Comparison of Three Different Derivative Approaches Aiming at Estimation of Image Movement

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Abstract

The use of optical flow techniques to extract the velocity from the cardiac movement has to take into account the computational inaccuracy raised from the derivative operator over discrete data. This study presents a comparison of three different derivative approaches (two based on linear and other based on non-linear filtering) to find out the best solution. Results of the experiments are compared using a structural distortion based image quality metric.

1. Introduction

The diagnosis quality of cardiac diseases has a potential to be improved by extracting quantitative information from 3D image sequences of the heart.

Given the cardiac movements, a possible approach to estimate their velocity components for each voxel is the optical flow technique [1], and here extended for a 3D space. However, a critical problem to obtain this estimation lies in the fact that this process depends on derivative approximations from discrete data.

This work performs a comparison of three different approaches for calculation of image derivatives in the presence of different noise levels, applied to a simplified and well-controlled mathematical model. The aim is to determine which approach estimates the derivatives more accurately in the presence of noise.

2. Methodology

The first method, named "Traditional Derivative", follows the steps usually performed to calculate partial derivatives of noisy data: the noisy image is convolved with a Gaussian filter, followed by the application of a partial derivative operator [2].

The second method, named "Gaussian Derivative", is based on linear scale-space theory applied to discrete data: partial derivatives of a rescaled image can be obtained by the convolution of the original image with the corresponding derivatives of the Gaussian function [3].

Both methods tend to create very smooth flow fields and can reduce the precision of the velocity estimation. In the third method, the anisotropic diffusion filter proposed by Perona and Malik [4] was applied prior to the derivative operation. The main objective of using this non-linear smoothing filter is to preserve edges, so possible abrupt flow discontinuities may be preserved during the estimation process.

All filters and differential operators were implemented using the libraries of the open-source software system Insight Toolkit (ITK) [5].

3. Experiments

The experiments were performed using an image of a cube, which consisted of 10x10x10 voxels in a volume with spatial resolution of 64x64x64 voxels. This cube moved along all directions (X, Y, and Z) resulting in a sequence of 16 frames. The intensities of the image were 138 in the cube and 10 in the background. The original volume was corrupted by either Poisson or Gaussian noise with three different intensities: high, medium, or low. Poisson noise level is defined as the square root of the voxel count. In the present experiment, the noise levels were: 3 for the background and 12 for the cube. The Gaussian noise level was measured by the contrast-to-noise ratio (CNR) [2]. The CNR for the three levels of Gaussian noise were: 26 (low), 14 (medium), and 7 (high).

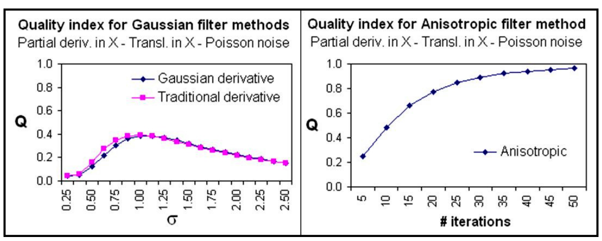
Let A be the ideal derivative image obtained from the application of the partial derivative operator on X, Y, Z, or T direction to the original noiseless volume. The three derivative approaches were applied in each direction to the noisy volumes resulting in a set of images (image B) that were compared to A. These comparisons were evaluated through a structural distortion based image quality measurement proposed by Wang *et al.* [6]:

$$Q = \frac{s_{AB}}{s_A \cdot s_B} \cdot \frac{2 \cdot \overline{\mathrm{A}} \cdot \overline{\mathrm{B}}}{\left(\overline{\mathrm{A}}\right)^2 + \left(\overline{\mathrm{B}}\right)^2} \cdot \frac{2 \cdot s_A \cdot s_B}{s_A^2 + s_B^2} \tag{1}$$

where: $\overline{\mathrm{I}}$ represents the average of all voxels of the image I, s_I^2 is the variance of I, and s_{AB} is the covariance between A and B.

This equation models any distortion as a combination of three different factors: loss of correlation, mean and variance distortion between images A and B. The range of Q is [-1, 1] and the best value 1 is achieved when $A_i = B_i$ for every voxel i [6].

Figure 1 shows two chart samples obtained in the experiments for the proposed quality index. The curves for all other charts, including other partial derivatives and noise intensities, present similar behaviors.



(a) (b)

Figure 1: Quality index vs. (a) σ for two linear approaches or (b) number of iterations for non-linear approach referred to partial derivative in X, translation in the X direction, and in the presence of Poisson noise.

There are no noticeable differences between the application of Traditional and Gaussian Derivative methods (Figure 1a). Q is maximized for a determined value of σ that depends on the noise level. When dealing with real images of the same spatial resolution with unknown noise level, the results suggest that a value for σ around 1.4 can be tried out to optimize the quality of the partial derivative estimation. However, considering the overall experiments using Gaussian methods, the best result for Q is limited to, approximately, 50% of the ideal noiseless case.

Better results are reached with non-linear anisotropic method (Figure 1b). Q tends asymptotically to 1 as the number of iterations increases, which means optimal quality or minimal distortion. For all experiments two filter parameters were previously set: the time step to 1/32 and the conductance to 9. According to Figure 1b, less than 10 iterations are enough to get better quality (over 40%) than using Gaussian methods. Fixing the same quality level for the worst tested case, observed with Gaussian high noise, the required number of iterations does not exceed 35.

4. Conclusion

In this work three methods to calculate image derivatives were compared by applying them to sets of noisy images of a simple mathematical model. The choice of this model was due to its simplicity that allows interpretation and analysis of the results. Despite of its simplicity, the simulated images have the same spatial resolution of PET or SPECT images, and the cube volume has the same magnitude order of the human heart left ventricle.

For the experiments reported, the best performance among the three approaches was achieved with the anisotropic method.

In the sequence of this work, which intends to calculate the velocity by using the optical flow technique, new experiments shall allow a direct comparison between real and the estimated velocity data. The aim is not only to commit to the most suitable methodology, but also to set the ideal range of parameters according to the image resolution and structures size. Getting the best possible precision of the derivative parameters will lead to finding out more reliable velocity field for clinical application.

5. References

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