A directional and parametrized transition detection algorithm based on morphological residues

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Abstract. The boundary identification represents an interesting and difficult problem in image processing, mainly if two flat zones are separated by a gradual transition. The most common operators work very well for sharp edges, but fail for gradual transitions. In this work, we have done a characterization for gradient by a study of the opening residues. This characterization is useful to identify and classify sharp and gradual transitions between two consecutive flat zones according to a size criterion.

1 Introduction

The boundary identification represents an interesting and difficult problem in image processing, mainly if two flat zones are separated by a fuzzy region (gradual transition). The most common operators work well for sharp edges, like Sobel and Roberts [1], but fail for gradual transitions. The gradual transitions can be detected by a statistical approach proposed by Canny [2]. Another approach to cope with this problem is through mathematical morphology operators based, for example, on the notion of morphological gradient and multi-scale morphological gradient [3]. From this approach, it is not possible to classify the detected transitions according to a parameter of size, i.e., we can not identify the transitions of a specific size because these transitions, identified at a certain scale i, correspond to all the transition with size smaller than i.

To cope with these problems, we will consider a hierarchical analysis of an image, with respect to a size parameter, as it was proposed by Leite and Guimarães [4]. The method, based on this representation, takes into account the behavior of the residues related to the transitions defined at the different hierarchical levels of the image representation. Also, in this work, we are interested only in directional information, more specifically, in 1D image. Thus, we propose a directional and parametrized transition detection algorithm based on morphological residues that considers linear structuring elements. Here, the parametrization is useful to classify the different kinds of transitions according to the given size parameter.

This paper is organized as follows. In Sec. 2, we define some basic concepts of mathematical morphology. In Sec. 3, we describe the multi-scale gradient operator proposed by Soille and we also propose a new variant of this operator. In Sec. 4, we define the hierarchical decomposition. In Sec. 5, we propose a new algorithm to identify the boundaries according to a parameter of size. Finally, some conclusions and discussions are given in Sec. 6.

2 Basic concepts

In this section, we describe some basic morphological operators considered in this work (see [3, 5] for more details). Let B be a flat structuring element (SE) and let $\lambda \mathrm{B} = \{\lambda b \mid b \in \mathrm{B}\}, \lambda \geq 0$ denote the structuring element B of size (i.e., the radius) λ.

Definition 2.1 (Morphological gradient) Let δ_λ and ε_λ be the dilation and erosion with a structuring element B of size λ, respectively. The morphological gradient ρ_λ of size λ is defined by

$$\rho_\lambda = \delta_\lambda - \varepsilon_\lambda \qquad (1)$$

The morphological gradient, also called thick gradient, gives the maximum variation of the function in a neighborhood of radius λ. If the size λ equals the radius of the transition between regions of homogeneous grayscale, the morphological gradient will output the contrast value between these regions. However, the output of this gradient is represented by thick edges. Another interesting morphological operator that extracts light regions of the image, smaller than a certain SE, is the white top-hat defined as follows.

Definition 2.2 (White top-hat) Let γ_λ be an opening by a structuring element B of size λ ($\gamma_\lambda = \delta_\lambda \varepsilon_\lambda$). The white top-hat WTH_λ of size λ corresponds to the residue of the opening, and is defined by

$$\mathrm{WTH}_\lambda = Id - \gamma_\lambda \tag{2}$$

where Id is the identity operator.

The white top-hat represents the residues related to the specific size, λ, of the SE. If we generalize this concept to a range of SE, then we can define the granulometry as follows.

2.1 Granulometry

The granulometry, $(\psi_\lambda)_{\lambda \geq 0}$, first conceived by Matheron [6], describes quantitatively the "coarseness" of an image and is a basic morphological concept used, for example, in pattern recognition. The granulometry decomposes the image in classes of components according to the used structuring element.

Definition 2.3 (Granulometry [6]) Let $(\psi_\lambda)_{\lambda \geq 0}$ be a set of image transformations depending on a parameter λ. This set constitutes a granulometry iff the following properties hold:

$$\forall \lambda \geq 0, \psi_\lambda \text{ is increasing} \tag{3}$$

$$\forall \lambda \geq 0, \psi_\lambda \text{ is anti-extensive} \tag{4}$$

$$\forall \lambda \geq 0, \mu \geq 0, \psi_\mu \psi_\lambda = \psi_\lambda \psi_\mu = \psi_{max(\lambda,\mu)} \tag{5}$$

Equation 5 above implies an idempotent operation, that is, $\psi_\lambda[\psi_\lambda] = \psi_\lambda$. Typically, the set of transformations ψ_λ is a decreasing set of algebraic openings given by any convex structuring element B and its homotetic representation ($\lambda \mathrm{B} = \{\lambda b \mid b \in \mathrm{B}\}, \lambda \geq 0$) [5].

2.2 Image simplification

Another important operator explored here, namely ultimate erosion [3], is associated with the simplification of the images. Its definition is given next.

Definition 2.4 (Ultimate erosion) The ultimate erosion represents the set of all components of an image that disappears from one erosion step to the other, when we consider increasing SE. It is defined for a binary image X and a grayscale image f as follows.

$$\mathrm{ULT}(X) = \bigcup_\lambda \{ {\varepsilon_1}^{(\lambda)}(X) \backslash G_{{\varepsilon_1}^{(\lambda)}(X)}[{\varepsilon_1}^{(\lambda+1)}(X)] \} \tag{6}$$

$$\mathrm{ULT}(f) = \bigvee_\lambda \{ {\varepsilon_1}^{(\lambda)}(f) - G_{{\varepsilon_1}^{(\lambda)}(f)}[{\varepsilon_1}^{(\lambda+1)}(f)] \} \tag{7}$$

where G is the morphological reconstruction by the dilation operation [3, 5]. "\" and "−" mean set difference and grayscale difference, respectively.

3 Multi-scale gradient based on WTH

The possibility to detect smooth variations between flat zones by considering morphological gradient operators provides an important tool to identify fuzzy transitions. Nevertheless, the result of these operators yields thick edges as a result and, also, false edges (merge of the gradual boundaries) that can be produced if the distance between two transitions is smaller than the width of the used structuring element. According to [3], these problems can be avoided if we consider the multi-scale gradient proposed by Soille in [3]. In this section, we describe this gradient and propose a variant of this operator based on the morphological ultimate erosion.

3.1 Soille's gradient

The multi-scale gradient model proposed by Soille can be represented by the block diagram in Fig. 1. According to this model [3], the problem of thickness introduced by the morphological gradient (Equation 1) is avoided by the application of erosions on the thick image, the size of the SE used by these erosions depends on the size of the SE used by morphological gradient. To avoid the merge of boundaries, a white top-hat is applied to the morphological gradient before these erosion operations, the WTH will identify basically the peaks of the image of the gradient. For example ([3], p. 117), the edges originated by both sides of a small region merge because the width of these transitions (edges) is larger than of the region itself, and a subsequent erosion would not split them back. In fact, when morphological gradients coming from two distinct transitions the resulting thickness is larger than the width of the considered SE, and to obtain the regions with size related to the SE, we can use the white top-hat.

Definition 3.1 (Soille's morphological multi-scale gradient) The Soille's morphological multi-scale gradient at scale λ is given by:

$$\rho^S_\lambda = \rho_\lambda * L_1 \varepsilon_{(\lambda-1)} \mathrm{WTH}_\lambda \rho_\lambda \tag{8}$$

where ρ_λ, $\varepsilon_{(\lambda-1)}$ and WTH_λ represent the morphological gradient of size λ, the erosion of size $\lambda - 1$ and the white top-hat of size λ, respectively. L_k represents the thresholding operation at level k. "*" means multiplication operation.

The main problems of this approach are closely related to the choice of the SE family. The first of these problems concerns the quality of the detection. For example, let us consider a 1D signal (Fig. 2(a)) and a homotetic family of SE. If we apply the Soille's gradient on such a configuration the transition is not well identified at the corresponding scale (Fig. 2(b)

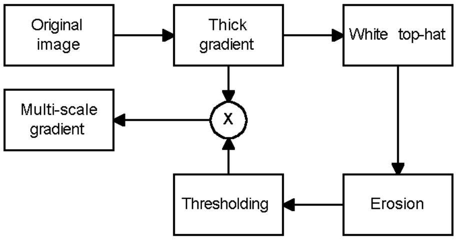


Figure 1: Soille's multi-scale gradient.

and (c)). In general, this operator produces good results when the image presents linear transitions thanks to the homotetic family of SE, but in other cases, the results are not interesting, as in Fig. 2(c), in which this result corresponds to the supremum of the gradient values at levels in the range $[1, 4]$. The second problem concerns the elimination of regions due to the introduction of the erosion operation, as illustrated in Fig 2(b). Finally, from this approach, it is not possible to classify the transition according to a size parameter, i.e., we can not identify the transitions of a specific size k because these transitions, identified at a certain scale i, are included in the set of transitions smaller than i, where $k \leq i$. Intuitively, this problem can be avoided by the difference between two consecutive levels, but the gradient values depend on the morphological gradient that may vary at consecutive scales.

To deal with the above problems, we propose to replace the erosion by the ultimate erosion operation. In this case, we preserve the same properties of the Soille's gradient, that is, the computation of thin edges without merging gradual boundaries. We also preserve all regions smaller than the corresponding SE.

3.2 Multi-scale gradient based on ultimate erosion

Here, we replace the erosion in Fig. 1 by the ultimate erosion.

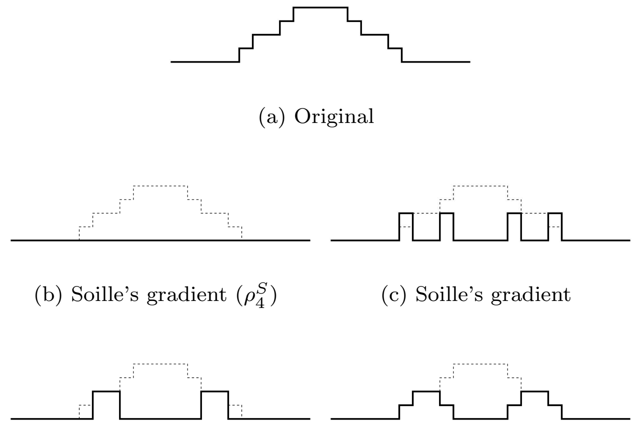
Definition 3.2 (Multi-scale gradient based on ultimate erosion) Our morphological multi-scale gradient ρ^U_λ based on ultimate erosion at scale λ is defined by:

$$\rho^U_\lambda = \rho_\lambda * L_1 \mathrm{ULT}(\mathrm{WTH}_\lambda \rho_\lambda) \tag{9}$$

In Fig. 2(d) and (e) we illustrate the application of the multi-scale gradient based on ultimate erosion, considering a certain scale and a range of scales ($[1, 4]$), respectively. The problem identified here is associated with noise sensitivity, and with the fact that the transitions are not thin. Also, it is not possible to classify the transitions regions according to a size criterion.

4 Gradient based on morphological residues

The classification of the transitions according to a size information can be obtained by a decomposition of the



(a) Original

(b) Soille's gradient (ρ^S_4)

(c) Soille's gradient

(d) Gradient based on ultimate erosion (ρ^U_4)

(e) Gradient based on ultimate erosion

Figure 2: Morphological multi-scale gradient: (b,d) correspond to the gradient values at a specific level $\lambda = 4$, and (c,e) correspond to the supremum of the gradient values at different levels ($\lambda = [1, 5]$).

image that is associated with a certain size parameter. Leite and Guimarães [4] proposed a general framework, based on the notion of morphological residues, to segment an image by considering information of a hierarchical decomposition of this image with respect to a size parameter λ. Here, we will study the relationships between the size of the transitions and the morphological residues related to these transitions.

4.1 The transition size information

The basic idea to detect transitions between flat zones is to apply the morphological gradient (Equation 1). In this work, we are interested only in directional information and consequently, we consider a linear SE. Next, we define some basic concepts.

Definition 4.1 (Flat zone, n-flat zone and regional maximum) Let g be a 1D image. A flat zone of g is a maximal set (in the sense of inclusion) of consecutive points x with the same grayscale value. A regional maximum is a flat zone N of level k such that the points adjacent to N have a grayscale strictly lower than k. An n-flat zone F_i is a flat zone with size greater than or equal to n, i.e., $|F_i| \geq n$ where $|X|$ means set cardinality.

Definition 4.2 (k-Transition) Let g be a 1D image. We denote by F the set of k-flat zones. A transition T between two k-flat zones, F_i and F_j, is the range $[p_0..p_{n-1}]$ such that $p_0 \in \mathrm{F}_i$, $p_{n-1} \in \mathrm{F}_j$, $\forall m, 0 < m <$

$n-1, p_m \notin \mathrm{F}_i \bigcup \mathrm{F}_j$ and $\forall l \neq i, j$ $\mathrm{F}_l \not\subset [p_0..p_{n-1}]$.

Definition 4.3 (Dome) Let g be a 1D image. We denote by M the set of regional maxima of g. A dome of g is a connected component of the thresholding of g at level 1. A single dome is a dome which contains only one maximum $M_i \in \mathrm{M}$.

Proposition 4.1 (Dome-transition) Let g be a 1D image. We denote by $\rho_\lambda(g)$ the morphological gradient of size λ of the image g. Let T be a k-transition of g having two adjacent k-flat zones, namely ${\mathrm{R}^k}_1$ and ${\mathrm{R}^k}_2$. The k-transition T is represented by a dome on $\rho_\lambda(g)$ if $|\mathrm{T}| < \min\{|{\mathrm{R}^k}_1|, |{\mathrm{R}^k}_2|\}$ and $\lambda > \lceil(|\mathrm{T}|-1)/2\rceil$, where $\lceil x \rceil$ is the integer ceiling value of x.

Proof

A k-transition T is represented by a dome in $\rho_\lambda(g)$ if $\exists p \in {\mathrm{R}^k}_1$ such that $\varepsilon_\lambda(g)(p) = \delta_\lambda(g)(p)$ and if $\exists q \in {\mathrm{R}^k}_2$ such that $\varepsilon_\lambda(g)(q) = \delta_\lambda(g)(q)$. These conditions are true, if the radius λ of the SE used by the erosion ε_λ and dilation δ_λ is $\lambda < (|{\mathrm{R}^k}_1| - 1)/2$ and $\lambda < (|{\mathrm{R}^k}_2| - 1)/2$. Considering that the morphological gradient compute the maximum grayscale variation of a neighborhood of radius λ, and we are interested in computing the variation of the grayscale between two k-flat zones separated by a k-transition T, the radius λ must be $\lambda > (|\mathrm{T}| - 1)/2$. Thus, there is at least a point in this dome with this maximum variation. So, $\lambda < (|{\mathrm{R}^k}_1| - 1)/2$ and $\lambda < (|{\mathrm{R}^k}_2| - 1)/2$ imply in $\lambda < \min\{(|{\mathrm{R}^k}_1| - 1)/2, (|{\mathrm{R}^k}_2| - 1)/2\}$. Finally, $(|\mathrm{T}| - 1)/2 < \min\{(|{\mathrm{R}^k}_1| - 1)/2, (|{\mathrm{R}^k}_2| - 1)/2\}$, i.e., $|\mathrm{T}| < \min\{|{\mathrm{R}^k}_1|, |{\mathrm{R}^k}_2|\}$. □

Unfortunately, in real cases, the n-transitions between two n-flat zones are noisy and we need to introduce a filtering operation on the images, like the alternated morphological filters [3, 5], to produce monotone n-transitions. The size of SE used here can be associated with the size of the n-transitions that we would like to analyze. In this work, we are interested only in single domes. Each single dome can be represented by two values, the top and the basis area numbers. The top area and basis area correspond to the size of the regional maximum of the single dome and the number of points that belongs to the single dome, respectively.

Proposition 4.2 (Single dome basis and top areas) Let g be a 1D image. We denote by $\rho_\lambda(g)$ the morphological gradient of size n of the image g. Let ${\mathrm{R}^k}_1$ and ${\mathrm{R}^k}_2$ be two k-flat zones separated by a monotone k-transition T of size ($|\mathrm{T}| < \min\{|{\mathrm{R}^k}_1|, |{\mathrm{R}^k}_2|\}$ and $\lambda > \lceil(|\mathrm{T}| - 1)/2\rceil$. The top and basis areas of a single dome that corresponds to a monotone k-transition T are $\mathrm{Top} = 2\lambda - |\mathrm{T}|$ and $\mathrm{Basis} = 2\lambda + |\mathrm{T}|$, respectively.

Proof

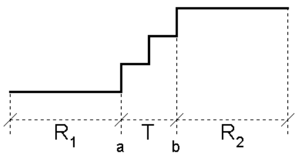
From our hypothesis, a monotone k-transition T, located between the positions a and b, that separates two k-flat zones, R_1 and R_2, is represented by a dome D in morphological gradient $\rho_\lambda(g)$ of the image g (see Proposition 4.1), so there is at least a point $p \in \mathrm{D}$ such that its grayscale value is equal to the difference h between the grayscale values of R_1 and R_2. Thus, the point p belongs to a regional maximum of D. The erosion and dilation of size λ shift T by λ positions (in Fig. 3(b) and (c) we show an example of these operations), and consequently the adjacent k-flat zones are also shifted. So, the k-transition T of g is related to the k-transition T′ in the eroded image that is located between the positions a' and b', and the k-transition T of g is related to the k-transition T″ in the dilated image that is located between the positions a'' and b''. To compute the morphological gradient, we subtract the eroded image from the dilated image producing a new image. As R_1 and R_2 are k-flat zones, the subtraction between the shifted k-flat zones in the eroded and the dilated image produce a regional maximum of grayscale h, and as the k-transition is monotone, the dome D contains only a regional maximum. Thus, the monotone k-transition of g is represented by a single dome in the morphological gradient of g. The size of the regional maximum, called top area, and the number of points that belongs to the single dome, called basis area, correspond to the minimal and maximal distance between the transitions T′ and T″ in the dilated and eroded image (Fig. 3(d) shows an example of the morphological gradient), so

$$\mathrm{Top} = a + \lambda - (b - \lambda) = 2\lambda - |\mathrm{T}| \tag{10}$$

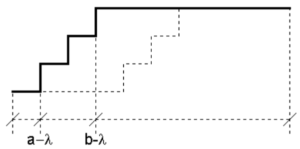
$$\mathrm{Basis} = b + \lambda - (a - \lambda) = 2\lambda + |\mathrm{T}| \tag{11}$$

□

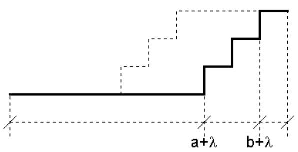
Considering that the top and the basis areas are associated with the size of the n-transition, and that the n-transition can be represented by a single dome, we can use the morphological residue information to characterize the properties of the n-transition taking into account that each n-transition on g (see Proposition 4.1) is represented by a single dome on the gradient image. The decomposition in morphological residues can be useful in classifying the single domes, more precisely, depending on the size of SE, two transitions can be merged to form a single dome, i.e., according to the features of this decomposition (as we will see in the next section), a single dome can not represent a n-transition.



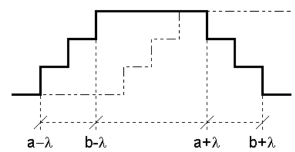
(a) |T| = 2



(b) δ_3



(c) ε_3



(d) ρ_3

Figure 3: Example of basis and top areas computed from a linear structuring element of radius $\lambda = 3$.

4.2 Morphological residues

The morphological residues characterize the information extracted from an image by considering a set of granulometric transformations. The residues are given by the difference between two consecutive granulometric levels, as follows:

Definition 4.4 (Morphological residues [5]) Let $(\psi_\lambda)_{\lambda \geq 0}$ be a granulometry. The morphological residues of residual level λ, $\mathcal{R}_\lambda$, are given by the difference between the result of two consecutive granulometric levels, that is,

$$\forall \lambda \geq 1, X \in Z^2, \mathcal{R}_\lambda(X) = \psi_{\lambda-1}(X) \backslash \psi_\lambda(X) \quad (12)$$

$$\forall \lambda \geq 1, f \in Z^2, \mathcal{R}_\lambda(f) = \psi_{\lambda-1}(f) - \psi_\lambda(f) \quad (13)$$

where X and f represent a binary and a grayscale images, respectively. The morphological residues represent the components preserved at level $(\lambda - 1)$ that are eliminated at the granulometric level λ.

According to the transformation ψ, the set of residues corresponding to $(\mathcal{R}_\lambda)_{\lambda \geq 1}$ contains the complete granulometric information and defines a complete hierarchical representation of an image in the sense that the original image can be exactly reconstructed from its residues:

$$X = \bigcup_{\lambda \geq 1} \mathcal{R}_\lambda(X) \quad (14)$$

$$f = \sum_{\lambda \geq 1} \mathcal{R}_\lambda(f) \quad (15)$$

where X and f represent a binary and a grayscale image, respectively. Furthermore, we can limit the amount of information to be analyzed by considering only the data defined at a certain resolution. To follow, we define the residual levels in which the top and the basis areas are preserved.

Definition 4.5 (Residue level numbers) The residue levels Inf and Sup, for a single dome, correspond, respectively, to the levels in which the top and the basis areas are preserved in the morphological residue representation. These values correspond to $\text{Inf} = \lceil(\text{Top}/2)\rceil$ and $\text{Sup} = \lceil(\text{Basis}/2)\rceil$.

5 Parametrized transition detection algorithm

As stated before, an image can be completely decomposed into morphological residues. Here, we consider this decomposition for the morphological gradient image to study the behavior of its single domes, that is, we will verify if a single dome in the morphological gradient image corresponds to a n-transition in the original image. To facilitate the analysis of this decomposition, we consider the residue mapping [4].

Definition 5.1 (Residue mapping [4]) Let g be a 1D image. We denote by $(\mathcal{R}_\lambda)_{\lambda \geq 0}$ the family of morphological residues of g. Let M be a set of regional maxima of g. For all points $p \in \text{M}$, we define a residue mapping, $\mathcal{M}$, as follows

$$\mathcal{M}(p) = (\mathcal{M}_1(p), \mathcal{M}_2(p), \cdots, \mathcal{M}_\lambda(p)) \text{ where}$$

$$\mathcal{M}_i(p) = \begin{cases} 1, & \text{if } \mathcal{R}_i(p) \geq 1 \\ 0, & \text{if } \mathcal{R}_i(p) = 0 \end{cases} \quad (16)$$

where λ represents the last level in which the morphological residues are greater than zero.

If $\mathcal{M}_k(p) = 1$ then the point p is preserved in the residue of level k. Fig. 4(a) shows an example of morphological gradient of size 4. If we compute the residue mapping, $\mathcal{M}(.)$ for the regional maxima in Fig. 4(a)-right, we have the following (for $\lambda = 1, 2, ..., 7$):

$$\forall p \in A, \mathcal{M}(p) = (0, 0, 0, 1, 1, 0, 0)$$
$$\forall p \in B, \mathcal{M}(p) = (0, 0, 1, 1, 1, 1, 0)$$
$$\forall p \in C, \mathcal{M}(p) = (0, 0, 0, 1, 0, 0, 0)$$

Fig. 4(b) shows an example of morphological gradient of size 10. If we compute the residue mapping, $\mathcal{M}(.)$ for the regional maxima in Fig. 4(b)-right, we have the following (for $\lambda = 1, 2, ..., 16$):

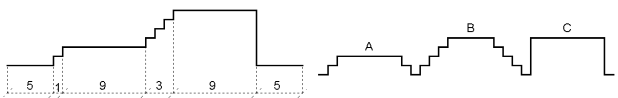
$$\forall p \in D, \mathcal{M}(p) = (0, 0, 0, 0, 1, 1, 1, 0, 0, 0, 0, 0, 1, 1, 1, 0)$$
$$\forall p \in E, \mathcal{M}(p) = (0, 0, 0, 0, 1, 0, 1, 1, 0, 1, 0, 1, 0, 1, 1, 0)$$

Finally, from this mapping information, we can easily extract some features of the analyzed components.

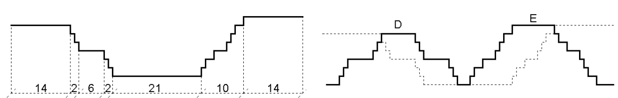
Algorithm 5.1 (Parametrized transition detection algorithm by residue)

Input: |T|: the size of transition to detect
ξ: permitted error
f: 1D original image
k: the minimum size of each flat-zone
- Compute the morphological gradient of size $\lambda \geq |\mathrm{T}|$ of the original image
- Compute the mapping, $\mathcal{M}(.)$, for the regional maxima of the above output image
- Compute the Top and Basis values according to Equation 10 and Equation 11
- Compute the First(.), Last(.) and k-Last(.) numbers based on the mapping $\mathcal{M}(.)$
- Compare the First(.) number with the Top value, the Last(.) and k-Last(.) number with the Basis value considering error ξ

Output: Transitions of size |T| are detected if both comparisons are true.



(a)



(b)

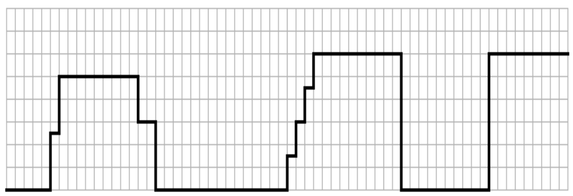
Figure 4: Example of 4-transitions. The image illustrated in (a)-left contains 3 4-transitions. The image illustrated in (b)-left contains 3 4-transitions. (a)-right and (b)-right illustrate the morphological gradient of (a)-left and (b)-left of size 4 and 10, respectively.

Definition 5.2 (First, last and n-last numbers) Let $\mathcal{M}(p)$ be a mapping of a point p and let n be the minimum permitted size of a flat zone. We call first number the index of the first appearance level of p in $\mathcal{M}(p)$, i.e., the first level in which p is preserved. We call last number the index of the last disappearance level of p in $\mathcal{M}(p)$, i.e., the last level in which p is preserved. We call n-last number the index of the last disappearance level of p before the first $\lfloor (n-1)/2 \rfloor$ consecutive zeros in $\mathcal{M}(p)$, if there is $\lfloor (n-1)/2 \rfloor$ consecutive zeros in $\mathcal{M}(p)$, otherwise the n-last number is equal to last number.

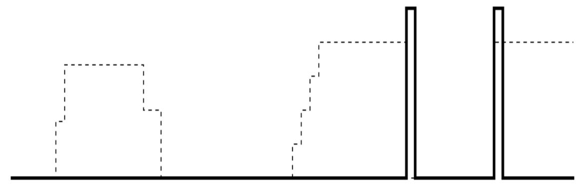
For some examples of mapping illustrated above, we have the following: First$(B) = 3$, Last$(B) = 6$ and 4-Last$(B) = 6$; First$(D) = 5$, Last$(D) = 15$ and 4-Last$(D) = 7$; First$(E) = 5$, Last$(E) = 15$ and 4-Last$(E) = 15$. The last and n-last numbers are used to differentiate the shape of the single domes, more precisely, a single dome can be associated with a n-transition if the last value is equal to the n-last value. Also, the first and n-last numbers are closely related to the top and basis areas of a dome, Top and Basis, respectively. While the computation of these areas depends on the transitions that we want to detect (see Proposition 4.2), the first and n-last values depend on the morphological decomposition information. So, to find a n-transition T with Top and Basis measures, according to a family of SE used, we need simply to identify the points p of the image with First(p) equal to Inf, n-Last(p) equal to Sup and Last(p) equal to Sup. For example, to find the 4-transition of size $|\mathrm{T}| = 3$ illustrated in Fig. 4(a), we need to compute the theoretical numbers (top and basis areas) that corresponds to the single dome associated with this kind of transition. Considering that the size of morphological gradient is $\lambda = 4$, so Top $= 2\times4-3 = 5$ and Basis $= 2\times4+3 = 11$, and consequently Inf $= 3$ and Sup $= 6$. Finally, we must compare these values to the first, last and 4-last values computed from the residue decomposition. In this example, the single dome which contains the regional maximum B corresponds to the 4-transition of size 3. The same process can be realized to find the 4-transition of size 13 illustrated in Fig. 4(b). In this example, we can verify that the two single domes have the same top and basis values, but according to the features of the residue decomposition, only the dome which contains the regional maximum E corresponds to a transition.

Algorithm 4.1 summarizes our approach for detecting transitions of a certain size |T| on a gradient image. Due to the presence of noise in real images, we consider a range $[|\mathrm{T}|, |\mathrm{T}| + \xi]$ of allowable size.

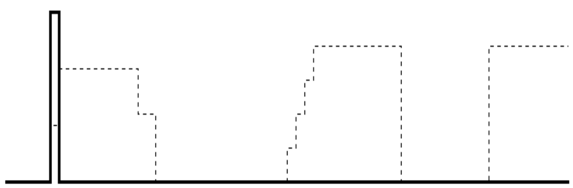
In Fig. 5 we show an example of k-transition identification, where there are 2 sharp k-transitions, 3 gradual k-transitions with different sizes (1, 2 and 3), where $k = 4$. We apply our parametrized transition de-



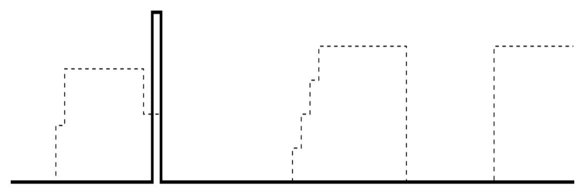
(a) Original



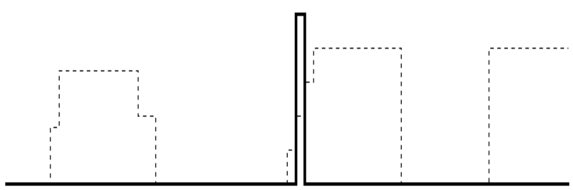
(b) $|\mathrm{T}| = 0$ and $\xi = 0$



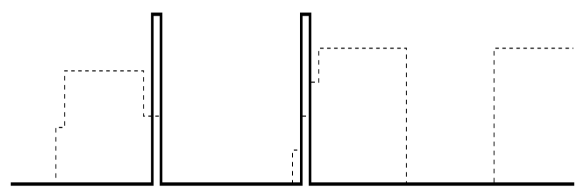
(c) $|\mathrm{T}| = 1$ and $\xi = 0$



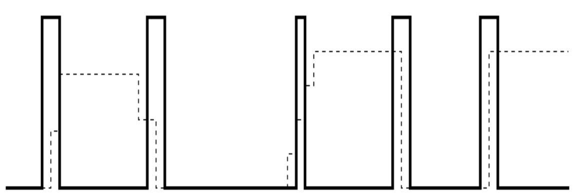
(d) $|\mathrm{T}| = 2$ and $\xi = 0$



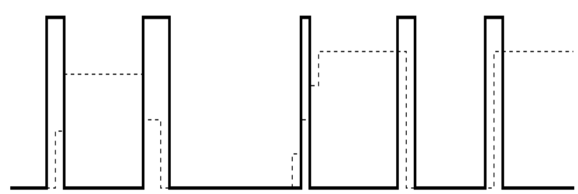
(e) $|\mathrm{T}| = 3$ and $\xi = 0$



(f) $|\mathrm{T}| = 2$ and $\xi = 1$



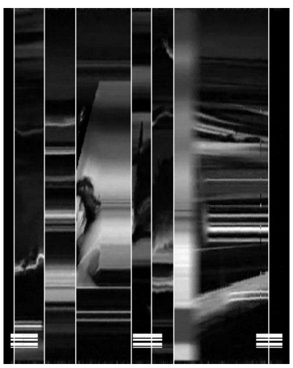
(g) Soille (ρ_3^S)



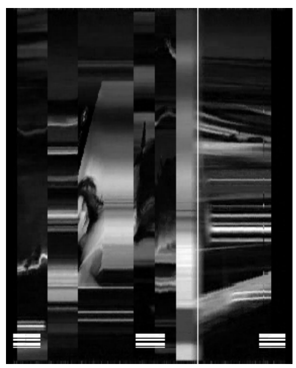
(h) Ultimate (ρ_3^U)

Figure 5: Examples for computation of gradients: (a) original image; (b-f) results of our algorithm applied to the image illustrated in (a); (g) and (h) illustrate the thresholded results (L_1) of the gradient proposed by Soille and gradient based on ultimate erosion, respectively.

(a)



(b) $|\mathrm{T}| = 0$ and $\xi = 0$



(c) $|\mathrm{T}| = 6$ and $\xi = 0$

Figure 6: Example of video transition detections (see [7, 8]): (a) visual rhythm by sub-sampling; (b) detected cuts and (c) detected gradual transition is represented by white vertical bars.

tection algorithm to identify and classify the transitions according to their size. In Fig. 5(b) only the sharp k-transitions are identified, in Fig. 5(c), Fig. 5(d) and Fig. 5(e) the k-transitions of size 1, 2 and 3 are detected, respectively. In Fig. 5(f) we identify k-transitions within an interval of permitted size, according to a given error. Finally, in Fig. 5(g) and Fig. 5(i) we illustrate the thresholded result (L_1) of the Soille's gradient and based on ultimate erosion, respectively, in which the parameter of size is 3.

6 Discussions and conclusions

In this work, we propose a new method to characterize and identify the directional boundary between consecutive flat zones according to a parameter of size. Our method is based on a hierarchical decomposition of the image components in morphological residues which allow the analysis of their transitions at different levels of this decomposition. According to the parametrization of our method, we can easily identify the transitions of a specific size, while this identification is not so easy from other methods, like Soille's gradient. Also, differently of the Soille's gradient and its variant, which considers each scale separately, our method consider all hierarchical decomposition data to obtain the transition information. This is possible thanks to analysis of morphological residues.

An interesting application of the method proposed here concerns the video segmentation problem in which we can classify the transitions according to a parameter of size. An example of this application is given in Fig. 6 in which we apply the algorithm proposed in [7, 8] to video segmentation. In this algorithm, we can replace the gradient computation by our 1D parametrized transition detection algorithm.

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