Analysis of Medical Image Sequences by Recursive Polynomial Registration

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Abstract. This paper presents a new technique of polynomial image registration, applying recursive least squares techniques for the estimation of the mapping function coefficients. The described technique can be used with efficiency for registration of images of non rigid bodies with low frequency local variations. In addition, the incorporation of weights for the samples is foreseen, reflecting the visual trust in the acquired control points. The technique is applied in the construction of a system for aid in the differential diagnosis of lung diseases, with low computational cost and larger interaction with the operator, since the recursive nature of the algorithm facilitates the update of the registration during the acquisition of the control points.

1 Introduction

It is common, in medical practice, to interpret lung x-rays by visual comparison with previous radiographs of the same patient. These comparisons help the specialist to identify abnormalities and detect pathological changes. Kano et al [KANO 94] report that, in the hospitals of the University of Chicago, about 80% of the x-rays are interpreted visually side by side, in comparison with previous x-rays. However, it has also been reported [GREENE 92], [AUSTIN 92] that important changes in time have not been observed during the analysis of sequential lung x-rays.

This work results of the study, design and implementation of a system to aid the medical diagnosis, capable of enhancing differences between images acquired in different times. The implemented system performs the image registration, which is the operation responsible for the alignment or overlap of the patient's images, and posterior subtraction of the aligned images enhancing their differences. In the course of this work we analyzed broncodisplasy cases, with x-rays images provided by the Radiology Service of the Faculdade de Medicina de Ribeirão Preto - USP, partner entity in the development of this project.

In the theoretical side, we present a new approach for polynomial image registration, capable to work with non rigid objects. A new methodology is presented for the inference of the coefficients in the mapping function, besides the traditional technique of pseudo-inverse (calculation of the least square error), where the inference is made recursively and interactively with the system operation. Finally, the system also provides the incorporation of weights to different control points, assigned by the operator, based on the visual confidence on those points.

2 The Diagnosis Process

In practice, the diagnosis process usually comprises four steps: (1) identification of the problem or problems that require investigation; (2) consideration of the diseases or syndromes that may be potential causes of the specific problem; (3) formulation and initiation of a diagnostic strategy where tests are used as diagnostics maneuvers, in a sequential and systematic way, that will identify the specific etiology of the problem, reducing the delay, the cost, the pain and the risk; and (4) evaluation of the accumulated data to determine if diagnosis was established.

In most cases of lung disease is advisable to acquire preliminary basic data, including a history, physical exams, standard radiological studies and blood/urinary "routine" exams, before taking a decision with respect to the selection of the problem or problems that will constitute the reference point for future research. In most cases, it will be a radiology change that will show the anomaly, for example a lung nodule.

The radiological exam is probably the most important tool in the detection, study, differential diagnosis and handling of lung diseases. In any other field of the medicine, with the possible exception of orthopedical surgery, radiology came to have a dominant role. In fact, what is revealed by the x-ray of the thorax is the primary basis for the elaboration of a differential diagnosis in most cases of lung disease.

When the breathing symptoms and the physical signals of abnormality are minimum or absent, the thorax radiography can supply the only proof of any lung disease existence. In some cases, when the symptoms and physical signals supply unequivocal proofs that we have a lung disease, the radiological exam indicates the location and the extension of the disease with incomparable precision and provides valuable tracks for the more probable diagnostic hypotheses. Sequential lung x-rays demonstrate the tiniest progression or regression of lung lesions, with a sensitivity that doesn't have similarity with any other diagnosis modality.

The analysis and the interpretation of the x-rays of the thorax is an art and a science thoroughly described in dedicated books to the subject as [LILLINGTON 79], which served as the basic reference for this section.

3   Image Registration

3.1   Introduction

Image registration techniques are researched and applied in several areas as remote sensing or medicine, for example. In this last one, a great number of digital techniques requests the combination of multiple images [ALTHOF 97], as in the case of this study, that involves the analysis and subtraction of images acquired at different times. The image registration process may be divided in several steps [BROWN 92]: feature detection, determination of a mapping function and interpolation of the observed image. These steps also depend on several techniques, models or specific knowledge for their implementation. This section presents, in brief way, some terms (or techniques) used in this work, seeking a global presentation of the developed study and the image registration field.

The image registration corresponds to the determination of a mapping function between two images, that constitutes a mathematical model of the geometric distortion. By registering two images of the same scene we mean the alignment of them so that the points of the images that correspond to the same objects coincide [ERTHAL 85].

An usual method to perform the image registration, is through the arbitrary choice of a set of points in the first image and, for each one of them, determine its correspondent in the other. This set of points receives the denomination of control points. It is through them that rotational, translational and shearing differences are determined, allowing an image to be aligned to the other correctly.

The objective of the registration process is, then, to perform the overlap of the images in the closest possible way. Translations, rotations and scale changes and also more complex geometric transformations are used for this.
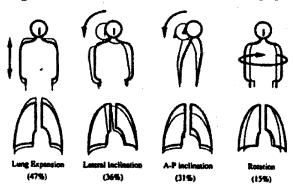
Mascarenhas and Velasco [MASCARENHAS 89], explain the importance of image registrations in several situations. Its use is common in the area of remote sensing, as shown by several authors, for example Richards [RICHARDS 86] and more recently Fonseca [FONSECA 98], among several others. For instance, when images are obtained by different sensors and one wants to know the response of the same point to different sensors. Other examples occur when someone wants to compare images obtained on different dates; when we have images acquired from different positions and one intends to obtain three-dimensional information of the scene and when we want to register an image obtained by sensors with a map. Other applications are in the analysis of images sequences or in computerized tomography.

3.2   Distortion Correction

When one image is acquired, it can contain geometric distortions that change its geometry, so that the position, the size and the form of the pixel can be changed. Several sources of such geometric errors can be cited, as the rotation of the object during the acquisition process, among others [RICHARDS 86]. Specifically in the area of remote sensing, systematic distortions, as those caused by the rotation of the earth and non-polar orbit of the satellite, can be corrected using orbital models and data calibration.

In the medical case, we should consider the distance and the rotation between the source of ray-X and the film, besides the fact that the human body can not be considered as a rigid body. Kano et al. [KANO 94]

studied the causes of the distortions in thorax radiographs, classifying them in four types of distortions: lung expansion, lateral inclination, A-P (anterior-posterior) inclination and rotation. The displacement of the patient's body parallel to the film was not considered. The illustration below demonstrates the four types of distortions and the frequencies of incidences of each one. In some cases the geometric distortion is caused by the combination of two factors, that results in a total frequency larger than 100%. The frequency of distortions due to at least one of the mentioned factors is 86% [KANO 94].
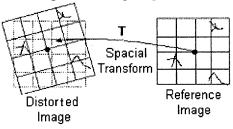
Figure 3-1. Causes of distortions in thorax radiographs

Lung Expansion (47%)   Lateral Inclination (36%)   A-P Inclination (31%)   Rotation (15%)

It is also apparent that the geometric distortions are complicated, in general, due to the fact that the x-rays correspond to two-dimensional projections of three dimensional objects. Therefore, in the case in study, it is necessary to use a non-linear model of distortions (warping) of an image relatively to the other to obtain a precise registration.

As mentioned before, the registration of two obtained images of a same scene can be understood as the operation that matches this pair of images. The image considered as the reference pattern, on which the second is superimposed, it is called reference image, and the image to be registered with the reference image is called distorted image, as is illustrated in the picture below.

Figure 3-2. Image Registration

Distorted Image

Reference Image

Therefore, to correct the distortion, we need essentially to reallocate the pixels from their current position on grid data to a grid of specified reference. The basic techniques involve three steps [SCHOWENGERDT 97]: (1) selection of one (or several) mathematical model of distortion, (2) space transformation and (3)resampling or interpolation. The union of the three steps described above is known as warping [WOLBERG 92].

By defining two images as two-dimensional functions I_1 and I_2 where $I_1(x,y)$ and $I_2(x,y)$ represent the pixels values of intensity, we can express the mapping between these two images as:

$$I_2(x, y) = g(I_1(f(x, y))) \qquad (3.2\text{-}1)$$

where f is a 2D transformation function in spatial scale. This means that f is a transformation that maps two space coordinates, x and y in two new coordinates x' and y'.

$$(x', y') = f(x,y) \qquad (3.2\text{-}2)$$

and g is the radiometric intensity of the function.

$$I_2(x, y) = I_1(f_x(x, y), f_y(x, y)) \qquad (3.2\text{-}3)$$

After the acquisition of the control points we have one group of point pairs that can be used to determine the transformation function, described above, that models the distortion mathematically between the reference and distorted images.

The mapping functions used to align two images can be global or local. A global transformation is given by a single equation that registers in a optimum way all the pixels of the two images. The local transformation is described by several equations, where each one is used for a certain area of the image. Local transformations, depending on the type of distortion, can be more precise, but they demand a larger computational cost.

3.3 Spatial Transforms

The fundamental characteristic of any registration technique is the type of the space transformation or mapping used to match the two images properly[BROWN 92] [RICHARDS 86].

The most general transformations are: rigid, affine, projective, perspective and global polynomial.

Rigid transformations involve movements of sensors or objects, where the objects keep their relative size and appearance (objects do not deform). A rigid body transformation is composed by rotations, translations and scale changes.

Affine transformations are more general than the rigid transformations and they can tolerate more complicated distortions, maintaining some mathematical properties as parallelism between lines of the two images.

Projective and perspective transformations consider the distortion due the projection of the objects at different distances of the acquisition sensor into the image plane.

Polynomial transformations are the more generic global transformation studied, and they can handle many types of distortions since they do not have a great variation over the image.

3.4 Polynomial Transform

The polynomial model described by Wolberg [WOLBERG 92] is an important tool used in this work. This model has some characteristics [BROWN 92] that make it more appropriate to handle the distortions found in our problem. We shall suppose that the registration of two images can be made by the pair of mapping functions f and g so that:

$$u=f(x,y) \text{ and } v=g(x,y) \qquad (3.4-1)$$

Usually the form of the mapping function, described above, is not known and a general mapping function is necessary. It is common practice the use of simple polynomials transformations with first, second or third degree[RICHARDS 86]. The generic form of these functions are

$$u = \sum_{i=0}^{N} \sum_{j=0}^{i} a_{ij} x^{i} y^{j-i} \qquad (3.4-2)$$

$$v = \sum_{i=0}^{N} \sum_{j=0}^{i} b_{ij} x^{i} y^{j-i}$$

where a_{ij} and b_{ij} are the constant polynomials coefficients. The number of polynomial coefficients is given by $(N+1)(N+2)/2$ for some N, where N is the non-negative degree power of the polynomial. The coefficients, in order, are associated with the terms:
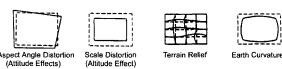
$$1, x, y, x^2, x*y, y^2, ..., x^N, x^{(N-1)}*y, ..., x*y^{(N-1)}, y^N$$

The formulation of this transformation was used initially in the area of remote sensing. However, the use of this proposed theory has been studied in other areas by several authors as [KANO 94] in medical imaging or [ROSENFELD 82] in computer vision.

The polynomial transformation given above is a global mapping function, acting on the whole image. It is plausible to use it in the correction of several distortions caused by acquisition errors. The inference is the major problem in this method, and some techniques will be described later in this section. Bernstein in [BERNSTEIN 71] classified the effect of some errors, caused by distortions in the acquisition process, that can be treated with the polynomial transformation technique. They are shown below:

Figure 3-3. Common geometric distortions

Centering　Size　Skew　Scan Nonlinearity　Radially Symmetric　Tangentially Symmetric

TYPICAL SENSOR INTERNAL DISTORTIONS

Aspect Angle Distortion (Attitude Effects)　Scale Distortion (Altitude Effect)　Terrain Relief　Earth Curvature

TYPICAL EXTERNAL IMAGE DISTORTIONS

The polynomial model is sufficiently general to represent several types of geometric transformations. The model of first degree can represent an affine transformation composed of rotation, scale and translation of the axes. The model of order 2 is appropriate to handle, for instance, perspective distortions.

Since, in this work, we found distortions of low frequency, the modeling using low order polynomials is justified. Common values for the degree N of the polynomial that are usually applied include N=1 to N=4. For most applications the second or third degree polynomials are enough [LILLESTRAND 72][NACK 77]. Polynomials of high degree generate results with good precision in the neighborhood of the control points, but they can generate distortions in the rest of the resulting image.

3.5 Inference of the polynomial coefficients

For the inference of the polynomial mapping function coefficients, the knowledge of some additional information is necessary as, for example, the data supplied by the control points. There are several techniques for the solution of the problem of coefficients inference.

3.5.1 Pseudoinverse Solution

The result usually used in practice for obtaining the polynomial coefficients is known by pseudo-inverse solution, and uses the method of the least square error to estimate the desired parameters [WOLBERG 92]. That technique is described in this section.

Let the correspondence be established among **M** points in the distorted and reference images. The space transformation that approximates the correspondence is given by a **N** degree polynomial. The number **K** of coefficients is given by:

$$K = \sum_{i=0}^{N} \sum_{j=0}^{N-i} 1 = \frac{(N+1)(N+2)}{2} \qquad (3.5-1)$$

For example, a second degree approach requests the determination of six coefficients. In that case, N=2 and K=6. We have then for M>=6.

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ \vdots \\ \vdots \\ u_M \end{bmatrix} = \begin{bmatrix} 1 & x_1 & y_1 & x_1 y_1 & x_1^2 & y_1^2 \\ 1 & x_2 & y_2 & x_2 y_2 & x_2^2 & y_2^2 \\ 1 & x_3 & y_3 & x_3 y_3 & x_3^2 & y_3^2 \\ & & \cdots & & & \\ & & \cdots & & & \\ 1 & x_M & y_m & x_M y_M & x_M^2 & y_M^2 \end{bmatrix} \cdot \begin{bmatrix} a_{00} \\ a_{10} \\ a_{01} \\ a_{11} \\ a_{20} \\ a_{02} \end{bmatrix} \quad (3.5-2)$$

A similar equation is built for v and b_{ij}. Both expressions can be written in matrix form, as:

$$u = H.a \quad (3.5-3)$$
$$v = H.b$$

Multiplying both sides by H^T

$$H^T u = H^T H.a$$
$$H^T v = H^T H.b$$

Solving for a and b, leads to:

$$a = (H^T H)^{-1} H^T u \quad (3.5-4)$$
$$b = (H^T H)^{-1} H^T v$$

That technique is known as pseudoinverse solution for the linear least square error problem. In the next sections, we will use the following notation:

- θ indicates the vector of parameters to be estimated (in the previous case described by a or b)

- x indicates the vector of samples (in the previous case described by u or v)

We will also use the formulation of linear weighted least squares given by:

$$\hat{\theta} = (H^T WH)^{-1} H^T Wx \quad (3.5-5)$$

that is an extension of the expression given above by the pseudoinverse solution. The W matrix represents a weighting factor, usually given by the inverse of covariance matrix of the noise of the observations [BOX 92] [KAY 93].

3.5.2 Sequential Least Squares

In many signal processing applications the received data are obtained sequentially. As the data are obtained simultaneously with the processing, an increasing amount of data is available for statistical inference of the parameters. In the case in study, a similar process of data acquisition happens, because the user acquires the control points and wishes to see the result immediately. An option to the traditional method of calculation by the pseudoinverse solution would be the use of the sequential processing or recursive computation in time.

Assume that we have the least squares error estimator $\hat{\theta}$ based on the data series {x[0],x[1],...,x[N-

1]}. In face of a new observation x[n], we will describe a form to update $\hat{\theta}$ (starting from the new sample and the current result) without the need for solving the system of equations described by (3.5-4). That methodology will be called sequential least squares error method, differing of the original approach that needs to process all points for the parameters estimation. The original approach will be referred as "batch processing".

The complete derivation and detailed theory about the sequential linear least squares estimator, can be found in the appendix 8A of [KAY 92]. We have the following result: consider the minimization of the minimum square error criterion (J) in the algorithm of the linear least square error, where we have $W=C^{-1}$, where C denotes the covariance matrix of the observation noise with zero mean or

$$J = (x - H\theta)^T C^{-1}(x - H\theta) \quad (3.5-6)$$

We know that the expression for the estimator is given by (3.5-5):

$$\hat{\theta} = (H^T WH)^{-1} H^T Wx$$

or

$$\hat{\theta} = (H^T C^{-1} H)^{-1} H^T C^{-1} x \quad (3.5-7)$$

Where C is the covariance matrix of the noise. We also have

$$C_{\hat{\theta}} = (H^T C^{-1} H)^{-1} \quad (3.5-8)$$

where $C_{\hat{\theta}}$ is the covariance matrix of the $\hat{\theta}$ estimator. We wish to use the sequential computation of the $\hat{\theta}$ estimator. Define:

$$C[n] = diag(\sigma_0^2, \sigma_1^2, ..., \sigma_n^2)$$
$$H[n] = \begin{bmatrix} H[n-1] \\ h^T[n] \end{bmatrix} = \begin{bmatrix} n \times p \\ 1 \times p \end{bmatrix}$$
$$x[n] = [x[0] \quad x[1] \quad ... \quad x[n]]^T$$

and denoting the least squares estimator by $\hat{\theta}$, based on x[n] or on (n+1) data samples, then the batch estimator is given for

$$\hat{\theta}[n] = (H^T[n]C^{-1}[n]H[n])^{-1} H^T[n]C^{-1}[n]x[n]$$

with covariance matrix

$$C_{\hat{\theta}} = \Sigma[n] = (H^T[n]C^{-1}[n]H[n])^{-1}$$

We must keep in mind that C[n] it is the covariance matrix of the noise, while $\Sigma[n]$ is the covariance matrix of the estimator. The resulting estimator is given by:

Estimator Update

$$\hat{\theta}[n] = \hat{\theta}[n-1] + K[n](x[n] - h^T[n]\theta[n-1]) \qquad (3.5\text{-}9)$$

where

$$K[n] = \frac{\Sigma[n-1]h[n]}{\sigma_n^2 + h^T[n]\Sigma[n-1]h[n]} \qquad (3.5\text{-}10)$$

Covariance Update

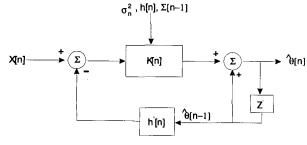$$\Sigma[n] = (I - K[n]h^T[n])\Sigma[n-1] \qquad (3.5\text{-}11)$$

The gain factor $K[n]$ is a p x 1 vector, and the covariance matrix $\Sigma[n]$ has dimensions p x p. It is of great interest that no matrix inversions are required. The estimator update is summarized in Figure 3-4. To start the recursion we need to specify initial values for $\hat{\theta}[n-1]$ and $\Sigma[n-1]$, so that $K[n]$ can be determined from (3.5-10) and then $\hat{\theta}[n]$ from (3.5-9). Graupe [GRAUPE 76] suggests the use of a high value for α on the diagonal matrix $\Sigma[-1] = \alpha I$ and $\hat{\theta}[-1] = 0$ to minimize the polarization effect, so we have:

Starting values:

$$\hat{\theta}[-1] = 0 \qquad (3.5\text{-}12)$$

$\Sigma[-1] = \alpha\,I$, where $\alpha \to \infty$.

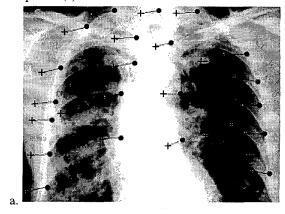**Figure 3-4.** Sequential least squares estimator

4 Results

As a result of this work we have the implementation of a tool capable to register images of non rigid bodies, in a fast and interactive way, due the characteristics of the recursive method. Moreover, a more precise method was proposed, as a result of the possibility of the association of weights by the specialist to different control points, according to his(her) confidence on them.
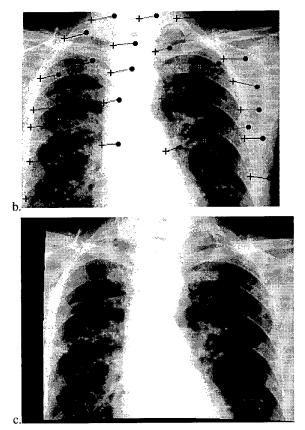
The developed method allows the operator to mark control points in the two images and to visualize the registration result immediately. As soon as the user is satisfied with the result, he can interrupt the process of points acquisition. Experience have shown that the necessary number of control points for obtaining a good result depends on the complexity of the differences between the images.

We are planning to use the implemented system for diagnosis of lung pathologies by specialists of the Hospital das Clínicas de Ribeirão Preto. However, the presented theory is general and it can be applied for registration and change detection in images from several areas of the image processing field. We will show now examples of results obtained by the use of the implemented system.

The test images were scanned with 150 dpi resolution and 256 gray levels. In the first example case, patient number 285440, we have an adult patient whose images were acquired on August 12th,1993 and October 21st, 1993 respectively. The observed distortions are due to translation, rotation and patient's thorax expansion. The illustrations in Figure 4-1 show the selection of control points in the images and the resulting registered image. In the following pictures the cross marks represent control points acquired in the reference image and dot marks the correspondent points in the distorted imaged.
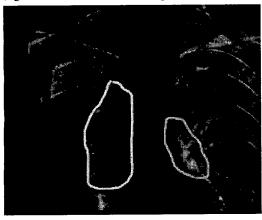
Figure 4-1 - Images (a)285440-08/12/1993, (b)285440-10/21/1993 and the respective acquired control points. Image (c) shows the result of the registration of (b) with respect to (a).
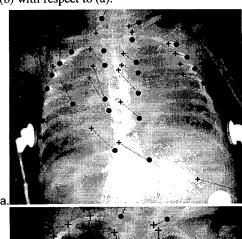
b.

c.

Visual analysis, during the execution of the program, clearly shows the evolution of the disease. A general decrease of the lesions was noticed, during the treatment. Figure 4-2 below presents the subtracted and enhanced image. In the right area there was a small increase in the concentration of the disease. The area on the left presents a reduction of the illness.
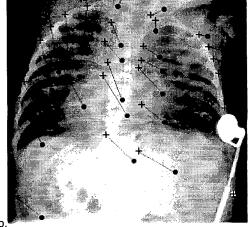
Figure 4-2 – Subtracted and contrast enhanced image (figure 4.1-c subtracted from figure 4.1-a).
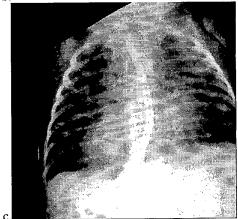

The next presented case shows the registration of a child x-ray. The registration was performed using third degree polynomials.

Figure 4-3 - Images (a) 0253910-19/09/1990, (b) 0253910-01/10/1990 and the respective acquired control points. Image (c) shows the result of the registration of (b) with respect to (a).

a.

b.

c.

5 Conclusions

Although the use sequential techniques is common in the estimation theory and signal processing fields, this work demonstrated its application in the area of image registration, and its ability to implement a registration system that is faster and more interactive for the final user.

The sequential method has a number of technical advantages when compared with the classic pseudoinverse method. We can mention as benefits:

1. The method does not require matrix inversions. The computational cost of the inversion algorithm is high and this fact turns the sequential algorithms more effective.

2. The gain factor $K[n]$ depends on our confidence in the new acquired data sample. This makes possible the visual association of weights during the selection of the control points resulting in a more accurate registration.

3. It is not necessary to store all the control points for the calculation. In the proposed method the vector of estimated coefficients matrix is updated while new points are acquired and its dimension does not increase with new data.

4. The accuracy, robustness and reliability of the discussed method is the same of the traditional method described in section 3.5.1, except when the user aquires a very small number of control points. As the numbe of control points increase, the sequential method converges for the same result.

The next step in this work is to implement the recursive outdating of control points, that will provide the user with the possibility of removing points without the need of recalculation of the entire system. Also under preparation is the recursive bayesian estimation of the polynomial coefficients.

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