# Analysis of the Robustness of Iterative Restoration Methods with Respect to Variations of the Point Spread Function

Vital C. Ferreira*, Nelson D. A. Mascarenhas*

*Universidade Federal de São Carlos - Depto. de Computação - CP 676 - CEP 13565-905 - São Carlos - SP - Brazil
(vital, nelson)@dc.ufscar.br

Abstract: This paper presents an empirical comparison of the Landweber iterative image restoration method with the direct CLS method, with the goal of determining which is more robust against errors in the estimated point spread function.

1. Image Formation Process
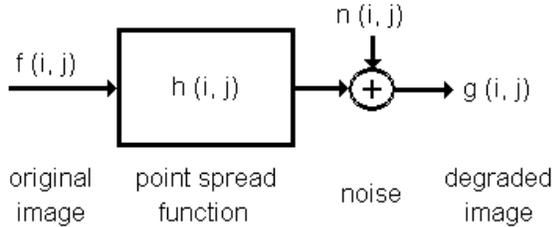
2. Restoration Methods

❑ Constrained Least Squares (direct)

$$\mathbf{h_{cls}} = \left(\mathbf{H^T H} + a\mathbf{C^T C}\right)^{-1}\mathbf{H^T}$$

❑ Landweber (iterative)

$$\hat{\mathbf{f}}_{\mathbf{k+1}} = \hat{\mathbf{f}}_{\mathbf{k}} + b\mathbf{H^*}(\mathbf{g} - \mathbf{H}\hat{\mathbf{f}}_{\mathbf{k}})$$

❑ Landweber with Regularization

$$\hat{\mathbf{f}}_{\mathbf{k+1}} = b\mathbf{H^* g} + \left(\mathbf{I} - b\left(\mathbf{H^* H} + a\mathbf{C^* C}\right)\right)\hat{\mathbf{f}}_{\mathbf{k}}$$

❑ Landweber with Regularization and Deterministic Constraints

$$\hat{\mathbf{f}}_{\mathbf{k+1}} = \mathbf{P}\left\{b\mathbf{H^* g} + \left(\mathbf{I} - b\left(\mathbf{H^* H} + a\mathbf{C^* C}\right)\right)\hat{\mathbf{f}}_{\mathbf{k}}\right\}$$

where α is the regularization parameter, β is a convergence parameter, and $\mathbf{C}$ is a Laplacian operator.
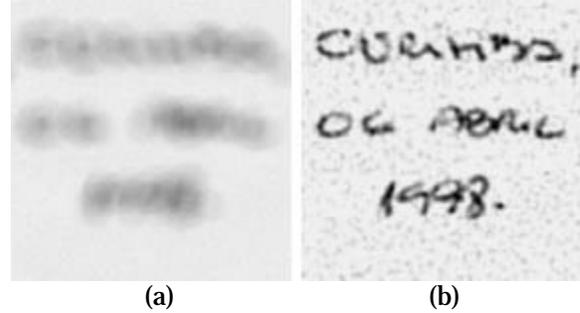
3. Quality Measure

$$\text{Improvemt in Signal to Noise Ratio} = 10\log_{10}\left\{\frac{\sum_{i,j}[g(i,j) - f(i,j)]^2}{\sum_{i,j}[\hat{f}(i,j) - f(i,j)]^2}\right\}$$

4. Results

(a)   (b)
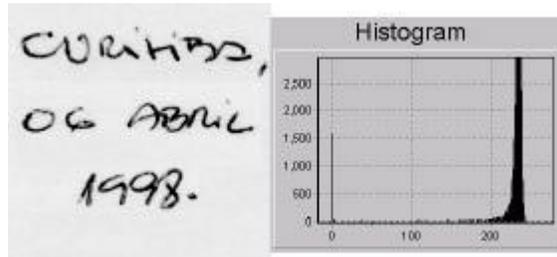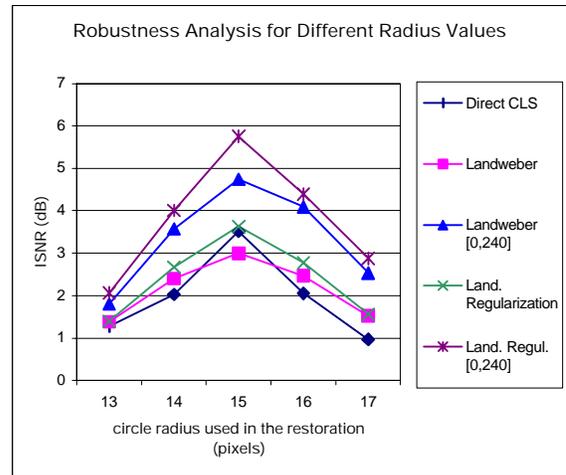
(a)   Bitmap test image in gray scale (256 x 256 pixels)
(b)   Image histogram

(a)   (b)

(a)   Image degraded with an out-of-focus blur (circle radius equal to 15 pixels), with additive gaussian noise (variance = 16)
(b)   Restored image (ISNR=5.77 dB) by the Landweber iterative method with regularization and with inequality constraints ($0 < f(i,j) < 240$).

ISNR versus circle radius of the used point spread function (PSF) in the restoration procedure where the best values were plotted tuning the α parameter and stopping the iterations at the optimum ISNR.

5. Conclusions

• Iterative methods are more robust than the direct CLS method with respect to using PSF with different circle radius sizes.
• Regularization in the iterative methods tends to increase ISNR.
• The use of inequality constraints, besides improving ISNR, increases robustness with respect to the reconstruction with incorrect PSF radius.

6. Acknowledgment