A Boundary Extraction Approach Based on Multi-resolution Methods and the T-Snakes Framework

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Abstract. In this paper we present a new approach which integrates the T-Snakes model and a multi-resolution method in a unified framework for segmentation and boundary extraction. In a first stage, a local scale property of the objects is used to define a triangulation of the image domain and a sampling (coarsest resolution) of the image field. The low resolution image is threshold to get a 0-1 field which is processed by a simple continuation method to generate polygonal curves whose interior contain the desired objects. If the polygonal curve envolves more than one object, then the resolution is increased in that region and the method will be applied again. This stage gives a rough approximation of the desired boundaries which will be improved by the T-Snakes to get the final result. We demonstrate the method for 2D medical imaging in the experimental results and indicate how it can be extended to 3D in the future works.

1 Introduction

In image analysis, the boundaries of the objects of interest are important features. These boundaries may be applied in shape analysis, motion tracking and segmentation [4].

The Active Contour Models, also called snakes, are well known techniques for boundary extraction and tracking [14]. Basically, there are two types of snake models: the implicit ones and the parametric ones [27].

Implicit models, such as the formulation used in [15, 23], consist basically of embedding the snake as the zero level set of a higher dimensional function and to solve the corresponding equation of motion.

The parametric snake models consist basically of an elastic curve (or surface) which can dynamically conform to object shapes in response to internal forces (elastic forces) and external forces (image and constraint forces). Such approach is more intuitive than the implicit models. Its mathematical formulation makes easier to integrate image data, an initial estimate of the boundary, desired contour properties and knowledge-based constraints, in a single extraction process [4].

In the original snake model, proposed by Kass at al. [14], the topology of the structures of interest must be known in advance since the mathematical model can not deal with topological changes.

To address this limitation McInerney and Terzopoulos [16, 18, 17] proposed the T-snakes model. The basic idea is to embed the snake model within the framework of a simplicial domain decomposition to provide a way of doing topological changes (splits and merges) of the snakes during their evolution. One advantage of the T-Snakes framework is its generality as the same approach can be applied over both 2D and 3D scenes.

Besides the domain triangulation, the T-Snakes depends on some threshold to define a normal force which is used to drive the snake(s) towards the targets [16, 18, 17].

In our work, we also assume that the image intensity patterns of the objects can be characterized by a threshold or some statistics of the image field [16, 18, 17].

More specifically, we assume a *local scale property* for the objects of interest and use this property to reduce the resolution of the image. Also, we suppose that the boundaries have the properties of *closedness, orientedness* and *connectedness* [20].

In a first stage, we use the local scale property to define a coarsest image resolution that guarantees not split the objects. From the corresponding grid we make a simple CF triangulation of the image domain. The low resolution image field is thresholded to get a binary function, which we call an *Object Characteristic Function.* Then, a simple continuation method, the *door-in-door-out* algorithm, is used to generate a set of close polygonal curves whose interior may contain zero, only one or more than one object.

Now, a set of scalar features and corresponding lower and upper bounds, priorly defined, are used to analyse these curves [21]. The simplest features are the area enclosed and

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the polygonal perimeter.

The regions containing zero objects are in general easily discarded by the area lower bound. Regions whose areas are larger than the upper bound may contain more than one object of interest. Hence, the grid resolution in those regions is increased to improve the segmentation.

The polygonal curves so extracted are in general rough approximations of the boundaries of interest. We improve these approximations by using the T-Snakes model [16] whose framework is the basic one for our method.

We have tested the method in 2D images. However, our approach can be extended for 3D images as we indicate in the section 6. The method proposed shares the basic elements of our work [7] for initializing snake models.

This paper is organized as follows. The next section discusses multi-resolution/multigrid methods in image analysis related to this work. In section 3 we present the T-Snakes model. The segmentation/boundary extraction framework which is the contribution of this paper is presented on section 4. On section 5 we discuss 2D experimental results. Finally, we present our conclusions and future works.

2 Related Works

The method presented in this paper adopts the basic philosophy of some non-parametric multi-resolution methods used in image segmentation based on Pyramid and Quadtree approaches [5, 2, 12]. These are data structure able to represent an image at different levels of resolution.

The basic idea of these approaches is that as the resolution is decreasing, small background artifacts become less significant relative to the object(s) of interest. So, it can be easier to *detect* the objects in the lowest level and than propagate them back down the structure. In this process, it is possible to *delineate* the boundaries in a coarser resolution and to re-estimate them when increasing the resolution [12, 2].

The work described here uses a multiresolution approach which starts at a coarsest resolution defined through the prior knowledge of scale and is adaptive in the sense that resolution is only increased at the locations where this is considered necessary. To increase the resolution we just refine the coarser grid and sample the image over the grid nodes corresponding. The refinement structure has the form of a *local uniform nested refinement* [3] pictured on Figure 1.

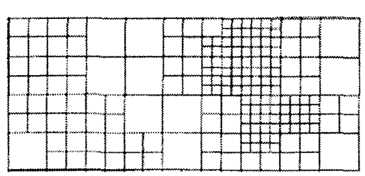


Figure 1: Representation of the multiresolution scheme.

Multi-resolution methods can be useful for problems in image analysis and computer vision which can be expressed as the minimization of energy functions. In [10] we found such an approach, called "multiscale relaxation" which have been used for snake models [19]. In that approach, the global optimization problem is solved within a sequence of particular subspaces of the original space of possible configurations. Each subspace defines a new "coarse" energy function whose parameters are derived from the original (full resolution) objective function. The constrained optimization is implemented in [10] using a coarse-to-fine procedure on a pyramidal structure.

On the other hand, we have the multigrid relaxation methods like the one presented in [24]. In this case, the resolution of the image is not reduced (it is always the full one). However, the grid resolution used to discretize the minimization problem goes from a coarsest one to a finest one.

In this paper, the approach is multigrid/multi-resolution when segmenting the image but not when applying the T-Snakes. This deformable model is used at the end of the boundary extraction stage. We do not need to use multiscale relaxation methods in this final step because we take the full resolution of the image for evolving T-Snakes. Besides, when appling T-Snakes it is supposed that the grid resolution is enough for completing the boundary extraction. Hence, we do not use multigrid relaxation methods during the T-Snakes evolution.

3 T-Snakes Framework

The T-Snakes approach is composed basically by three components [16, 18, 17]: (1) a triangulation (simplicial decomposition) of the domain of interest, in our case a closed subset $D \subset \Re^2$; (2) a particle model of the snake; (3) a *characteristic function* χ defined on the grid nodes which distinguishes the interior ($Int(s)$) from the exterior ($Ext(s)$) of a snake s:

$$\chi : D \subset \Re^2 \rightarrow \{0, 1\} \quad (1)$$

where $\chi(p) = 1$ if $p \in Int(s)$ and $\chi(p) = 0$, otherwise, where p is a node of the triangulation.

Following the classical nomenclature, a vertex of a triangle is called a *node* and the collection of nodes and triangle edges is called the (simplicial) grid Γ_s.

An edge in which the characteristic function χ changes its value is called a *completely labeled* edge. A triangle σ which contains a completely labeled edge is called a *traverse triangle*.

In this framework, the reparameterization of a contour is done by [16, 18, 17]: (1) taking the intersections points of the snake with the triangulation; (2)Update the characteristic function χ; (3) carrying out topological changes by using the characteristic function χ to distinguishing the inside from the outside of the snake(s).

As an example, consider the characteristic functions χ_1 and χ_2 relative to the circle and ellipse on Figure 2, respectively. The functions are defined on the grid nodes of a CF-triangulation of the plane. The vertices marked are those where $\max\{\chi_1, \chi_2\} = 1$. Observe that they are enclosed by a merge of the contours.

This merge can be approximated by a (polygonal) curve which can be easily obtained by a simple numerical continuation method based on the following property [1]:

Property 1 (Door-In-Door-Out Principle). A triangle has either zero or exactly two completely labeled edge.

Based on this property we can design an algorithm to generate the frontier of the region where $\chi(p) = 1$. The idea is to find an initial traverse triangle and then use one of its completely labeled edge as the *door-out* to go towards the next traverse triangle. The sequences of triangles $\sigma_0, \sigma_1, ..., \sigma_q$ and edges $\tau_0, \tau_1, ..., \tau_q$ so generated, have pairwise different elements [1]. The algorithm can be summarized as follows.

Continuation Algorithm (*Door-In-Door-Out*)
begin
  find an initial traverse triangle $\sigma_0 \in \Gamma$
  find a completely labeled edge τ_0 of σ_0;
end;
Repeat:
  find $\sigma_n \in \Gamma$, $\sigma_n \neq \sigma_{n-1}$ such that $\tau_{n-1} = \sigma_n \cap \sigma_{n-1}$;
  find a completely labeled edge τ_n of σ_n such that $\tau_n \neq \tau_{n-1}$;
  stop when returning to the initial triangle $\sigma_0 \in \Gamma$.

The same algorithm can be used for more than two contours (and obviously for only one) and for splits and merges. The procedure complexity is $O(N)$ where N is the number of triangles. Also if D is compact, we can prove the following property [1]:

Property 2. The sequences $\{\sigma_i\}$, $\{\tau_i\}$ are finite or there is a finite integer $q > 0$ such that $\sigma_0 = \sigma_q$.

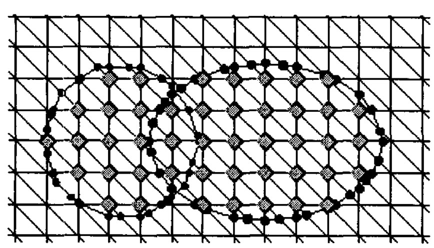


Figure 2: Two snakes colliding with the inside grid nodes and snaxels marked.

3.1 Discrete Snake Model

A T-Snake [16, 17] is a discrete form of the classical snake [14]. It is defined as a set of N particles (snaxels), whose positions $\{v_i = (x_i, y_i), i = 0, ..., N-1\}$ are connected to form a closed contour. Each pair of points v_i, v_{i+1} is called a "model element ". The snaxels are linked by springs defined by a stiffness parameter a_i, and a natural length l_i. The corresponding elastic force is added to a rigidity (smoothing) force and an external (image) force [18, 17]. These forces are given respectively by the following expressions:

$$Elastic\ Force : \alpha_i = a_i e_i r_i(t) - a_{i-1} e_{i-1} r_{i-1}(t). \quad (2)$$

where $\|r_i(t)\| = \|v_{i+1} - v_i\|$ and $e_i = \|r_i(t)\| - l_i$,

$$Rigidity\ Force : \beta_i = b_i \left(v_i - \frac{1}{2}(v_{i-1} + v_{i+1}) \right). \quad (3)$$

$$Image\ Force : f_i = \gamma_i \nabla P, \quad (4)$$

where b_i, γ_i are force scale factors, $P = -\|\nabla I\|^2$ (I is the image field).

A normal (balloon-like) force is also used to push the snake towards image edges until it is opposed by the external image forces. This force is given by:

$$Normal\ Force : F_i = k_i (sign_i) n_i, \quad (5)$$

where n_i is the normal at the snaxel v_i, k_i is the force scale factor, and $sign_i = 1$ if $I(v_i) \geq T$ and $sign_i = -1$ otherwise (T is a thresholded for the image I).

The T-Snake position is updated according to the following evolution equation:

$$v_i^{(t+\Delta t)} = v_i^t + h_i \left(\alpha_i^t + \beta_i^t + F_i^t + f_i^t \right), \quad (6)$$

where h_i is an evolution step.

The T-Snake model incorporates also an *entropy condition*: "once a node is *burnt* (passed over by the snake) it stays *burnt* " [16, 18, 17]. A termination condition is defined based on the number of deformations steps (*temperature*) that a triangle remains as a traverse one. A T-Snake is considered to have reached its equilibrium state when the temperature of all the snaxels fall below a pre-set "freezing point".

The T-Snake model can be summarized as follows [18, 17]. Until the temperature of all snaxels fall bellow the freezing point: Compute the external and internal forces and update the snaxels positions using equation (6). Compute the intersection between the grid and the model elements. Next, update the characteristic function (1) and through it determine the corresponding set of traverse triangles. For each traverse triangle find a model element which separates the inside from the outside nodes. Discard the other ones.

To update the characteristic function χ given by expression (1), a classification algorithm [18] is designed to determine which nodes were burnt during a deformation step given by equation (6).

The threshold T used in the normal force (5) plays an important role in the T-Snakes model. If it was not chosen properly, the T-Snake can be frozen in a region far from the target(s) [16, 18, 17].

The choice of T is more critical when two objects to be segmented are too close like in Figure 3. In this figure the grid nodes marked are the ones whose image intensity fall bellow the threshold T.

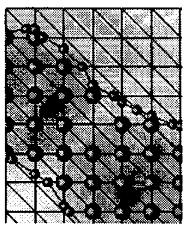


Figure 3: T-Snake and grid nodes marked.

For the T-Snake to separate the objects pictured it has to burn the grid nodes marked. However, the normal force given by expression (5) changes its signal near these grid nodes. So, the force parameters in expressions (2)-(4) have to be chosen properly to advance the T-Snake over these grid nodes. However parameters choice remains an open problem in snake models [6].

Having in mind this limitations and the topological capabilities of the T-Snakes model we propose the following method for segmentation and boundary extraction.

4 Our Segmentation Framework

The segmentation and boundary extraction method that we propose in this paper is based on the following steps: (1) Extract region based statistics; (2) Coarser image resolution and Triangulation; (3) Define the *Object Characteristic Function*; 4) Boundary approximation by *door-in-door-out algorithm*; 5) Apply T-Snakes model.

The region based statistics can be extracted by image histograms or pattern recognition techniques [25] (see [11] for a recent review). These statistics can be represented by a mean μ and variance σ of the image field I (always supposed a grey level one) or any other field defined over the image domain [13]. That means:

$$p \in O \Longrightarrow |I(p) - \mu| \leq k\sigma \qquad (7)$$

where k is an used defined parameter [17].

In some appications, a threshold T could be defined to characterize the object(s) of interest:

$$p \in O \Longrightarrow I(p) \geq T \qquad (8)$$

Given a point $p \in O$, let r_p be the radius of a hyperball B_p that contains p and lies entirely in the object region; that is, if $q \in B_p$ then $I(q)$ satisfies equation (8) (or equation (7)). We assume that for the objects of interest there is an $r_l > 1$ such that $r_p \geq r_l$; $\forall p, p \in O$. We call this a *local scale property* of the objects.

Also, we suppose the following properties for the objects boundaries: (a) Closedness, (b)Orientedness, (c) Connectedness.

The *Object Characteristic Function* of step (3) is similarly defined as equation (1):

$$\chi : D \subset \Re^2 \rightarrow \{0, 1\} \qquad (9)$$

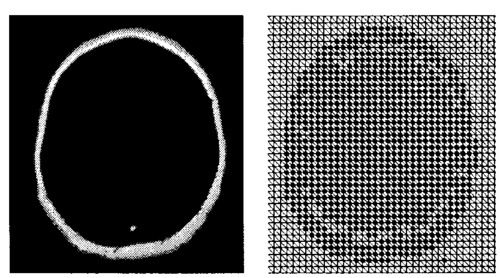
where $\chi(p) = 1$ if $I(p) \geq T$ and $\chi(p) = 0$, otherwise, where p is a node of the triangulation.

We must observe that due to the local scale properties above it is possible to reduce the resolution of the image by $r_l \times r_l$ without spliting the objects of interest.

The Figure 4 shows this fact. In that figure we have a MRI slice of a head. A simple inspection shows that $T = 5.0$ is enough for defining function (9) and that the local scale property can be characterized by $r_l = 5$.

The corresponding object characteristic function is pictured in Figure 4.b. Observe that we can search the grid nodes until we find a traverse triangle and then apply the *door-in-door-out* algorithm to get the corresponding polygonal curve. As we are assuming properties (a)-(c), we do not need to concern with the interior of the closed curves so generated (Figure 5).

As we do not have object splits due to the local scale property, we do not need to concern with merge of regions also. Hence, the closed curves generated gives a rough segmentation of the image. That is the main point of this work.



(a) (b)

Figure 4: (a)Original image. (b)Triangulation and Object Characteristic Function.

Observe that according to the property 2 of section 3, the *door-in-door-out* algorithm generates closed curves or curves which start and end in the image frontiers [1]. So,

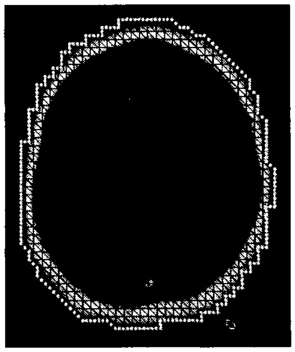


Figure 5: Grid and boundary approximation.

a simple inspection discards open curves. Also, both properties 1 and 2 of section 3 assure that the polygonal curves generated are simple [1], then (b) is always satisfied.

The remaining curves satisfy (a)-(c) above, as we can see on Figure 7, which is the result obtained by applying steps (1)-(4) to the image pictured on Figure 6. However, some of those curves may be too small because they were generated due to small artifacts or noise in the background. Hence, they have to be discarded. Also, some curves may contain more than one object of interest (see Figure 7).

Now, we can use a set of pre-defined features [21]. For each feature we define upper and lower bounds based on a prior knowledge of the objects. We have used only the area enclosed by the curve. From the local scale property and the trangulation used we can set the area lower bound as $4\,(r_l)^2$. The definition of the upper bound is an application dependent task. For instance, in medical imaging analysis, anatomical elements can be used.

It is important to stress that the upper bound(s) is not an essential point for the method. It's role is only to avoid expending time computation in regions were the boundaries enclose only one object.

Among the remaining curves, a simple inspection would discard those ones which are bellow the area lower bound. The ones whose interior have areas larger than the upper bound will be processed in a finer resolution.

In fact, in images like in Figure 6, the *outer* scale corresponding to the separation between the objects may be finner than the local scale property of the objects of interest. The corresponding problem falls into a category known as *two-scale problems.*

The commom characteristic feature of these problems is that - the spatial scales vary dramatically from one region to another. This happens in the regions on Figure 7 where the segmentation is not completed yet. In those regions the outer scale may be finer than the local scale of the object (*local scale property* defined above). Hence, the coarsest resolution could not *separate* the objects. To correct that result we increase the resolution in those regions to account for more details.

For uniform meshes like the one in Figure 1 this can be accomplished by a multi-resolution scheme whose data structure is that one of *Adptive Mesh Refinement* methods [3]. In these structures each node in the refinement level l splits in η^n nodes in level $l+1$, where η is the refinement factor and n is the space dimension ($\eta = 2$ and $n = 2$ in our case). Hence, in $2D$, the node structure has η^2 pointers to the soons, η^2 pointers to the neighboors and one pointer for the corresponding parent. Such scheme have been explored in the context of *Level Sets* methods also [23].

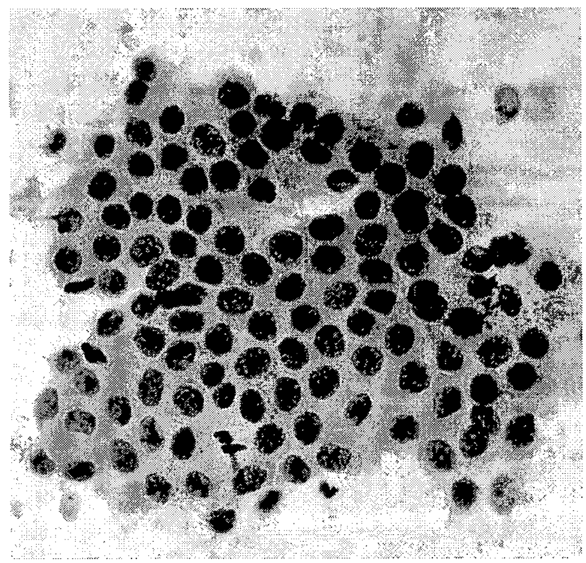


Figure 6: Endocervical Columnar cells.

In some cases, despite the fact that the full resolution was used, some curves may have the interior with area larger than the upper bound. If we have a statistics like in expression (7) we can relax the threshold ($T \rightarrow T \pm \Delta T$) and apply steps (2)-(4) again, but only in the regions detected.

Among the remaining curves it may happen that some of then still correspond to small artifacts too close in the background. In general, these curves can be discarded only by increasing the resolution.

After applying steps (1)-(4) we have a rough approximation of the object boundaries. The idea is to use a snake model to get the final result. The T-Snakes model was chosen firstly, because it can deal naturally with the self-intersections that may happen during the evolution of those curves. Secondly, by choosing T-Snakes, the steps (1)-(5) encompass an unified framework based on the triangulation, threshold and the Characteristic Functions for segmentation and boundary extraction.

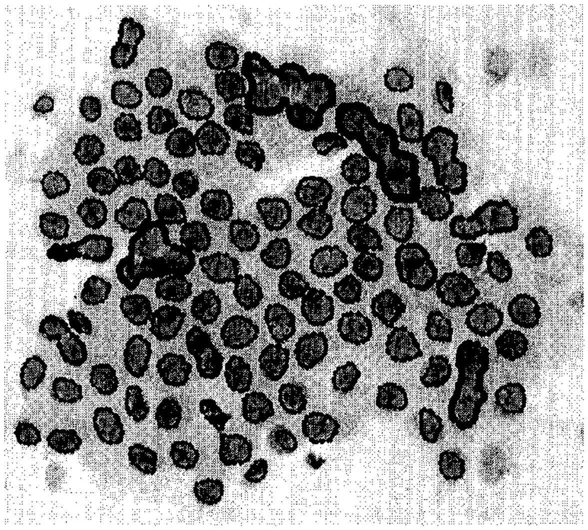


Figure 7: Solution for resolution 5×5.

The continuation *door-in-door-out* algorithm used shares the basic elements used in iso-surface generation methods in hierarchical 3D meshes [9]. But, these methods in general do not use the scale and topological restrictions which formalize our *prior knowledge* of the structures of interest.

5 Experimental Results

The result, corresponding to the three cells observed inside a T-Snake near the upper-right corner of Figure 7, obtained by using a grid with a finer resolution is shown in Figure 8. In this case, only the full image resolution was enough to segment the objects.

A good point of our method is that we do concern with the outer scale only in run time. When we realize that a boundary approximation obtained involves more than one object, we have realized also that the scale used may be larger than that one of the space between the objects.

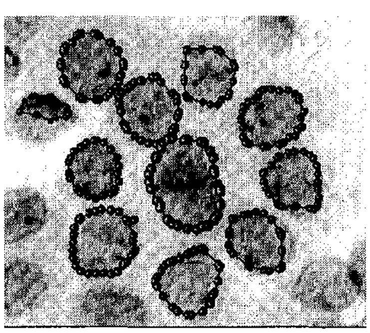
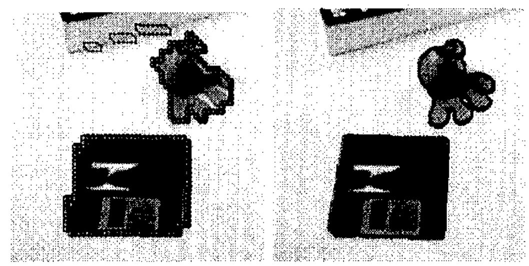


Figure 8: Solution for resolution $1x1$.

The Figure 9 shows another example. The *outer* scale is finner than the inner one due to the separation between some parts of the upper right object. The resolution 9×9 used was enough to separate the objects (9.a) but only the finnest resolution (Figure 9.b) could resolve the regions with higher curvature.



(a) (b)

Figure 9: (a)Original image and boudaries approximation with 9×9 resolution. (b)T-Snake solution with 1×1 grid.

It is important to highlight that the T-Snakes model is used only as a boundary extraction method. The segmentation process is given by the steps (1)-(4). So, the problem pictured on Figure 3 is avoided as the T-Snakes is applied only after the segmentations is (at least roughly) completed. It is important to observe that the same idea was used by Sarti at al [22] in the context of implicity snake models for cytology analysis. However, their segmentation algorithm based on Hough Transform, gradient-weighted thresholding and watershed is very different from the one summarized by steps (1)-(4) above.

The step h^i in the equation (6) can be small (one or two pixels at a time) without bring performance restrictions as we are close the boundaries (in general). The proximidity of the boundaries makes the parameters choice easier wich is another advantage of our method. Also, the entropy condition used stops the contraction force component of the internal forces (2)-(5) which is a desired feature.

The Figure 10 shows the final result corresponding to Figure 4. This figure is a particular slice from a set of 79 ones. We have applied our method for all of then with grid resolution of 5×5 and threshold $T = 5.0$. The results obtained were as good as the ones presented on Figures 11 and 12.

If desired, we can apply the Dual-T-Snakes model [8] for thinning the frontier corresponding to the head bonne. This is a further directions of this work.

A simple observation of Figure 8 shows that there is a cell whose boundary is incorrect. This happens due to the threshold used. To correct this result we have to stablish a method to relax the threshold to give a better result. However, we have to be carefull because when relaxing the threshold ($T \rightarrow T \pm \Delta T$) we may split objects and so a merge will be necessary.

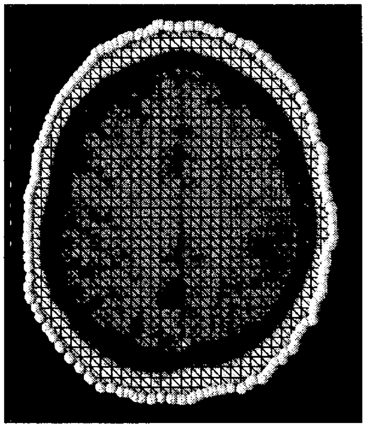
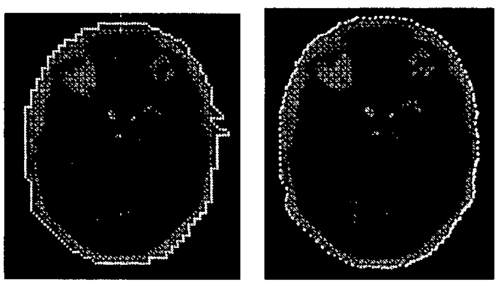
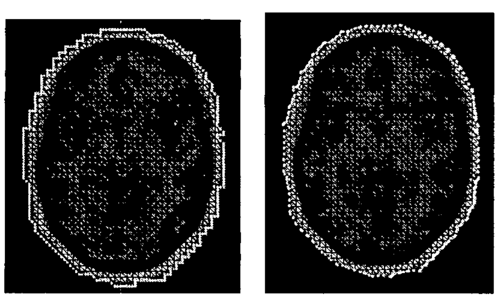


Figure 10: Solution for MRI slice.



(a) (b)

Figure 11: (a)Boundary approximation with 5 × 5 grid. (b)Final solution.



(a) (b)

Figure 12: (a) Boundary approximation. (b)T-Snake result with 5 × 5 grid

6 Conclusions and Future Works

In this paper we present a new approach which integrates the T-Snakes model and a multi-resolution method in a unified framework for segmentation and boundary extraction.

The method uses region based statistics, domain triangulation and characteristic functions to initialize a T-Snakes model closer to the boundaries of interest. The framework can be used to process 2D images with target objects immersed in a background.

Further directions are to extend the method for 3D by using the T-Surface approach [17] and an extention of the *door-in-door-out* algorthm for higher dimensions [1]. Also, we need pattern recognition methods to stablish the local scale property automaticaly [11].

The local scale property is a fundamental point for our method. Without this property the method can not be applied efficiently. An approach to accomplish this limitation could be the fuzzy connectedness segmentation [26, 7]. This method gives an alternative way to define the field to be thresholded. We will investigate its efficience in further works. Also, a policy to relax the threshold has to be designed to correct some results.

Acknowledgments

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