Image decomposition in morphological residues: An approach for image filtering and segmentation

SILVIO JAMIL FERZOLI GUIMARÃES[1], NEUCIMAR JERÔNIMO LEITE[2]

[1] Departamento de Ciência da Computação - DCC/UFMG
Av. Antônio Carlos 6627 - Campus Pampulha - Prédio do Icex - sala 4010
CEP 31270-010 Belo Horizonte - Minas Gerais - Brasil
sjamil@dcc.ufmg.br
[2]Instituto de Computação - UNICAMP - Caixa Postal 6176, 13083-970, Campinas, SP, Brasil
neucimar@dcc.unicamp.br

Abstract Morphological residues represent an image in an hierarchical way by means of a decomposition of its structures and according to a size parameter λ. From this decomposition, we can obtain a relation between the different residual levels associated with the complexity of the image structures. In this work, we introduce a method for filtering of components in gray-scale images based on the morphological residue decomposition which takes into account a size parameter and a certain level of complexity of the different structures we want to be filtered.

1 Introduction

Since the 1960s, Mathematical Morphology [1], [2], [3] has become increasingly popular in the community of digital image processing due to its rigorous mathematical description and capacity to extract information based on shape transformations.

Many works developed in this area concern mainly pre-processing and segmentation applications. Preprocessing consists in enhancing the "image syntax" to increase the success of the other operations. One example of this technique is filtering [4], [5], [6], [7], [8]. The aim of segmentation is to partition the image in its constituent parts. Segmentation techniques and problems are treated, for example, in [1], [2], [5], [6], [9], [10], [11], [12].

The computational cost of the Mathematical Morphology techniques is relatively high. Most of the time, the need to extract the information in a cost-effective way yields the use of a set of images from the same scene containing different types of information. In [13] is defined an image multi-resolution decomposition scheme, the pyramid scheme, which illustrates this kind of multiple representation of an image. This scheme encompasses both the morphological concept of granulometry [14] and morphological residues [8].

In this work, we consider the problem of filtering and segmentation through the decomposition of an image in morphological residues. This decomposition is directly associated with the notion of granulometry that describes quantitatively the coarseness of an image. Informally, as we will see in Section 2, the granulometry characterizes an image as a collection of grains that can be sieved in a grain size distribution process. It decomposes an image in classes of components, according to a size parameter, whereas the morphological residues constitute a complete hierarchical representation of the image [1], [2].

This paper is organized as follows. In Section 2, we introduce the multi-resolution scheme considered in [13], associated with the concept of granulometry and morphological residues. In Section 3, we present a new filtering method based on an image decomposition scheme, as well as some of its basic properties. In Section 4, we discuss some results of the proposed method by considering synthetic and real images. Finally, some conclusions are drawn in Section 5

2 Multi-resolution decomposition

The multi-resolution decomposition scheme represents an image by different levels of resolution or "coarseness" which, in turn, can be associated with the different amount of information we want to analyze.

A general framework for image decomposition, the pyramid scheme, is defined in [13]. The method considers two basic operations: (a) analysis, that simplifies the image representation by reducing the amount of information, and (b) synthesis, which tries to recover the information lost in the analysis step.

The combination of these operations produces an approximation of an original image X due to a partial recuperation of the image by the synthesis step. In this case, it is possible to obtain an *image of details* Y, containing the information not recovered by the synthesis step, and given by the difference

[1]This work was supported by CAPES and the FINEP/PRONEX/IC project, n. 76.97.1022.00

between the original image X and the image defined by the combination of the above operations.

We can easily see that the granulometry and the morphological residues [14] can be represented by this framework, since, as we will see next, the granulometry decreases the amount of information of an image, according to a size parameter, while the morphological residues, which can be seen as the image of details, contain the information lost between two successive granulometric levels.

2.1 Granulometry

The *granulometry*, $(\psi_\lambda)_{\lambda\geq 0}$, which depends on a size parameter λ [14], describes quantitatively the "coarseness" of an image and is a basic morphological concept used, for example, in pattern recognition. The granulometry decomposes the image in classes of components according to the used structuring element. It can be defined as follows:

Definition 2.1 (Granulometry [14]) *Let $(\psi_\lambda)_{\lambda\geq 0}$ be a set of image transformations depending on a parameter λ. This set constitutes a granulometry iff the following properties hold:*

$$\forall\lambda \geq 0, \psi_\lambda \text{ is increasing} \quad (1)$$

$$\forall\lambda \geq 0, \psi_\lambda \text{ is anti-extensive} \quad (2)$$

$$\forall\lambda \geq 0, \mu \geq 0, \psi_\mu\psi_\lambda = \psi_\lambda\psi_\mu = \psi_{max(\lambda,\mu)} \quad (3)$$

Equation 3 above implies an idempotent operation, that is,

$$\psi_\lambda[\psi_\lambda(X)] = \psi_\lambda(X) \quad (4)$$

The set of transformations ψ_λ is a decreasing set of algebraic openings given by any convex structuring element B and its homotetic representation ($\lambda B = \{\lambda b \mid b \in B\}, \lambda \geq 0$) [1].

Directly associated with the granulometry operation is the concept of morphological residues, defined below.

2.2 Morphological residues

The *morphological residues*, $\mathcal{R}$, [2] characterize the information extracted from an image by considering a set of granulometric transformations. The residues are given by the difference between two consecutive granulometric levels, as follows:

Definition 2.2 (Morphological residues[2]) *Let $(\psi_\lambda)_{\lambda\geq 0}$ be a granulometry. The morphological residues $\mathcal{R}_\lambda$, of residual level λ associated with the size parameters λ, are given by the difference between the result of two consecutive granulometric levels, that is,*

$$\forall\lambda \geq 1, X \in R^N, \mathcal{R}_\lambda(X) = \psi_{\lambda-1}(X)\backslash\psi_\lambda(X) \quad (5)$$

$$\forall\lambda \geq 1, f \in R^N, \mathcal{R}_\lambda(f) = \psi_{\lambda-1}(f) - \psi_\lambda(f), \quad (6)$$

where X and f represent a binary (set) and a gray-scale (function) image, respectively, and $\backslash$ stands for the difference between sets.

Equations 5 and 6 define, respectively, the morphological residues for binary and gray-scale images. They represent the components preserved at level $(\lambda - 1)$ that are eliminated at the granulometric level λ.

According to the transformation ψ, the set of residues corresponding to $(\mathcal{R}_\lambda)_{\lambda\geq 1}$ contains the complete granulometric information and defines a complete hierarchical representation of an image. Thus, for the binary case,

$$X = \bigcup_{\lambda\geq 1} \mathcal{R}_\lambda(X) \quad (7)$$

and, for the gray-scale case,

$$f = \mathcal{R}_1(f) + \cdots + \mathcal{R}_\lambda(f) + \cdots = \sum_{\lambda\geq 1} \mathcal{R}_\lambda(f) \quad (8)$$

Therefore, we can limit the amount of information to be analyzed by considering only the data defined in a certain resolution, as we will consider in the next section.

3 Filtering by morphological residues

The decomposition of an image in morphological residues (Definition 2.2) is very useful for the characterization of some image structure attributes, such as size, volume and shape [8]. Another attribute we will introduce in this work concerns the *vanishing level* of a point of the image. As we will see later, this attribute is related to the irregularity of the image structures, when considered as a topographic surface, and characterizes the persistence of the information between different residual levels. From these considerations we will define an image attribute based on the granulometric residues and, consequently, a new filtering method taking into account this information.

3.1 Basic definitions

Let $\Gamma_I \subset \mathcal{Z}^2$ be the domain of a binary or gray-scale image I, where each point can assume discrete values in the range $[0, L]$, L equals 1 for binary images. Let $\Phi_\phi(\Gamma_I)$ be a subset of points $p \in \Gamma_I$ obtained by a transformation, $\phi : \mathcal{Z}^2 \to \mathcal{Z}$, of image I, where the transformed points are greater or equal to zero. We can define the binary residues of an image as follows.

Definition 3.1 (Binary residue) *Let $\Phi_{\mathcal{R}_\lambda}(\Gamma_I)$, $\lambda \geq 1$, be a subset of points $p \in \Gamma_I$ so that $\mathcal{R}_\lambda(\Gamma_I(p))$ is greater or equal to zero. The binary residue of image I is given by*

$$\Phi_{\mathcal{R}_\lambda}(\Gamma_I)(p) = \begin{cases} 1, & \text{if } \mathcal{R}_\lambda(p) > 0 \\ 0, & \text{otherwise} \end{cases} \tag{9}$$

In this case, for a binary image X, we have that

$$\Phi_{\mathcal{R}_\lambda}(\Gamma_X) \bigcap \Phi_{\mathcal{R}_\mu}(\Gamma_X) = \emptyset, \lambda \neq \mu, \lambda, \mu \in N \tag{10}$$

which means that the details obtained at level λ are not present at a different level μ. Unlike the binary case, a gray-scale image G yields

$$\Phi_{\mathcal{R}_\lambda}(\Gamma_G) \bigcap \Phi_{\mathcal{R}_\mu}(\Gamma_G) \neq \emptyset, \lambda \neq \mu, \lambda, \mu \in N \tag{11}$$

which means that we do not necessarily have a successive suppression of points between two different levels of the binary residues. Instead, the value of these points can be only "smoothed" by the opening function ψ_λ.

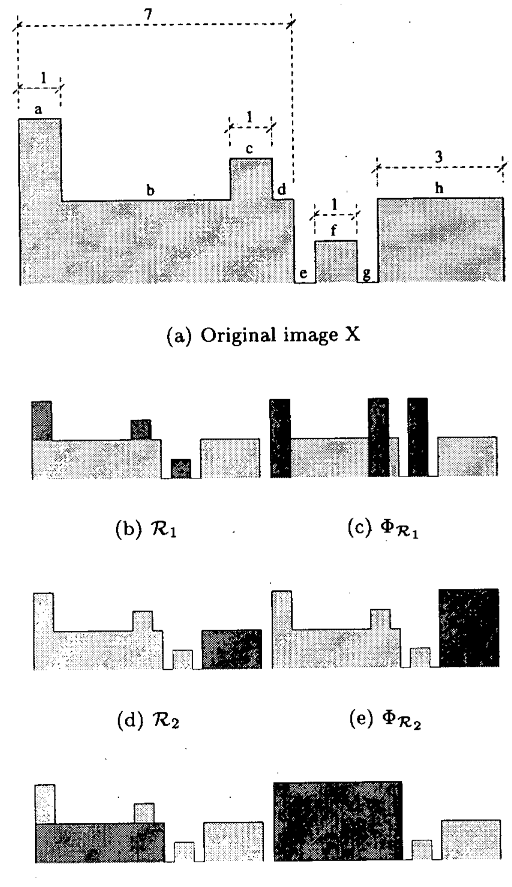
Figure 1 illustrates these aspects for a monodimensional case, where each residual level λ represents the size parameter λ associated, for example, with the radius of a flat structuring element. The dark parts in Figures 1(b), 1(d) and 1(f) are the residues $\mathcal{R}_\lambda$ of the original image X, (Figure 1(a)), whereas Figures 1(c), 1(e) and 1(g) represent the subsets $\Phi_{\mathcal{R}_\lambda}(\Gamma_X)$.

Based on the subsets $(\Phi_{\mathcal{R}_\lambda})_{\lambda \geq 1}$ of points present at different residual levels, we can associate to each point $p \in \Gamma_I$ the following residue mapping of the binary residues.

Definition 3.2 (Residue mapping) *Let $(\Phi_{\mathcal{R}_\lambda})_{\lambda \geq 1}$ be a set of binary morphological residues. For all points $p \in \Gamma_I$, we define a residue mapping, $\mathcal{M}$, containing information about a size parameter associated with each residual level λ, so that*

$$\mathcal{M}_\lambda(p) = \Phi_{\mathcal{R}_\lambda}(p) \tag{12}$$

By this mapping, we can take into account the parameter information related to the different levels in which the image points vanish (they change



(a) Original image X

(b) $\mathcal{R}_1$ (c) $\Phi_{\mathcal{R}_1}$

(d) $\mathcal{R}_2$ (e) $\Phi_{\mathcal{R}_2}$

(f) $\mathcal{R}_4$ (g) $\Phi_{\mathcal{R}_4}$

Figure 1: Morphological residues (left) and binary residues (right) of levels 1, 2 and 4.

their state from 1 to 0), according to the subsets $\Phi_{\mathcal{R}_\lambda}(\Gamma_f)$ defined from the sequence of residual operations $(\mathcal{R}_\lambda)_{\lambda \geq 1}$.

Definition 3.3 (Vanishing attributes) *The vanishing of a point is represented by the transition of its value from 1 to 0 in the residue mapping $\mathcal{M}$. Informally, the vanishing attributes are related to the moment a given point does not belong to the morphological residue anymore.*

According to Equation 11, we can see that a point in a gray-scale image can change its state from 1 to 0 many times in the residue mapping (since this point can belong to different residual levels as indicated by the binary residues, see Table 1 below).

Related to this vanishing information, we can consider two attributes on the image structures: one concerning the order, ϱ, in which the vanishing occurs in the residue mapping process, and the other associated with the number of occurrence, η, of this vanishing. The first attribute is based on the different moments a point vanishes at the different residual levels, while the second one concerns the number of transitions from 1 to 0 of the point in $\mathcal{M}$. As we will see later, this vanishing number can be related to the irregularity (complexity) of the image structures.

Table 1 illustrates the residue mapping $\mathcal{M}$ and the vanishing number η of the plateaus in Figure 1(a).

λ	a	b	c	d	e	f	g	h
1	1	0	1	0	0	1	0	0
2	0	0	0	0	0	0	0	1
3	0	0	0	0	0	0	0	0
4	1	1	1	1	0	0	0	0
5	0	0	0	0	0	0	0	0
η	2	1	2	1	0	1	0	1

Table 1: Residue mapping of the plateaus in Figure 1(a).

Based on this information, we can finally define residues by attribute.

3.2 Residues by attribute

By considering the persistence of the points of a gray-scale image all along the residual levels defined, we can use the parameter λ and/or the information on the vanishing of a point in these residual levels as basic attributes for filtering. Here, the parameter λ is closely related to the size of the image components (small (big) structures are represented by low (high) residual levels), and the vanishing information gives an idea about the complexity or regularity of their shape (very regular components tend to have small η values in $\mathcal{M}$).

Definition 3.4 (Residues by attribute - a general definition) *Let $(\mathcal{R}_\lambda)_{\lambda \geq 1}$ be a set of morphological residues and $(\mathcal{M})_{\forall p \in \Gamma_I}$ the residue mapping of all points in the domain Γ_I. The residues by attribute, Ω, represent the information concerning the residual level λ, $\mathcal{R}_\lambda$, relative to a size parameter λ, and/or to the vanishing of the image points in $\mathcal{M}$.*

The characterization of the significant structures of the image is related to the right choice of the corresponding attributes. Based on this definition, we can define, for example, the following types of filtering:

- Filtering by using the size parameter λ, independently of the residue mapping $\mathcal{M}$.
- Filtering by considering either the information of order, ϱ, or the number, η, of times a point vanishes in $\mathcal{M}$, independently of the size parameter λ.
- Filtering which depends both on the size parameter λ and on the information contained in $\mathcal{M}$.

Basically, the above filtering/segmentation processes consist mainly of three steps (which we will explain in details in Sections 3.3 and 3.4):

1. Attribute definition: conveyed by the kind of structures we want to extract. It depends on the size parameter and/or on the information contained in $\mathcal{M}$.
2. Marker definition: based on the considered attributes, it determines a set of markers representing the significant parts of the image to be extracted.
3. Reconstruction: compute the geodesic reconstruction of the original image, based on the markers previously defined [12].

From the above filtering operation, different images can be obtained and combined for the definition of the final result that highlights their components of interest. Here, we can define three types of residues by attribute:

Definition 3.5 (Residues by attribute of size) *Let $(\mathcal{R}_\lambda)_{\lambda \geq 1}$ be a set of morphological residues. The residues by attribute using the size parameter λ concerns directly the information about the size of the image structures (see, e.g., [2] and [7] for this kind of attribute)*

Definition 3.6 (Residues by attribute of vanishing) *Let $(\mathcal{M})_{\forall p \in \Gamma_I}$ be the residue mapping of the points in the domain Γ_I. The residues by attribute of vanishing are based on the information contained in the residue mapping $\mathcal{M}$, relative to the vanishing number and/or the occurrence order of this vanishing (indicated by the 1 to 0 transitions in $\mathcal{M}$).*

Definition 3.7 (Residues by attribute of size and vanishing) *Let $(\mathcal{R}_\lambda)_{\lambda \geq 1}$ be a set of morphological residues and $(\mathcal{M})_{\forall p \in \Gamma_I}$ the residue mapping of all point in the domain Γ_I. The residues by attribute of size and vanishing represent the information contained in the residual level λ, $\mathcal{R}_\lambda$, relative to the size parameter λ, and to the vanishing attributes in $\mathcal{M}$.*

The definition of the structures characterized by the above attributes is given by a set of markers identifying these structures in the image (as in the morphological segmentation paradigm [12]). The next section treats this aspect.

3.3 Marker definition

The information to be extracted by the filtering process depends on the markers used in the geodesic reconstruction [7], [12], since these markers define the image structures to be preserved or not. The markers are defined according to the type of filtering, which depends on the used attributes. To obtain the desired filtering, we define two sets, named *Preserve* and *Eliminate*, that represent the markers for the image structures that should be preserved and eliminated, respectively.

For each type of considered residues, we obtain different sets of markers, as we illustrate in the following.

3.3.1 Marker definition associated with the size parameter λ and independent of the residue mapping $\mathcal{M}$

To obtain the structures associated with the size parameter λ, we can define the sets of markers simply by

$$\begin{aligned}&\mathit{Preserve} = \{\text{all points } p \in \Phi_{\mathcal{R}_\lambda}\}\\ &\mathit{Eliminate} = \{\text{all points } p \in (\Phi_{\mathcal{R}_\mu} \backslash \Phi_{\mathcal{R}_\lambda}),\\ &\mu > \lambda\}\end{aligned}$$

where again \ stands for the difference between sets.

3.3.2 Marker definition associated with the residue mapping $\mathcal{M}$

As we have seen before, from the residue mapping $\mathcal{M}$, we can obtain information about the number and the order of occurrence of the vanishing of a point. For example, if we define $\Xi(p)$ as the number of vanishing of a point p, then we can obtain the following set of markers concerning the vanishing number η:

$$\begin{aligned}&\mathit{Preserve} = \{p \in \Gamma_I |\ \Xi(p) = \eta\}\\ &\mathit{Eliminate} = \emptyset\end{aligned}$$

Let η and ϱ be, respectively, the number and the order of the vanishing of a point, as before. An example of a marker definition related to the order value ϱ can be given by the sets

We notice that the order ϱ occurs only when the vanishing number η of the point is greater or equal to ϱ ($\eta \geq \varrho$).

$$\begin{aligned}&\mathit{Preserve} = \{p \in \Gamma_I |\ \Xi(p) \geq \varrho\}\\ &\forall t \notin \mathit{Preserve}, \mathit{Eliminate} = \{t \in \Gamma_I \mid \Xi(p) < \varrho\}\end{aligned}$$

3.3.3 Marker definition depending both on the size parameter λ and on the residue mapping $\mathcal{M}$

Let us define $\theta(\mathcal{M}(p), \eta)$ as the level λ in which the vanishing number of a point is equal to η. A set of markers based on the size parameter λ and on the value η can be given by

$$\begin{aligned}&\mathit{Preserve} = \{\forall p \in \Gamma_I \mid \Xi(p) = \eta \text{ and}\\ &\theta(\mathcal{M}(p), \eta) = \lambda\}\\ &\mathit{Eliminate} = \emptyset\end{aligned}$$

If we define $\theta(\mathcal{M}(\mathrm{p}), \varrho)$ as the level λ in which the vanishing of a point occurs in the order ϱ, then the set of markers concerning the vanishing order ϱ and the size parameter λ can be given by

$$\begin{aligned}&\mathit{Preserve} = \{p \mid \Xi(\mathrm{p}) \geq \varrho \text{ and } \theta(\mathcal{M}(\mathrm{p}),\ \varrho) = \lambda\}\\ &\forall t \notin \mathit{Preserve}, \mathit{Eliminate} = \{t \mid \Xi(\mathrm{p}) < \varrho\}\end{aligned}$$

After the definition of the suitable set of markers, we can execute the final filtering step represented by the geodesic reconstruction algorithm.

3.4 The reconstruction step

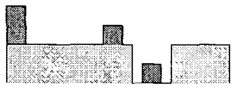
After the definition of the set of markers, we consider the morphological reconstruction algorithm [12] for the full retrieval of the significant structures of the image. Hence, the general filtering algorithm based on residues by attribute can be summarized as:

Algorithm 3.1 (Residues by attribute and filtering)
Input: image I, the size parameter λ and/or the information about the vanishing of the image structures.

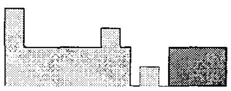
Output: the filtered image.

1. *Determine the residue mapping $\mathcal{M}$ for all point $p \in \Gamma_I$.*
2. *Determine the set of markers, Preserve and Eliminate, according to each attribute.*
3. *Extract the image components by the reconstruction method:*
   - (a) *Reconstruction of the image structures using the set of markers Preserve.*
   - (b) *Reconstruction of the image structures using the set of markers Eliminate.*
   - (c) *Subtraction of the image obtained in (3b) from the image obtained in (3a)*

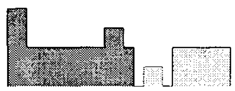
Figures 2, 3, 4 and 5 illustrate some filtering operations based on Figure 1(a) and on the residue mapping $\mathcal{M}$ shown in Table 1. The darker parts in these figures represent the different structures obtained in the filtering process.



(a) $\lambda = 1$

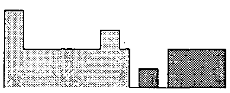


(b) $\lambda = 2$

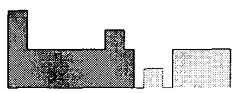


(c) $\lambda = 4$

Figure 2: Examples of structures obtained by filtering the image based on residues by attribute of size λ.



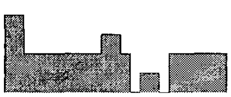
(a) $\eta = 1$



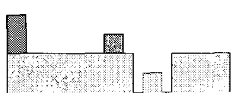
(b) $\eta = 2$

Figure 3: Examples of structures obtained by filtering the image based on residues by attribute of vanishing (number) of the regional maxima.

Figures 5(c) and 5(d) illustrate some structures that can be obtained only by a filtering of residues by attribute since these structures have different contrast and size parameters. Observe that the common characteristic of these structures concerns the vanishing information associated with the specific size parameter λ.

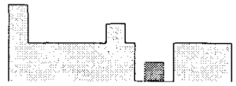


(a) $\varrho = 1$

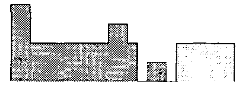


(b) $\varrho = 3$

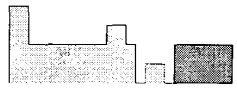
Figure 4: Examples of structures obtained by filtering the image based on residues by attribute of vanishing (order) of the regional maxima.



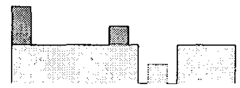
(a) $\lambda = 1$ and $\eta = 1$



(b) $\varrho = 1$ and $\lambda = 1$



(c) $\lambda = 2$ and $\eta = 1$



(d) $\varrho = 2$ and $\lambda = 4$

Figure 5: Examples of structures obtained by filtering the image based on residues by attribute of number, order and size parameter λ.

3.5 Properties

Let $\rho_T(X)$ be the morphological reconstruction of an image X with the set of markers T, and P and E the set os markers *Preserve* and *Eliminate*, respectively, obtained from X. We can express the residues by attribute filtering as

$$\Omega(X)(x) = \begin{cases} \rho_P(X)(x) - \rho_E(X)(x), \\ \text{ if } \rho_P(X)(x) \geq \rho_E(X)(x) \\ 0, \text{ otherwise} \end{cases}$$

As we explain next, this filtering is an idempotent, anti-extensive and connected operator.

Anti-extensive: $\rho_P(X) - \rho_E(X) \leq X \Longrightarrow \Omega(X) \leq X$.

Idempotent: Let us consider $P1$ and $E1$ as the sets of markers *Preserve* and *Eliminate*, respectively, obtained at the first iteration ($\Omega(X)$). The transformation $\Omega(\Omega(X))) = \rho_{P1}(\Omega(X)) - \rho_{E1}(\Omega(X))$ does not modify the result obtained at the first iteration that, originally, eliminates the undesired content of the image. Thus, $\Omega(\Omega(X))) = \rho_{P1}(\Omega(X))$, because $E1 = \emptyset$. Since $P1 = P$, we have that $\Omega(\Omega(X))) = \rho_P(\Omega(X)) = \Omega(X)$.

Connected: Informally, the method extends the image plateaus due to the morphological reconstruction considered in the filtering process.

The next section shows some examples of filtering on real images.

4 Some examples

Figures 6-7 illustrate the filtering based on residues by attribute by considering different classes of images.

(a) Original image.

(b) Filtering considering the second vanishing associated with the residual level 16 ($\varrho = 2$ e $\lambda = 16$)

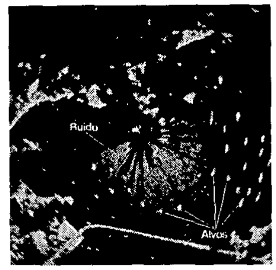
(c) Image (a) minus a filtered image given by the mapping of the image maxima and vanishing number between 2 and 7 ($2 \leq \eta \leq 7$).

Figure 6: Extraction of some image structures.

Figure 6 illustrates, for example, the extraction of different image components with the same gray-level and size parameter, but with different vanishing order(the strings "COMO COMPRAR OU VENDER UM CARRO PELA INTERNET" e "??"). Figure 7 illustrates the noise smoothing of a radar image as well as the elimination of some of its irrelevant components. Figure 8 illustrates the suppression of a component of the image (the strings). Finally, Figure 9 shows a segmentation example based on the threshold of a filtered image. In this case the significant components, which cannot be extracted by a simple thresholding, have the same size and vanishing order in the original image.

5 Conclusions

In this work, we introduced a new filtering technique based on the notion of residues by attribute. Basically, the method considers a decomposition of the image through the morphological residues which constitute a complete and hierarchical representation of the original image. From this decomposition, we



(a) Original image.

(b) Filtered image containing mainly the original targets representing ships.

Figure 7: Filtering by considering the mapping of the image maxima and $\varrho = 1$ and $3 \leq \lambda \leq 5$.

consider the concept of binary residues, vanishing attributes and its corresponding attributes of order and number. Based on these attributes we define a set of filtering scheme that has been proved to be useful in image smoothing and segmentation.

A common problem with this type of filtering refers to the algorithm performance. The computational time for the morphological residues definition is considerable, even when we consider a decomposition method of the structuring element. A future work in this sense concerns the algorithmic optimization of this method by considering, for example, the component trees [15] representation. Another aspect to be investigated is the introduction of other more complex attributes in the filtering scheme, such as the connected surface information of the components at a certain residual level.

References

[1] Jean Serra. *Image Analysis and Mathematical Morphology*, volume 1. Academic Press, 1982.

[2] Jean Serra. *Image Analysis and Mathematical Morphology: Theoretical Advances*, volume 2. Academic Press, 1988.

[3] Gerald Jean Francis Banon and Junior Barrera. *Bases da Morfologia Matemática para Análise de Imagens Binárias.* Livro Publicado na Escola de Computação, 1994.

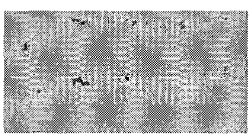
[4] Edmond J. Breen and Ronald Jones. Attribute openings, thinnings, and granulometries. *Computer Vision and Image Processing*, 64(3):377–389, November 1996.

(a) Original image.

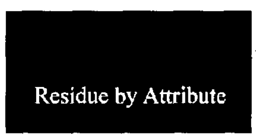
(b) Filtering by considering the image maxima with vanishing number greater than 14 ($\eta \geq 14$).

(c) Image (a) minus image (b).

Figure 8: Another segmentation example.

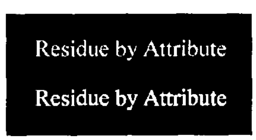


(a) Original image.



(b) A simple segmentation by thresholding (here, the best threshold level is 153).

(c) Filtered image.



(d) Segmentation with a threshold level equal to 1.

Figure 9: Filtering-based segmentation with $\varrho = 1$ and $\lambda = 3$).

[5] Azriel Rosenfeld and Avinash C. Kak. *Digital Picture Processing*, volume 1. Academic Press, INC, second edition, 1982.

[6] Rafael C. Gonzalez and Richard E. Woods. *Digital Image Processing.* Addison-Wesley Publishing Company, 1993.

[7] Luc Vincent. Grayscale area opennings and closings, their efficient implementation and applications. In Jean Serra and Pierre Salembier, editors, *Mathematical Morphology and Its Applications to Signal Processing*, pages 22–27. UPC Publications, May 1993.

[8] Corinne Vachier. *Extraction de Caracteristiques, Segmentation D'Image et Morphologie Mathematique.* PhD thesis, École Nationale des Mines de Paris, 1995.

[9] Robert Haralick and Linda Shapiro. Image segmentation techniques. *Computer, Vision, Graphics and Image Processing*, 35:100–132, 1985.

[10] Luc Vincent and Pierre Soillet. Watersheds in digital spaces: An efficient algorithm based on immersion simmulations. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 13(6):583–598, Junho 1991.

[11] S. Beucher and X. Yu. Road recognition in complex traffic situations. In *7th IFAC/IFORS Simposium on Transportation Systems: Theory and Application of Advanced Technology*, 1994.

[12] S. Beucher and F. Meyer. The morphological approach to segmentation: The watershed transform. In Edward R. Dougherty, editor, *Mathematical Morphology in Image Processing*, chapter 12, pages 433–481. Marcel Dekker, Inc., 1993.

[13] John Goutsias and Henk J. A. M. Heijmans. Multiresolution signal decomposition schemes. part1: Linear and morphological pyramids. Technical report, Center of Imaging Science and Department of Electric and Computer Engineering, 1997.

[14] George Matheron. *Random Sets and Integral Geometry.* John Wiley, 1975.

[15] Ronald Jones. Components trees for image filtering and segmentation. *Proceedings of IEEE Workshop on NonLinear Image and Signal Processing*, 13(6):583–598, September 1997.