The proposed algorithm focused on the capability of selecting the most relevant corners, i.e. the ground-truth corners and hence the reconstruction error was quite penalized. Bearing in mind that there is a compromise between CR and ISE measures, i.e. a great ability to compact implies a trend of the reconstruction error to increase in general, as our results confirmed it. Regarding the execution time, tests revealed that the proposed method is competitive when comparing it to CSS and Marji & Siy methods.

References

[1] F. A. Cheikh, A. Quddus, and M. Gabbouj, "Shape recognition based on wavelet-transform modulus maxima," *Image Analysis and Interpretation*, vol. 1, pp. 461–464, April 2000.

[2] T. Bernier and J. Landry, "A new method for representing and matching shapes of natural objects," *Pattern Recognition*, vol. 36, no. 8, pp. 1711–1723, August 2003.

[3] L. F. Costa and R. M. J. Cesar, *Shape Analysis and Classification: Theory and Practice*. CRC Press, 2001.

[4] I. C. J. De Paula, F. N. S. Medeiros, G. A. Mendona, C. J. P. Passarinho, and I. N. S. Oliveira, "Correlating multiple redundant scales for corner detection," in *Proceedings of VI International Telecommunications Symposium*. Fortaleza, Brazil: IEEE, September 2006, pp. 650–655.

[5] J. Hua and Q. Liao, "Wavelet-based multiscale corner detection," in *Proceedings of 5th International Conference on Signal Processing*, vol. 1. Beijing, China: IEEE, August 2000, pp. 341–344.

[6] X. Gao, F. Sattar, A. Quddus, and R. Venkateswarlu, "Multiscale contour corner detection based on local natural scale and wavelet transform," *Image and Vision Computing*, vol. 25, no. 6, pp. 890–898, June 2007.

[7] M. Awrangjeb and G. Lu, "An improved curvature scale-space corner detector and a robust corner matching approach for transformed image identification," *IEEE Transactions on Image Processing*, vol. 17, no. 12, pp. 2425–2441, December 2008.

[8] F. Mokhtarian and R. Suomela, "Robust image corner detection through curvature scale space," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 20, no. 12, pp. 1376–1381, December 1998.

[9] S. J. Wang, L. C. Kuo, H. H. Jong, and Z. H. Wu, "Representing images using points on image surfaces," *IEEE Transactions on Image Processing*, vol. 14, no. 8, pp. 1043–1056, August 2005.

[10] E. Rosten, R. Porter, and T. Drummond, "Faster and better: A machine learning approach to corner detection," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 32, no. 1, pp. 105–119, January 2010.

[11] J. S. Lee, Y. N. Sun, and C. H. Chen, "Multiscale corner detection by using wavelet transform," *IEEE Transactions on Image Processing*, vol. 4, no. 1, pp. 100–104, January 1995.

[12] M. Marji and P. Siy, "Polygonal representation of digital planar curves through dominant point detection a nonparametric algorithm," *Pattern Recognition*, vol. 37, no. 11, pp. 2113–2130, November 2004.

[13] X. Zhang, H. Wangb, M. Hong, L. Xu, D. Yang, and B. C. Lovell, "Robust image corner detection based on scale evolution difference of planar curves," *Pattern Recognition Letters*, vol. 30, no. 4, pp. 449–455, March 2009.

[14] X. Zhang, H. Wang, A. W. B. Smith, L. Xu, B. C. Lovell, and D. Yang, "Corner detection based on gradient correlation matrices of planar curves," *Pattern Recognition*, vol. 43, no. 4, pp. 1207–1223, April 2010.

[15] F. Mokhtarian and F. Mohanna, "Performance evaluation of corner detectors using consistency and accuracy measures," *Computer Vision and Image Understanding*, vol. 102, no. 1, pp. 81–94, April 2006.

[16] C. J. Harris and M. Stephens, "A combined corner and edge detector," in *Proceedings of The Fourth Alvey Vision Conference*, Manchester, England, August 1988, pp. 147–151.

[17] F. Mokhtarian and A. Mackworth, "Scale-based description and recognition of planar curves and two-dimensional shapes," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 8, no. 1, pp. 34–43, January 1986.

[18] ——, "A theory of multiscale, curvature-based shape representation for planar curves," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 14, no. 8, pp. 789–805, August 1992.

[19] B. Zhong and W. Liao, "Direct curvature scale space: Theory and corner detection," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 29, no. 3, pp. 508–512, March 2007.

[20] C. W. D. De Almeida, R. M. C. R. De Souza, and N. L. J. Cavalcanti, "A shape-based image retrieval system using the curvature scale space (css) technique and the self-organizing map (som) model," in *Proceedings of the 6th International Conference on Hybrid Intelligent Systems*, Auckland, New Zealand, December 2006, pp. 25–29.

[21] M. C. Roh, B. Christmas, J. Kittler, and S. W. Lee, "Gesture spotting in low-quality video with features based on curvature scale space," in *Proceedings of the 7th International Conference on Automatic Face and Gesture Recognition*, Southampton, U.K., April 2006, pp. 375–380.

[22] L. F. Estrozi, L. G. Rios-Filho, A. Bianchi, R. M. Cesar. Jr, and L. F. Costa, "1d and 2d fourier-based approaches to numeric curvature estimation and their comparative performance assessment," *Digital Signal Processing*, vol. 13, no. 1, pp. 172–197, January 2003.

[23] B. Zhong, K.-K. Ma, and W. Liao, "Scale-space behavior of planar-curve corners," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 31, no. 8, pp. 1517–1524, August 2009.

[24] A. Carmona-Poyato, F. J. Madrid-Cuevas, R. Medina-Carnicer, and R. Muñoz Salinas, "Polygonal approximation of digital planar curves through break point supression," *Pattern Recognition*, vol. 43, no. 10, pp. 14–25, 2010.

[25] F. Attneave, "Some information aspects of visual perception," *Psychological Review*, vol. 61, pp. 183–193, 1954.

[26] S. Mallat, *A Wavelet Tour of Signal Processing*, 2nd ed. Academic Press, September 1998.

[27] T. Pavlidis, *Algorithms for graphics and image processing*. Maryland: Computer Science Press, 1982.

[28] D. Sarkar, "A simple algorithm for detection of significant vertices for polygonal aproximation of chain-coded curves," *Pattern Recognition Letters*, vol. 14, no. 12, pp. 959–964, December 1993.

[29] L. Latecki, R. LakSmper, and U. Eckhardt, "Shape descriptors for non-rigid shapes with a single closed contour," in *Proc. IEEE Conf. on Computer Vision and Pattern Recognition*, 2000, pp. 424–429.