New Tensorial Representation of Color Images: Tensorial Morphological Gradient Applied to Color Image Segmentation

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Abstract

This paper proposes a new Tensorial Representation of HSI color images, where each pixel is a 2×2 second order tensor, that can be represented by an ellipse. A proposed tensorial morphological gradient (TMG) is defined as the maximum dissimilarity over the neighborhood determined by a structuring element, and is used in the watershed segmentation framework. Many tensor dissimilarity functions are tested and other color gradients are compared. The comparison uses a new methodology for qualitative evaluation of color image segmentation by watershed, where the watershed lines of the n most significant regions are overlaid on the original image for visual comparison. Experiments show that the TMG using Frobenius norm dissimilarity function presents superior segmentation results, in comparison to other tested gradients.

1. Introduction

The edge enhancement of an image by gradient computation is an important step in morphological image segmentation via watershed [3, 17]. For grayscale images, the morphological gradient [14] is a very good option and its computation is simple: for each point in the image, a structuring element is centered to it and the difference between the maximum and the minimum graylevels inside the structuring element is computed. Here, the dissimilarity information exploited is the intensity difference among pixels inside the structuring element.

Such concept does not extends naturally to color images. Although the dissimilarity information is richer in color images than in grayscale ones, the design of methods to edge enhancement in color images is complex. Note that the metric that measures the natural dissimilarity information of color images is unknown. Also note that if one considers the color space as a complete lattice [18, 6], the order relation is not total and even if a total order is imposed in this space, it will be not natural for the human eye.

One option to construct color gradients relies on the design of metrics to compute them [12, 11]. Such metrics exploit the dissimilarity information in color images, usually collected from each band, and then compute the gradient based on the distance of the colors inside a given connected region: the higher the dissimilarity among the colors inside this region, the higher its gradient. The dissimilarity metrics impose a total order relation and the gradient may be computed.

An alternative metric is the one based on tensorial algebra [7, 5]. Using tensors to represent colors in images bring us the possibility to make use of all the tensor theory. Given a tensorial representation of colors, it is possible to compute the gradient of a color image by computing the dissimilarity among the tensors. Some approaches of color representation based on tensors can be found in the literature. Most of them utilizes the Structure Tensor (or a modified version of it) to represent RGB color images and uses this representation to comply different tasks, such as: feature extraction [19, 20], computation of optical flow [4] and segmentation [8].

This paper proposes a new color representation given by the correspondence between the HSI color model and tensors. The tensorial morphological gradient(TMG) for color images is also a new proposal to compute color gradients based on tensorial algebra. Several ways to compute the dissimilarity between tensors have been published [1, 2, 21]. Three of them are used in this work to compute the tensorial morphological gradient. In order to analyze the proposed gradient, it is compared to some classical color gradients applied to segmentation of the n most significant regions of an image.

This paper is organized as follows: Section 2 describes an existing tensorial representation of color images and a new representation scheme is proposed. Section 3 presents dissimilarity measures commonly used to compare tensors and introduces the TMG, based on tensors and their dissimilarities. Section 4 describes the methodology utilized to compare segmentation results obtained by each tested gradient. Section 5 shows the segmentation results obtained by watershed applied on some color image gradients, including the TMG proposed in this paper. Finally, Section 6 concludes the paper.

2. Tensorial representation of color images

2.1. Tensors

Tensor is an abstract object, expressing some definite type of multi-linear concept. Their well-known properties can be derived from their definitions, as linear maps or more generally; and the rules for manipulation of tensors arise as an extension of linear algebra to multilinear algebra [7].

In practice a bi-dimensional second order tensor is denoted by a 2×2 matrix of values:

$$\mathbf{T} = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix}, \quad (1)$$

and can be reduced to principal axes (eigenvalue and eigenvector decomposition) by solving the characteristic equation:

$$\mathbf{T} - (\lambda \cdot \mathbf{I})e = 0, \quad (2)$$

where $\mathbf{I}$ is the identity matrix, λ are the eigenvalues of the tensor and e are the normalized eigenvectors. If the tensor is symmetric,i.e., $T_{12} = T_{21}$, the eigenvalues will always be real. Moreover, the corresponding eigenvectors are perpendicular [5]. In this case, the tensor can be represented by an ellipse, where the main axes lengths correspond to the eigenvalues and their direction to the respective eigenvectors (Fig. 1).

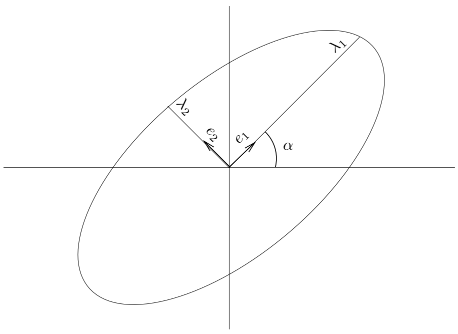


Figure 1. Ellipse representing a tensor

2.2. Structure tensor representation

Structure tensor (or second-moment matrix) [19] is a matrix representation of partial derivative information. In the field of image processing and computer vision, it is typically used to represent the gradient or to find features such edges and corners. The color structure tensor describes the bi-dimensional first order differential structure at a certain point in the image. The structure tensor matrix is formed as:

$$\mathbf{sT} = \begin{pmatrix} f_x^2 & f_x f_y \\ f_x f_y & f_y^2 \end{pmatrix}, \quad (3)$$

where f_x and f_y denote the partial derivatives of the grayscale image f along the x and the y axis, respectively. Also here eigenvalues (λ_1, λ_2) and eigenvectors (e_1, e_2) can be calculated by decomposition. These new gradient features allow a more precise description of the local gradient characteristics.

DiZenzo introduced the idea of using a *Color Structure Tensor* [22], given by:

$$\mathbf{sT}=\begin{pmatrix} R_x^2+G_x^2+B_x^2 & R_xR_y+G_xG_y+B_xB_y \\ R_xR_y+G_xG_y+B_xB_y & R_y^2+G_y^2+B_y^2 \end{pmatrix}, \quad (4)$$

where R, G and B are the three color components of the RGB color image $\mathbf{f} = (R, G, B)$ and the subscripts denote the partial derivatives. In this case, after eigenvalue and eigenvector decomposition, e_1 is a unit vector that indicates the prominent local orientation, which is equal to the orientation in the image with maximum color change.

2.3. Proposed tensorial representation

One way to describe an ellipse is to define its minor and major semiaxes and its rotation. Another possibility is to chose the ellipses attributes from the tensor represented by it. The ratio between eigenvalues determines the shape of the ellipse (eccentricity), their sum defines the scale of the ellipse (also called Trace) and its principal eigenvector direction defines the rotation of the ellipse:

$$Shape = \frac{\lambda_2}{\lambda_1}. \quad (5)$$

$$Trace = (\lambda_1 + \lambda_2), \quad (6)$$

By establishing a relation between these attributes and the attributes of the HSI color model: Hue, Saturation e Intensity, it is possible to represent a color in terms of a tensor. In other words, interpreting the Hue of a color as the direction of the principal eigenvector of the tensor, the Saturation as the shape of the ellipse and the Intensity as the trace, for each color of the HSI model there will be a tensor describing it. This representation proposal can be seen in Fig. 2.

Fig. 2(a) shows different colors ($0 \leqslant H \leqslant \pi/2$), with same saturation ($S = 0.5$) and same intensity ($I = 1$). Similarly to the HSI model, changing the color without changing saturation and intensity causes changes only in the direction of the ellipse. Fig. 2(b) shows one color ($H = 0$) with same intensity ($I = 1$) and different saturation values ($0 \leqslant S \leqslant 1$) . In this case, changes in saturation determine changes in the shape of the ellipse. More saturated is the color, more elliptical is its tensor. In one extreme ($S = 1$), the color is represented by a line segment. In the other extreme, color with no saturation is represented by a circle. Finally, Fig. 2(c) presents the same color ($H = 0$) with one fixed saturation ($S = 0.5$) and intensity varying between 0 and 1 ($0 \leqslant I \leqslant 1$). Colors with null intensity are represented by a point. As the intensity rises, the size of the ellipse also grows (without changing its shape).

3. Tensorial morphological gradient (TMG)

One way to compute the gradient of an image is via application of a dissimilarity function. Usually, the exploited dissimilarity information is the intensity difference among pixels inside the structuring element. This section proposes a color gradient that makes use of tensorial dissimilarity functions applied to the tensor representation described in Subsection 2.3.

Given two tensors $\mathbf{T}_i$ e $\mathbf{T}_j$, there are some dissimilarity measures that might be used to compare them [1]. The simplest one is the dot product between the principal eigenvector directions:

$$d_1(\mathbf{T}_i, \mathbf{T}_j) = |e_{1,i} \cdot e_{1,j}|, \tag{7}$$

where $e_{1,i}$ and $e_{1,j}$ are the principal eigenvectors of tensors $\mathbf{T}_i$ e $\mathbf{T}_j$, respectively. The absolute value of the dot product solves the problem with the sign ambiguity of the eigenvectors. Another simple comparison between two tensor quantities is the tensor dot product:

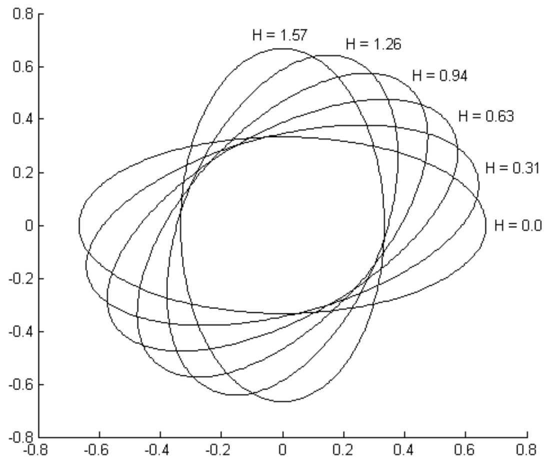
$$d_2(\mathbf{T}_i, \mathbf{T}_j) = \lambda_{1,i}\lambda_{1,j}(e_{1,i} \cdot e_{1,j})^2 + \lambda_{2,i}\lambda_{2,j}(e_{2,i} \cdot e_{2,j})^2. \tag{8}$$

It uses not only the principal eigenvector direction, but the full tensor information. Another dissimilarity measure that uses the full tensor information is the Frobenius norm:

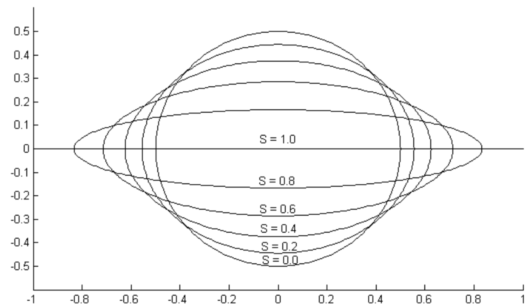
$$d_3(\mathbf{T}_i, \mathbf{T}_j) = \sqrt{\text{Trace}((\mathbf{T}_i - \mathbf{T}_j)^2)}, \tag{9}$$

Let $E = \quad \times \quad$ be the set of all points in the image. Let f be a color image under the HSI color model. The proposed (TMG) based on the tensorial representation of Subsection 2.3 is defined by

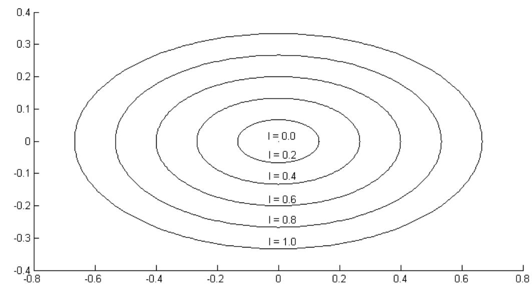
$$\nabla_B^T(f)(x) = \bigvee_{y \in \{B_x - \{x\}\}} d_n(\mathbf{T}_x, \mathbf{T}_y) \tag{10}$$



(a) Hue variation



(b) Saturation variation

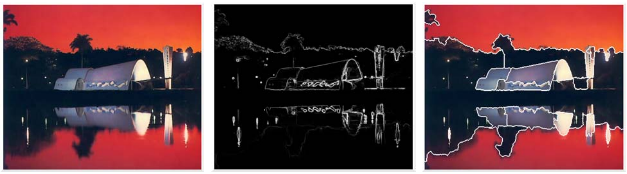


(c) Intensity variation

Figure 2. Tensorial representation of HSI color information

$\forall x \in E$, where d_n represent any of the three dissimilarity functions presented above (d_1 , d_2 or d_3), $B \subset E$ is a structured element centered at the origin of E, $\mathbf{T}_x$ is the tensor that represents the color $f(x)$, and $\mathbf{T}_y$ is the tensor that represent the color located in y (in the neighborhood of x, defined by E). ∇_B^T is the proposed TMG. Because the chosen metrics are already comparisons between neighbors, the proposed gradient is not the difference between the maximum and the minimum values, but only the maximum value. In other words, the computed gradient in a neighborhood given by an structuring element is the maximum dissimilarity between the neighbors of the given point.

An example of segmentation based on the proposed tensorial morphological gradient (TMG) can be seen in Fig. 3. Fig. 3(a) shows the original image, Fig. 3(b) presents the TMG calculated using the Frobenius norm dissimilarity function (Eq. 9 and Eq. 10) and Fig. 3(c) shows the segmentation result obtained applying the watershed transform on the proposed TMG.



(a) Original Image (b) TMG (c) Segmentation

Figure 3. An example of segmentation using the tensorial morphological gradient (TMG)

4. Comparison methodology

An usual comparison of color gradient metrics is made by quality assessment of the segmentation results provided by applying the watershed transform [16, 17]. The watershed segmentation are done by manual markers imposition and compared by analyzing how well the objects of interest are segmented. Another common methodology is to choose a set of parameters in watershed, specific for each gradient and for each image, in order to get the best segmentation result for that gradient. In both cases, the obtained watershed lines are then overlaid on the original image or on a mosaic image created from the segmentation result in order to allow this evaluation. The weakness of these kind of comparison methodology, it is not only the subjectiveness, but also the fact that the segmentation results that are compared are dependent of a good choice of parameters or markers.

An alternative to do the assessment of the segmentation results is to segment the color image in a fixed number of regions. By imposing markers to the basins with the greatest volume dynamics [13, 10], it is possible to evaluate how the watershed operator will segment the n most significant regions in the image. The least important basins in each gradient will be filtered and their borders suppressed; the way such least important basins are suppressed defines the accuracy of the segmentation result. The difference to the common used segmentation comparisons method is that the proposed methodology does not search for good segmentation results in order to compare them, but imposes restrictions (in this case, number of regions) and compares the performance of each gradient under such restrictions.

The watershed lines are then overlaid on the original image, instead on a mosaic image, in order to compare the results subjectively. It is done because it is harder to assess the results on mosaic images, since there are many ways to compute them and they interfere significantly in the visualization of the final result.

5. Experimental results

This section presents several experiments done in order to show the accuracy of the TMG. The first experiment compared several tensorial metrics in order to choose the most suitable metric to be applied in the TMG computation. The second experiment compared the TMG with other color gradients found in literature. In both experiments, the comparison methodology described in the Section 4 was applied. The images used in the experiments were obtained from "The Berkeley Segmentation Dataset and Benchmark" [15]. The watershed transform and other morphological functions used can be found in the "SDC Morphology Toolbox for MATLAB" [9].

5.1. Comparison of TMGs using different dissimilarity functions

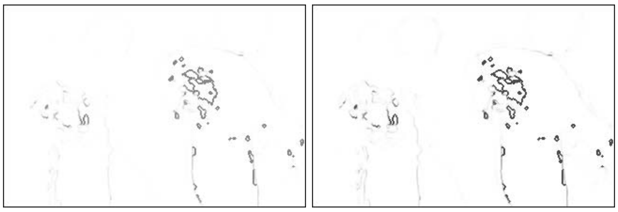
The first experiment was done in order to compare the segmentation results given by the three TMGs: dot product, tensor dot product and Frobenius norm. Several images and different numbers of regions were tested. Fig. 4 presents the image of parrots segmented using each TMG. In all cases, the color image was partitioned in 50 regions. Fig. 5 (a-c) shows, respectively, the TMGs computed by dot product metric, tensor dot product and Frobenius norm. The gradients were negated for a better presentation. The computation of the first two gradients provided smoother borders and it was expected that the segmentation result provided by them would not be good. The borders in the TMG Frobenius were stronger and should provide a better result.

It is evident the superiority of the segmentation result using the TMG from the Frobenius norm dissimilarity function, compared to the other two metrics. The blue wing

(a) Original Image (b) TMG - Dot product

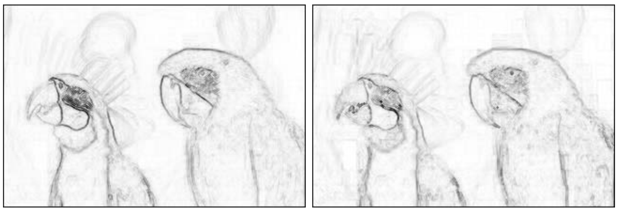
(c) TMG - Tensor dot product (d) TMG - Frobenius norm

Figure 4. Watershed segmentation of the "parrots" image with 50 regions using TMGs



(a) TMG - Dot product (b) TMG - Tensor dot product

(c) TMG - Frobenius norm (d) Structure tensor



(e) Morphological gradient (f) Weighted gradient

Figure 5. Compared gradients

of the first parrot, for example, was merged with the background in the dot product (Fig. 4(b)) and tensor dot product (Fig. 4(c)). It did not occured when the Frobenius norm metric was applied (Fig. 4(d)). This was observed also in other important regions of the image.

5.2. Comparison of the Frobenius norm TMG with other color gradients

Several gradients are used in color image processing to detect edges. One of them is called the morphological gradient, that depends on the size and shape of the chosen structuring element. Using a flat structuring element at each point the morphological gradient yells the difference between the maximum and the minimum values over the neighborhood at the point determined by the flat structuring element [14]:

$$\nabla_B^M(f) = \bigvee\{\nabla_B(f_1), \nabla_B(f_2), \nabla_B(f_3)\}, \qquad (11)$$

where $B \subset E$ is the structuring element and ∇_B is the morphological gradient. The resulting gradient is an image containing the supremum among the maxima differences in each band of f.

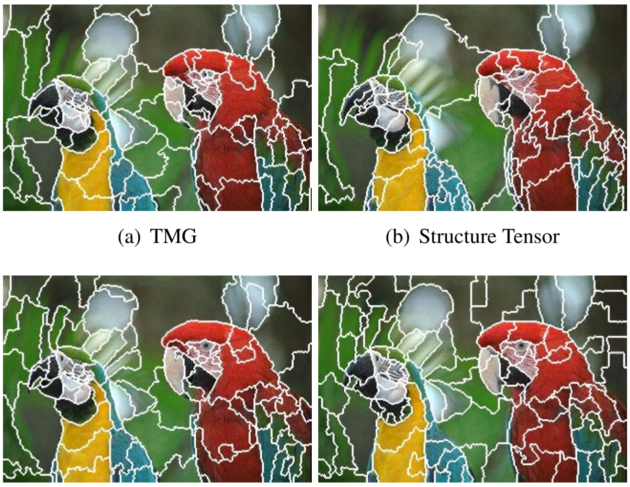
Another color image gradient is the weighted gradient [12, 11]. This operator is a transformation from a color image under the HSI color space to a grayscale one, by the linear combination of the gradients from each band. It is defined as follows:

$$\nabla_B^W(f) = [\omega_1 \nabla_B(f_1), \omega_2 \nabla_B^{\ominus}(f_2), \omega_3 \nabla_B(f_3)], \qquad (12)$$

where $B \subset E$ is the structuring element, f_1 , f_2 and f_3 represent respectively the intensity, hue and saturation color bands, $\omega_1, \omega_2, \omega_3 \in {}_{+}$ are their respective weights estimated by a distance function, $B \subset E$ is a structured element centered at the origin of E. ∇_B is the classical morphological gradient and $\nabla_B^{\ominus}$ is the angular gradient.

The second experiment had the purpose to compare the segmentation results obtained using the Frobenius norm TMG to the segmentation results provided by the morphological gradient, the weighted gradient and the structure tensor. Fig. 6, Fig. 7 and Fig. 8 show results obtained for three different number of regions: 100, 50 and 25.

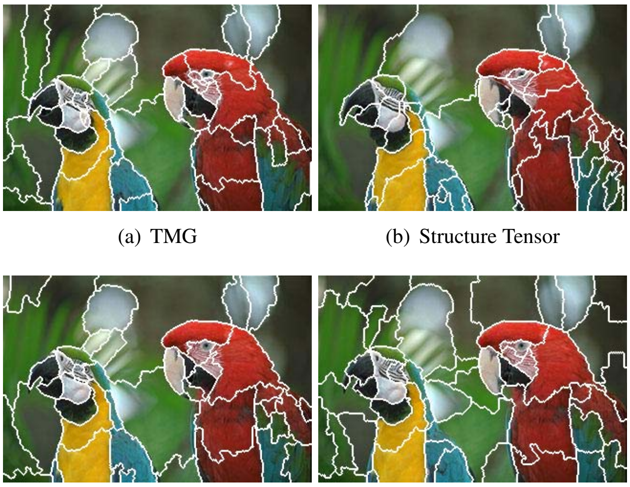
Fig. 5 (c-f) shows, respectively, the Frobenius norm TMG, the structure tensor, the morphological gradient and the weighted gradient. The gradients were negated for a better presentation. Structure tensor had strong borders and it was expected that it would provide good segmentation results in the next experiments, but it did not occur. Morphological and weighted gradients provided well enhanced borders, but the suppression of some important borders led to a bad segmentation in a few points. In overall, all gradients provided good segmentation results, specially for bigger numbers of regions (Fig. 6). Unless for small details, like the red parrot wing, the eyes or some spots in the background, their performance were comparable. As the number



(a) TMG (b) Structure Tensor

(c) Morphological Gradient (d) Weighted Gradient

Figure 6. Watershed segmentation of the "parrots" image with 100 regions using different color image gradients



(a) TMG (b) Structure Tensor

(c) Morphological Gradient (d) Weighted Gradient

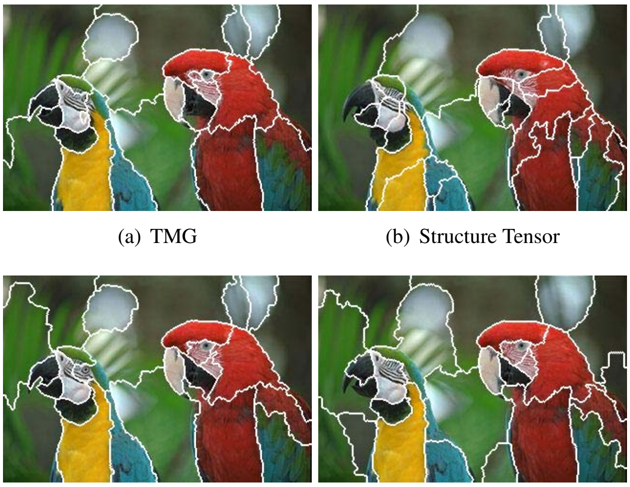
Figure 7. Watershed segmentation of the "parrots" image with 50 regions using different color image gradients

of regions becomes lower, their performance started presenting more perceptible differences. Fig. 7 shows that the structure tensor (Fig. 7(b)) was not able to segment important regions of the parrots, like the blue parrot's head or its neck. The weighted gradient (Fig. 7(d)) missed small parts, like the blue parrot's beak.

Fig. 8 shows the most substantial result, when the image was reduced to 25 regions. It is possible to perceive the degradation of the performance of both structure tensor and weighted gradient in Fig. 8(b) and Fig. 8(d). Their segmentation results show that important parts of the parrot were merged with the background. Also the morphological gradient (Fig. 8(c)) started missing some details, like the blue parrot's head. In this case, the Frobenius norm TMG (Fig. 8(a)) was the only one to preserve all the most important regions of the original image. The structure tensor in all cases provided the worst segmentation results (Fig. 6(b), Fig. 7(b) and Fig. 8(b)).

Another two images were also reduced to 25 regions using the four different gradients (TMG, structure tensor, morphological Gradient and weighted gradient): the airplane image shown in Fig. 9 and the peppers image shown in Fig. 10. In the experiment with the airplane image the structure tensor provided again the worst result (Fig. 9 (b)), segmenting badly the aeroplane, the shadow, the man and the ground. Weighted gradient provided a better result (Fig. 9 (d)) but several parts of the image were not correctly segmented.

Both morphological gradient and TMG provided good results (Fig. 9 (c) and (a), respectively), but the result pro-



(a) TMG (b) Structure Tensor

(c) Morphological Gradient (d) Weighted Gradient

Figure 8. Watershed segmentation of the "parrots" image with 25 regions using different color image gradients

vided by the morphological gradient failed to preserve a few important lines that appeared in the result provided by the TMG. Again, TMG provided the best result.

In Fig. 10(b) is possible to see that the structure tensor was not able to segment the original image at all. The segmented regions do not correspond to any object contained in the original image and most of watershed lines do not coincide with any border from the original image. The other three gradients were able to segment the most important regions, and is not possible to judge which presented the best result. While the TMG (Fig. 10(a)) inserted a useless line in the biggest red bell pepper, both morphological (Fig. 10(c)) and weighted gradients (Fig. 10(d)) segmented it as one single object. In the other hand, the TMG was the only gradient that was able to segment correctly the biggest green bell pepper. It missed only a small portion in the back of the pepper, while the morphological gradient missed a big portion of the bottom and the weighted gradient missed the upper right contour of it. In summary, Fig. 10 shows that all gradients (except the structure tensor) obtained equivalent segmentation results.

6. Conclusions

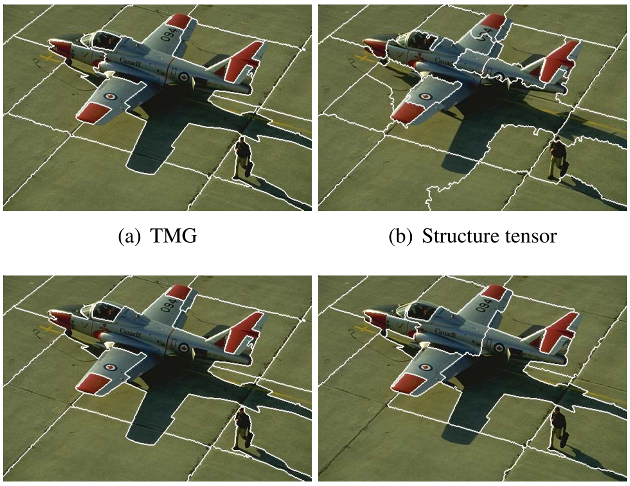
This paper brings three main contributions: (i) a new tensorial representation of a color image; (ii) the Tensorial Morphological Gradient (TMG); (iii) a new methodology for comparing segmentation results.

The new tensorial representation was obtained by establishing a relation between the attributes of the ellipse and the attributes of the HSI color model. Although in this paper segmentation was chosen to exemplify the utility of the tensorial representation of color images, several other applications could make use of it. Adopting the tensorial representation for color images allows one to apply tensorial techniques to solve a great variety of color images problems.

Based on this new tensorial representation and using tensor dissimilarity functions, a Tensorial Morphological Gradient (TMG) was proposed. Different tensorial dissimilarity functions were implemented (the dot product, the tensor dot product and the Frobenius norm) and the resulting TMGs were evaluated when applied on segmentation by the watershed transform.

For this evaluation, a new comparison methodology was proposed, where segmentation is done by applying the watershed transform and imposing the markers to the n basins with the greatest volume dynamics. As consequence, the proposed methodology does not search for good segmentation results in order to compare them, but imposes restrictions (in this case, number of regions) and compares the performance of each gradient under such restrictions.

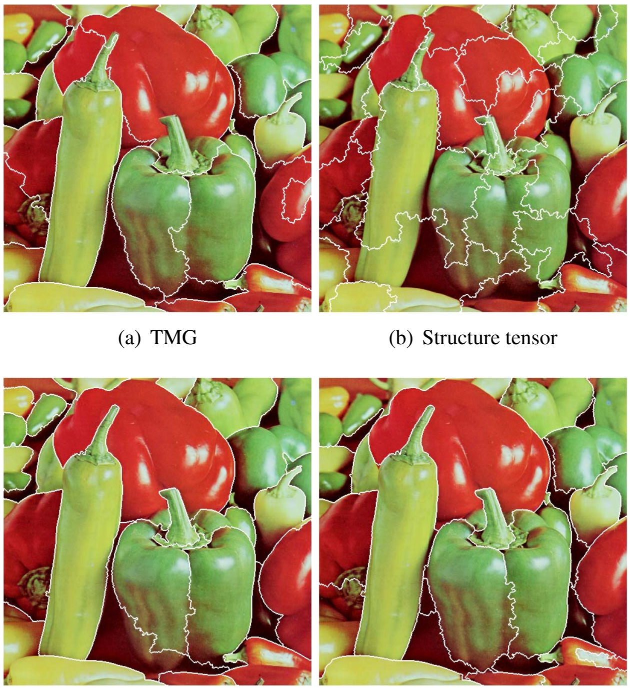
Segmentation experiments were conducted according to the proposed methodology. The TMG based on the Frobenius norm dissimilarity function presented the best segmentation results, in comparison to the other tensor dissimilarity functions. It also obtained better segmentation results when compared to other color image gradients.



(a) TMG (b) Structure tensor

(c) Morphological gradient (d) Weighted gradient

Figure 9. Watershed segmentation of the "airplane" image with 25 regions using different color image gradients



(a) TMG (b) Structure tensor

(c) Morphological gradient (d) Weighted gradient

Figure 10. Watershed segmentation of the "peppers" image with 25 regions using different color image gradients

Acknowledgments

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