Connected Filtering by Graylevel Classification Through Morphological Histogram Processing

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Abstract. This paper introduces a new connected filter which provides image simplification in terms of flat zone and graylevel reduction. This filtering consists in application of morphological operators to the histogram. It is done because each object in the image has a significative distribution of graylevels in the histogram. In other words, that is enough to classify the distributions in the histogram for simplification of that objects. It is also shown some experimental results, where the proposed method is compared to other classical ones applied to flat zone and graylevel reduction.

1 Introduction

Connected Filters [2, 13, 8, 11] belong to a family of morphological operators whose property is the reduction of flat zones in an image (connected regions of pixels with constant graylevels) without introduction of borders. However it can supress some borders, when they join two or more flat zones in only one.

These filters have a great number of applications. They can be, for example, applied to image compression and image segmentation [12, 1]. They are also applied to reduce the statistics of the image, in order to simplify the number of attributes used in pattern recognition techniques [10, 4].

Another important research area in digital image processing and computer graphics is color quantization. The importance of image simplification by color reduction is clear when dealing with problems of image display and image compression [5].

The objective of this paper is to propose a new connected filter which gives not only an image simplification in terms of flat zone reduction but also in terms of graylevel reduction. It is done by application of a composition of morphological operators to the image histogram. The main motivation behind the project of this operator is that each object in the image has a significative graylevel distribution. So, to simplify an object in the image, that is enough to classify its corresponding distribution in the histogram.

It were found in literature some works dealing with the problems presented above [15, 11, 6, 5]. It were done some experiments in order to compare their results under the point of view of quality result and flat zones and graylevel reductions.

Section 1 presents some preliminar definitions. Section 2 proposes the technique to connected filtering by graylevel classification and a variation when it is possible to choose the desired number of graylevels in the resulting image. Section 4 presents some experimental results and in the Section 5 we conclude this paper with a brief discussion.

2 Preliminary Definitions

Let $E \subset \mathbb{Z} \times \mathbb{Z}$ be a rectangular finite subset of points. Let $K = [0, k]$ be a totally ordered set. Denote by $Fun[E, K]$ the set of all functions $f : E \to K$. An *image* is one of these functions (called graylevel functions). Particularly, if $K = [0, 1]$, f is a binary image. An *image operator* (*operator*, for simplicity) is a mapping $\psi : Fun[E, K] \to Fun[E, K]$.

Let $N(x)$ be the set containing the *neighbourhood* [9, 3] of x, $x \in E$. We define a *path* [9] from x to y, $x, y \in E$ as a sequence $C = (p_0, p_1, ..., p_n)$ from E, where $p_0 = x$, $p_n = y$ and $\forall i \in [0, n-1], p_i \in N(p_{i+1})$.

A *connected subset* of E is a subset $X \subset E$ such that, $\forall x, y \in X$, there is a path C entirely inside X.

Let $f \in Fun[E, K]$. A *flat zone* of f is a connected subset $X \subset E$, such that $f(x) = f(y)$, $\forall x, y \in X$.

Definition 1 *Let $f \in Fun[E, K]$. An operator ψ is a connected operator if and only if (iff)*

$$f(x) = f(y) \Rightarrow \psi(f)(x) = \psi(f)(y),$$

$x, y \in E$, $x \in N(y)$.

Connected filters reduce the number of flat zones [2, 13, 8, 11] without introducting borders. Notice that when some flat zones in an image are reduced, some borders may be supressed, but it does not occur to appear new borders.

Definition 2 *The inf - reconstruction and sup - reconstruction operators are given, respectively, by,* $\forall f, g \in Fun[E, K]$,

$$\rho_{B,g}(f) = \delta^{\infty}_{B,g}(f)$$

$$\rho^{*}_{B,g}(f) = \varepsilon^{\infty}_{B,g}(f)$$

where $B \subset E$ *is the structuring element,* $n \in \mathbb{Z}_+$ *and* $\delta^n_{B,g}$ *and* $\varepsilon^n_{B,g}$ *are, respectively, the n-conditional dilation and the n-conditional erosion operators [14, 7].* $\delta^{\infty}_{B,g}(f)$ $(\varepsilon^{\infty}_{B,g}(f))$ *means that the dilation (erosion) is applied till idempotency.*

Let $\tau_i : Fun[E, K] \rightarrow Fun[E, [0, 1]]$, $i \in K$, be a threshold function, where $\tau_i(f)(x) = 1$, if $f(x) \geq i$, and $\tau_i(f)(x) = 0$, otherwise.

Definition 3 *Let* $f \in Fun[E, K]$. *A regional maximum is a flat zone* Z *such that* $f(z) > f(n)$, $z \in Z$, $n \in N$, $N \in \mathcal{F}_Z$, *where* $\mathcal{F}_Z$ *is a set of all flat zones adjacent to* Z *[3]. The regional maxima of* f *is found by application of a operator* $\mu^{\max}_{B_c} : Fun[E, K] \rightarrow Fun[E, [0, 1]]$, *given by*

$$\mu^{\max}_{B_c}(f) = \tau_1(\rho_{B_c,(f+1)}(f)) \vee \tau_k(f)$$

where $B_c \subset E$ *is the structuring element defining connectivity.*

3 The Proposed Technique

Let us introduce a new technique applied to connected filtering. One characteristic of the proposed filter is that, despite its connecting property, it does not require a connectivity parameter (4-connect or 8-connect) because all processing is done in the histogram of image.

As a consequence of such processing we have a reduction in the graylevels appearing in the image. In other words, the proposed filter is a mapping $\psi : Fun[E, K_1] \rightarrow Fun[E, K_2]$, where $|K_2| < |K_1|$.

Let $f \in Fun[E, K]$. Let us consider the histogram of the image as a function $h_f : K \rightarrow \mathbb{Z}_+$ (Fig. 1). Despite the domain of h_f, the morphological operators used in the graylevel classification were applied to in the same manner. In other words, we consider K as a subset of E.

Let $f \in Fun[E, K]$ and $h_f \in Fun[K, \mathbb{Z}_+]$ its histogram. Let $\max(h_f) = \max\{h_f(x) : x \in K\}$. Let

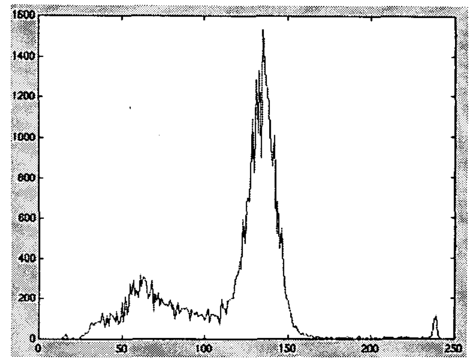


Figure 1: Histogram

$\kappa : Fun[K, \mathbb{Z}_+] \rightarrow Fun[K, \mathbb{Z}_+]$ be the mapping, given by, $\forall x \in K$,

$$\kappa(h_f)(x) = \begin{cases} h_f(x), \text{ if } \mu^{\max}_{B}(h_f)(x) = 1 \\ 0, \text{ otherwise} \end{cases},$$

where $B \subset K$ is the structuring element [7]. The mapping κ gives a function with only the regional maxima of h_f. Figure 2 shows the regional maxima of the histogram shown in Fig. 1.

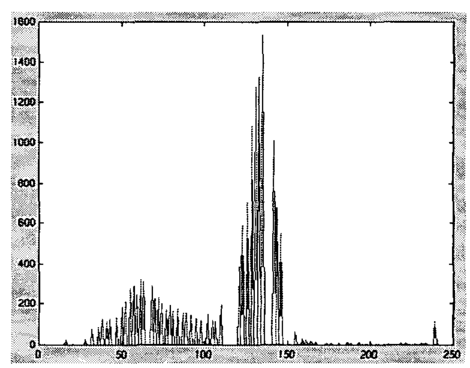


Figure 2: Histogram regional maxima

Let $\pi : Fun[K, \mathbb{Z}_+] \rightarrow Fun[K, \mathbb{Z}_+]$ be the mapping, given by, $\forall x \in K$,

$$\pi(h_f)(x) = \begin{cases} \max(h_f), \text{ if } \kappa(h_f)(x) = 0 \\ 0, \text{ otherwise} \end{cases}.$$

Let $\eta : Fun[K, \mathbb{Z}_+] \rightarrow Fun[K, \mathbb{Z}_+]$ be the sup-reconstruction of κ, denoted by,

$$\eta = \rho^{*}_{B,\kappa}(\pi),$$

where $B \subset K$ is the structuring element.

If exists in $\kappa(h_f)$ a sequence of increasing regional maxima followed by a sequence of decreasing regional maxima, its regional maxima must compose a curve whose regional maximum is the maximum among them. The operator η is applied to the function $\kappa(h_f)$, in order to preserve the regional maxima among the set of regional maxima of $\kappa(h_f)$ and construct the curves with the remaining regional maxima (Fig. 3).

The reason of the processing of the graylevel distributions in the histogram is that each object in the image is represented by a significative graylevel distribution. The idea is to filter the histogram in order to get new distributions where the objects are simplified and well represented. These new distributions are used to get a meaningful classification of graylevels by application of watershed operator introduced below.

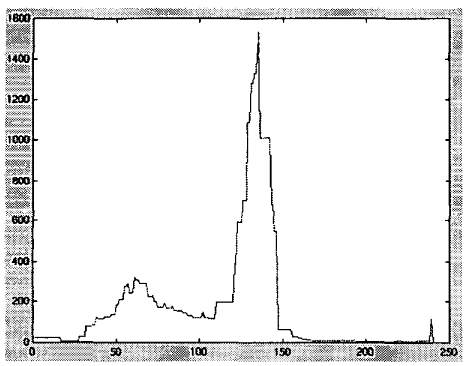


Figure 3: Reconstruction by application of η operator

Let $\omega : Fun[K, \mathbb{Z}_+] \to Fun[K, [0, 1]]$ be the watershed operator [1, 16], which detects the exact borders of objects, by flooding the image from the regional minima assigned to such objects. Let $\nu : Fun[K, \mathbb{Z}_+] \to Fun[K, \mathbb{Z}_+]$ be the negation operator.

Let $\lambda : Fun[K, \mathbb{Z}_+] \to Fun[K, \mathbb{Z}_+]$ be the mapping, given by, $\forall x \in K$,

$$\lambda(x) = \begin{cases} \max(h_f), \text{ if } \omega(\nu(\eta))(x) = 1 \\ 0, \text{ otherwise} \end{cases} .$$

The mapping λ gives a preliminary classification; the graylevel classes are separeted but not labeled (Fig. 4). The labeling of classes is given by the application of next steps.

Let $\varrho : K \to \mathbb{Z}_+$, such that $\varrho(x) = x$, $\forall x \in K$. Let $\theta : Fun[K, \mathbb{Z}_+] \to Fun[K, \mathbb{Z}_+]$ be the mapping, given by, $\forall x \in K$,

$$\theta(x) = \begin{cases} \max(h_f), \text{ if } \mu_B^{\max}(\eta)(x) = 1 \\ 0, \text{ otherwise} \end{cases} .$$

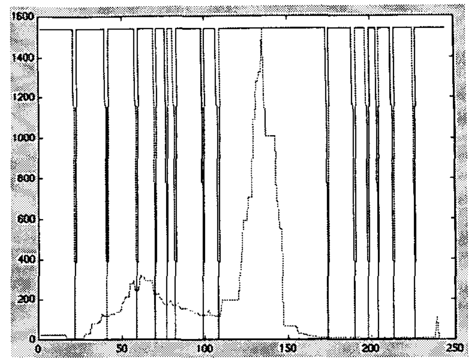


Figure 4: Pre-classification

Let $\beta : Fun[K, \mathbb{Z}_+] \to Fun[K, \mathbb{Z}_+]$ be the mapping, given by,

$$\beta = \varrho \wedge \theta.$$

The mapping β gives a function where each point of the regional maxima of $\eta(\cdot)$ is labeled by its corresponding graylevel. Its reconstruction conditioned to $\lambda(\cdot)$ gives the labeling of all graylevel classes. Figure 5 shows a composition of $\lambda(\cdot)$ (pre-classified graylevels) and $\beta(\cdot)$ (the labeled peaks inside each pre-class).

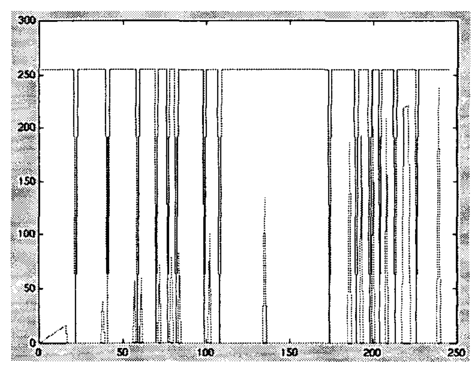


Figure 5: Function β assigning labels to each pre-class

Let $\zeta : Fun[K, \mathbb{Z}_+] \to Fun[K, K]$.

$$\zeta = \delta_B(\rho_{B,\lambda}(\beta)),$$

where $B \subset K$ is the structuring element, and $\delta_B(\cdot)$ is the dilation operator [14, 7].

Let $|K|$ be the number of distinct graylevels in K. We can say that the mapping ζ is a graylevel classificator. Given the histogram h_f, $f \in Fun[E, K]$, the classificator gives a new set of graylevels G, where $|G| < |K|$.

Definition 4 *Let $f \in Fun[E, K_1]$, $K_1 = [0, k_1]$, and $h_f \in Fun[K_1, \mathbb{Z}_+]$ the histogram of f. The* ***connected operator by graylevel classification*** *is a mapping $\psi : Fun[E, K_1] \to Fun[E, K_2]$, where K_2, $|K_2| < |K_1|$, is given by;*

$$K_2 = \zeta(h_f).$$

Since there is a reduction in the number of graylevels in the image, and it may causes the union of two or more flat zones, without split any flat zone, it is a connected operator. Figure 6 shows the graylevel classification provided by ζ, given the histogram shown in Fig. 1. The application of this operator is shown in Fig. 7. Figure 7 (a) shows the original image and Fig. 7 (b) shows the resulting one.

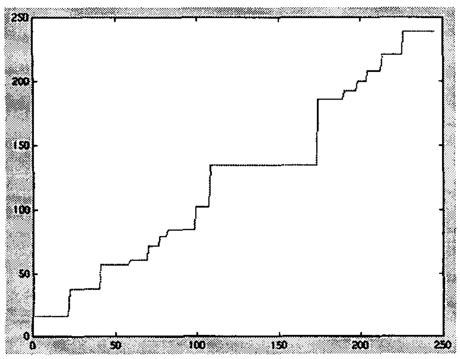
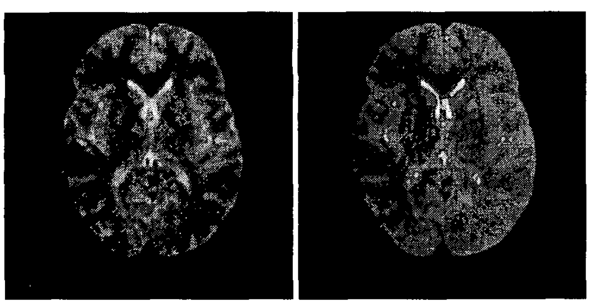


Figure 6: Classified graylevels



(a) (b)

Figure 7: (a) Original image (b) Simplified image

3.1 Reduction to n Colors

When the operator ψ is applied to an image f, it reduces the colors appearing in f to the number of regional maxima of $\eta(\cdot)$. However, it is possible to reduce the colors to a smaller number, by adding a parameter n which gives the number of colors to appear in $\psi(f)$.

We will denote by $\psi_n : Fun[E, K_1] \to Fun[E, K_2]$, $|K_2| < |K_1|$, $|K_2| = n$, the operator which performs the connected filtering by reduction of the colors in the image to n.

Let m be the number of regional maxima in $\eta(\cdot)$. Let Q be the set defined by

$$Q = \{q_i \in K : \mu_B^{\max}(\eta)(q_i) = 1 \text{ and } \eta(q_i) \geq \eta(q_{(i+1)}), i = 1, \cdots, m\}.$$

Let $\sigma_n : Fun[K, \mathbb{Z}_+] \times n \to Fun[K, \mathbb{Z}_+]$ be the mapping, given by, $\forall x \in K$, $\forall n \in \mathbb{Z}_+$,

$$\sigma_n(x) = \begin{cases} \max(h_f), \text{ if } \exists q_i \in Q : q_i = x \text{ and } i \leq n \\ 0, \text{ otherwise} \end{cases}.$$

Figure 8 shows a composition of $\eta(\cdots)$ and $\sigma_n(\cdots)$. In this example, $n = 5$.

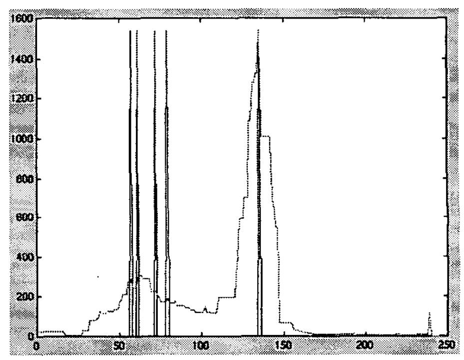


Figure 8: Markers assigned to the n highest regional maxima

Let $\eta_n : Fun[K, \mathbb{Z}_+] \times n \to Fun[K, \mathbb{Z}_+]$ be the mapping, given by, $\forall x \in K$, $\forall n \in \mathbb{Z}_+$,

$$\eta_n = \nu(\rho^*_{B,\nu(\eta)}(\sigma_n)).$$

The operator η_n is used to select the highest n regional maxima and to eliminate the remaining ones (Fig. 9). The function $\eta_n(\cdot)$ contains just n regional maxima and they are responsible for the classification of n classes (given by application of watershed operator). Compare Fig. 3 and 9 to see what were eliminated from $\eta(\cdots)$.

The connected operator ψ_n is defined by the same way that ψ, except for you must exchange $\eta(\cdot)$ for $\eta_n(\cdot)$ in the definition of λ operator.

The number of graylevels appearing in Fig. 7 (a) is 245. The application of ψ reduced the number of

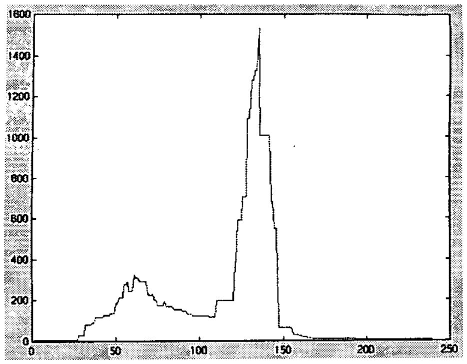


Figure 9: $\eta_n(\cdot)$ filtered with its remaining 5 regional maxima

graylevels to 15 (Fig. 7 (b)). Figure 10 shows the application of ψ_3 to Fig. 10, whose result contains 3 graylevels.

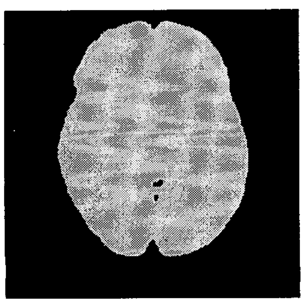


Figure 10: Simplified to 3 graylevels

4 Experimental Results

In this Section, we will introduce some experimental results. In the first experiment, we show a comparison among various connected operators applied to flat zones reduction. It were observed the reduction and the quality of the image. In the second one, we compared the proposed method to two others applied to reduction of colors.

4.1 Other Connected Operators

In this Subsection we introduce the connected operators applied to the first experiment.

4.1.1 Opening / Closing By Reconstruction

Let $f \in Fun[E, K]$. The *opening and closing by reconstruction* are defined, respectively, by,

$$\gamma_{B_{ero},B}(f) = \rho_{B,(\varepsilon_{B_{ero}}(f))}(f)$$

and

$$\phi_{B_{dil},B}(f) = \rho^*_{B,(\delta_{B_{dil}}(f))}(f),$$

where $B, B_{ero}, B_{dil} \subset E$ are structuring elements.

4.1.2 Area Opening / Area Closing

The *area opening* is a connected filter applied to binary images, aiming to eliminate flat zones valued 1 and with area lower than a threshold a. The area opening applied to graylevel images is done by a stack filter, given by the sum of binary area opening results (see below).

Let $\mathcal{B}$ be the set of all flat zones valued 1 in a binary image. Let $a \in \mathbb{Z}_+$ be an area threshold. Let $\mathcal{B}_a \subset \mathcal{B}$ be the set of all flat zones $F \in \mathcal{B}$ whose area is greater than a. The area opening operator is given by,

$$\gamma_a^{\text{area}}(f) = \bigvee_{F \in \mathcal{B}_a} \gamma_F(f),$$

where γ_B is the morphological opening [8, 3].

Let $f \in Fun[E, K]$. Let $f^i \in Fun[E, [0, 1]]$ be a binary image, defined by, $\forall x \in E$,

$$f^i(x) = \begin{cases} 1, \text{ if } f(x) = i \\ 0, \text{ otherwise} \end{cases}.$$

The area opening applied to a graylevel image f is a mapping $\Gamma_a^{\text{area}} : Fun[E, K] \to Fun[E, K]$, defined by,

$$\Gamma_a^{\text{area}}(f) = \sum_{i=0}^{k} \gamma_a^{\text{area}}(f^i).$$

Here, the domes which base area is lower than a are eliminated.

The dual operator of area opening is the *area closing*. This filter is a mapping Φ_a^{area}, given by,

$$\Phi_a^{\text{area}}(f) = \nu(\Gamma_a^{\text{area}}(\nu(f))).$$

4.1.3 Levelings

Another class of connected morphological operators proposed by Meyer [11] has the property of image simplification by by application of adaptive inf and sup-reconstructions, i.e., the reconstructions are done only in certain regions of the image. The resulting image from application of such filters, which reduce the flat zones and do not create regional maxima and minima are called *levelings*.

4.2 Comparison Among Flat Zones Reductions

The first experiment consists in applying various connected operators and to compare the reduction of flat zones among their results. It were applied four operators:

- Connected filtering by graylevel classification.
- A composition of opening and closing by reconstruction, where B_{ero} and B_{dil} are a 10×10 sized box structuring element and B is a elementar box structuring element.
- A composition of area opening and area closing, which eliminates 8-connected flat zones with area lower than $a = 100$.
- A leveling operator, used to eliminate 8-connected flat zones. The marker function chosen is the original image filtered by an alternate sequential filter[14, 7], using a 9×9 sized box structuring element and iterated 3 times.

	Flat Zones	Graylevels
Original Image	43984	232
Connec. Filt. Gray. Class.	3963	16
Open. / Close. by Rec.	6545	70
Area Opening / Closing	18289	205
Leveling	8118	65

Table 1: Comparison among flat zones reduction methods

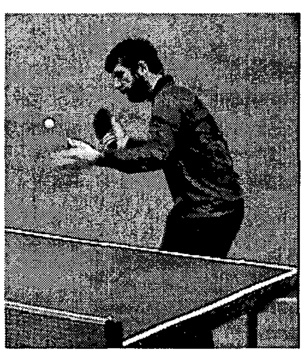


Figure 11: Original image

The four operators are applied to the image shown in Fig. 11. The results are shown in Fig. 12. Table 1 shows the reduction of flat zones in each result, and the reduction of graylevels as well.

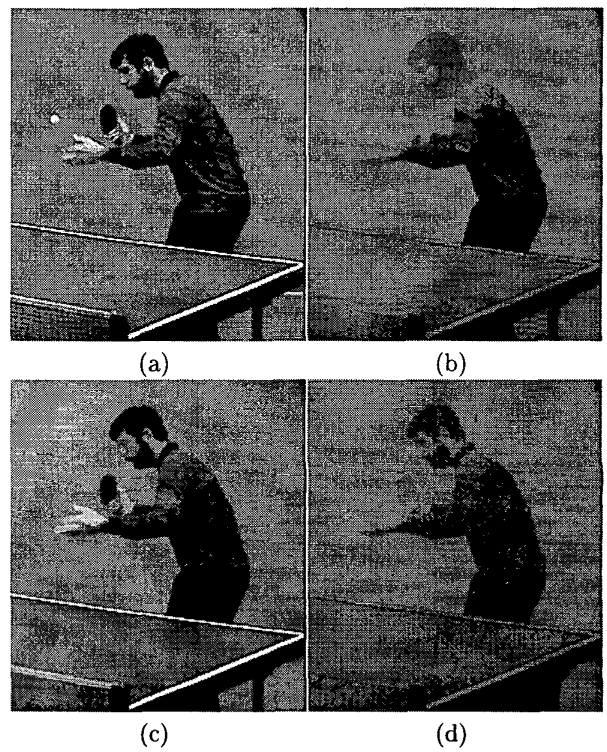


Figure 12: (a) By connected filtering by morphological histogram processing (b) By opening / closing by reconstruction composition (c) By area opening / area closing composition (d) By leveling operator

4.3 Comparison Among Graylevel Reduction Methods

This experiment consists in applying three techniques to reduce the number of graylevel of Fig. 13 (a) from 220 to 10 and to compare their results, evaluating their quality and the false borders reduction. It were applied three techniques:

- Connected filtering by graylevel classification.
- The uniform partitioning of $K = [0, 255]$, denoted by K_2, where, $\forall x \in K$,

$$K_2(x) = \begin{cases} ((\lceil x/25 \rceil) + 1) \times 25, \text{ if } x < 250 \\ 250, \text{ if } x >= 250 \end{cases}.$$

- The partitioning of $K = [0, 255]$ by a technique proposed by Heckbert, called *Median Cut*[6], where the new graylevels are given by the partitioning of the cummulative histogram, where each set of partition has the same area. Here, the cummulative histogram is partitioned in ten sets, and each set defines an interval in K, and each graylevel in one interval is changed to the average graylevel in this interval.

Figure 13 shows the result of each technique applied to Fig. 13 (a). Figure 13 (b) shows the result of the connected filtering by morphological histogram processing. Figure 13 (c) shows the result of the uniform partitioning of K. And Fig. 13 (d) shows the result of median cut technique.

Note that Fig. 13 (c), despite the reduction of colors (K were divided to 10 subsets but just 9 colors appear) it is added a lot of false borders due the quantization. More, the image is strongly simplified but some important regions are lacking; the hair is lost, mixed with the background.

Figure 13 (d) shows a better result but despite the lower number of false borders, it still have a lot of them. In addition, the contrast among the false borders is not so smooth.

The connected filter applied to histogram presents the best result (Fig. 13 (b)). It gives the desired reduction with a reduced number of false borders and without object losses. The advantage of our approach compared to median cut is that the second technique ignores the distributions in the histogram with low density [5]. It does not occur in our approach: it classifies all significative distributions in the histogram, even the lowest densities ones.

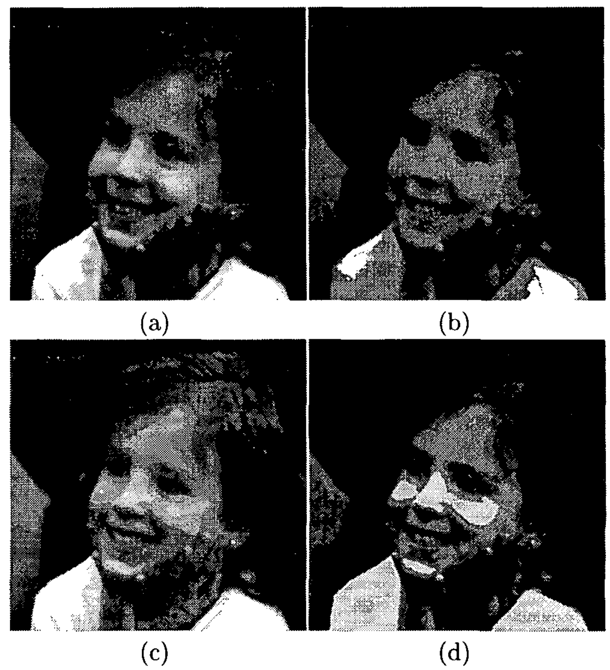


Figure 13: (a) Original Image (b) By connected filtering by morphological histogram processing (c) By uniform partitioning of the graylevels (d) By application of median cut

5 Conclusion

This paper proposes a new connected filter used to simplify the image in terms of flat zones and graylevels.

The technique consists in the classification of graylevels through application of morphological operators to the histogram. Its results is a new image with a fewer regions and a simplified colormap, compared to the original image.

It were done some experiments in flat zones and graylevel reductions and were obtained good results. The idea behind the good functioning of this operator is that each object in an image is represented by a significative distribution of graylevels. The classification of such objects is simplified to the classification of their distributions in the histogram.

The proposed technique shows itself as a promising way to simplify an image both spatially and in the graylevels. Further work includes the extension of this technique to color images.

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