Determination of the reference point of a fingerprint based on multiple levels of representation

Jorge L. A. Samatelo, Evandro O. T. Salles
Graduate Program on Electrical Engineering
Federal University of Espirito Santo
Vitoria, Brazil
{jlaching,evandro}@ele.ufes.br

***Abstract*—In this article we propose a technique to detect the reference point of a fingerprint based on the main supposition that the crests belonging to the superior area of reference point usually have a orientation quantized of 0^o. To take advantage of this fact it is accomplished an analysis of the orientation image considering their multiple levels of representation in relation to their values of quantization. This is a new approach that allows the use the morphologic characteristics of the quantized orientation image for each value of quantization. The technique presents some important characteristics: adaptability according to the quality of the image and the possibility of detecting the reference point of the fingerprints A and TA with acceptable error rate for these type of fingerprint. The technique is tested in the whole database of DB4 of NIST getting an estimation error of the reference point of 8.32% for a total of $4,000$ images.**

***Keywords*-fingerprint; singular points; orientation image.**

I. Introduction

A fingerprint is constituted of two types of oriented structures called crests and valleys. The valleys are conformed by the spaces among the crests. Therefore, the valleys (white color) are the negative of crests (black color). These structures contain two types of information; the global characteristics called singularities, which are invariant to translation, rotation, expansion and contraction of the fingerprint, and the local characteristics called minutiae that provide sufficient information to determine the individuality of a fingerprint. The lines that constitute the structures of crests are neither continuous nor straight, changing its direction gradually around of the singular points. It has two types of singular points: the *core* and *delta* points. A *core* point is defined as the area where the curvature of the crests is convergent to the local maximum and a *delta* point is defined as the area where the curvature of the crests is convergent to the local minimum [1][2]. Therefore, the singular points are physical centers of convergence and divergence of the structures of crests and valleys.

In a recognition system based on fingerprint, the singular points are useful to the task of classification [3][4][5] (reducing the search space in a database) and for the determination of the reference point of a fingerprint (eliminating the translation that a fingerprint could present). It is defined

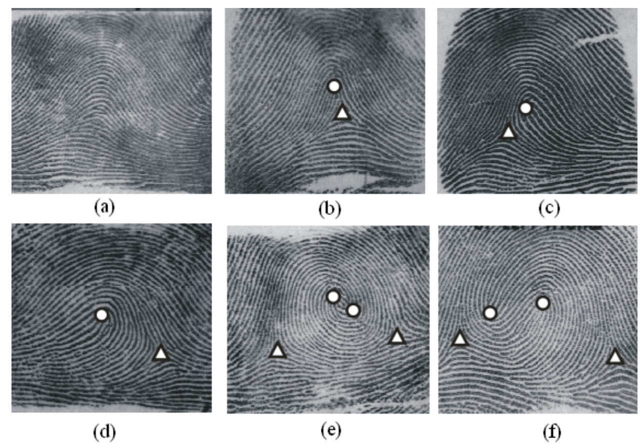

Figure 1. Types of fingerprints (a) A; (b) TA; (c) RL; (d) LL; (e) W; (f) TL. Where the *core* point is indicated by the circle and the *delta* point is indicated by the triangle.

as the reference point of a fingerprint the point that has maximum curvature in the most internal crests. Usually, the *core* point is used as reference point. The number of *core* points varies in relation to the fingerprint type. The types *Tented Arch* (TA), *Right Loop* (RL), and *Left Loop* (LL) has one *core* point (see Fig. 1.b, 1.c and 1.d respectively). The types *Plain Whorl* (W) and *Twin Loop* (TL) has two *core* points (see Fig. 1.e and 1.f respectively). The type *Arch* (A) doesn't present a *core* point (see Fig. 1.a).

A variety of techniques has been proposed for the detection of the singular points. Most of them operates in the orientation image, Θ, also known as field directional. The image Θ is defined as a matrix of size N_{Fil}xN_{Col} where $\Theta(i,j)$ represents the average orientation of the crests belonging to the block (i,j) of the image I (see Fig. 2.b). In addition, it is also used the region image Π to delete false singularities, where the region image indicates the category of block (i,j) of the image I. A block (i,j) with value 0 indicates a corrupted region and a block with value 1 indicates a well defined region (see Fig. 2.c). Another usefull information is the coherence matrix *Coh*. *Coh* is determined from the orientation image. It contains the homogeneity

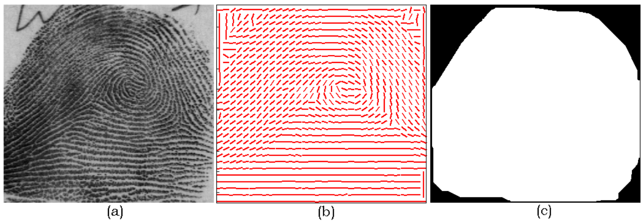

Figure 2. (a) fingerprint I; (b) orientation image Θ; (c) region image Π.

information in a region of the orientation image. In other words, those regions with dominant orientation have a coherence close to 1, while the constant regions (isotropic) have a coherence close to 0.

Some of the approaches given in literature are: In [6] the Poincaré index is used for the detection of the singular points *core* and *delta*. However, despite being efficient, this index is sensitive to noise in Θ (noisy areas in the fingerprint generating neighborhoods in Θ where the orientations change quickly, giving false singularities). Also, this technique does not detect the reference point for fingerprint type *Arch* given that the point with maximum curvature is not the *core* point in a strict sense.

In [7] is proposed a technique based on sinusoidal component of Θ, that locates the reference point by applying a spatial mask. This technique is robust to noise, but is sensitive to the fingerprint rotation. Furthermore, this technique does not effectively detects the reference point for fingerprint type *Arch*. In [8] is proposed a technique based in Θ labeling, allowing the detection of the reference point type *Arch*. However, this technique is sensitive to the noise and possible rotations of a fingerprint. In [9] is proposed a technique based on multiscale analysis of the consistence matrix (the consistency matrix is equivalent to coherence matrix) by finding a local minimum. This technique is robust to the rotation, however as it depends on the consistence matrix it becomes sensitive to the fingerprint noise. In [1] it is proposed a technique based on analysis of a sequence of line calls *fault lines*, obtained from the orientation image, such that the direction of these lines and their intersection defines the position of singularities. This technique works on blocks (each block is of size 2x2) and uses an algorithm of thinning and expansion based on a pyramid model to extract the singular points. This technique is insensitive the rotation. However, it requests an orientation image quite smoothed and a complex heuristic that establishes which crossing points corresponds to singular points.

The techniques based only on the orientation image are efficient since that the image is smoothed enough. In this case, the singularities generated by noise are attenuated and only singular points *core* and *delta* are present on final orientation image. However, these techniques present a low precision in the calculation of the reference point for

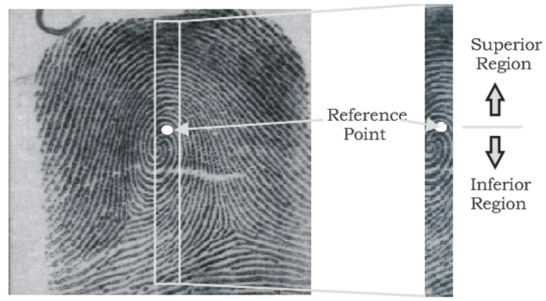

Figure 3. Distribution of the crests on a neighborhood of the reference point.

fingerprint type A.

For those reasons, we propose a technique based not only in the detection of singularities of the orientation image, but has the capacity to detect the reference point of any fingerprint type. The proposed technique is based on a new conception which was called “multiple levels of representation of the orientation image".

II. Algorithm

A great difference exists among the orientations of the lines of flow of crests in the superior and inferior regions to the reference point. As shown in Fig. 3 the lines of flow of ridges in the superior region of the reference point are nearly straight lines with a slope of 0^o, and the lines of flow of crests in the inferior region don't have a constant orientation. Therefore, we can suppose that the crests of the superior area to the reference point usually have an orientation quantized of 0^o. In [8] is proposed a technique of detection of the reference point that uses this characteristic considering that the reference point is located in the most inside element of the superior part of Θ whose orientation quantized is 0^o. Considered only this information, generates a loss of accuracy in estimating of the reference point (not always the reference point is in a block quantized to a 0^o) and creates great trouble in detecting the reference point of a fingerprint type A. To overcome these problems, is proposed the analysis of Θ at multiple levels of representation according to their values of quantization. Thus, the blocks of the neighboring orientations of 0^o are also used, we see below.

Suppose that Θ is quantized in Q values, i.e. it is only possible to have Q spaced orientations π/Q rad, Θ could be represented as the sum of matrices Θ_0, Θ_1, $\cdots$, Θ_{Q-1}:

$$\Theta(i,j) = \Theta_0(i,j) + \sum_{k=1}^{Q-1} \frac{k\pi}{Q} \Theta_k(i,j), \qquad (1)$$

such that:

$$\Theta_k(i,j) = \begin{cases} 1 & k = \left\lfloor \Theta(i,j)\frac{Q}{\pi} \right\rfloor \\ 0 & \text{otherwise} \end{cases}, \qquad (2)$$

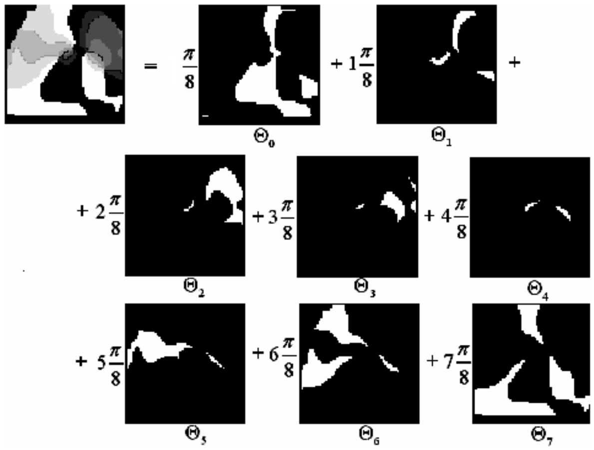

Figure 4. Orientation image constituted by their components of quantization, when $Q = 8$.

where: $\lfloor \bullet \rfloor$ is the greatest integer operator.

Each Θ_k is a binary image where its elements equal to 1 indicates the position of those elements in Θ that has the orientation $k\pi/Q$. So, we can say that Θ_k is the k^{th} component or representation (in relation to the value of quantization) of the orientation image. (See Fig. 4).

Multiple levels of representation analysis will involve the mixing of the matrix Θ_k's (at some level of analysis) so that some characteristic of our interest present in Θ is emphasized. A mixture that allows emphasizing the characteristics of Θ related to the reference point is given by:

$$\Theta_d^B(i,j) = \Theta_m(i,j) + \Theta_n(i,j), \tag{3}$$

such that: $m = \text{mod}(-d,Q)$ e $n = \text{mod}(Q-1-d,Q)$, where, d is a displacement parameter that defines the considered analysis level. Θ_d^B is a binary image (see Fig. 5), where only the blocks with orientations $m\pi/Q$ and $n\pi/Q$ are 1.

For fingerprints type TA, RL, LL, TL and W the reference point is located at the innermost corner of the binary object located in the superior part of Θ_d^B (see Fig. 6.b - 6.f) and for the fingerprint type A the reference point is located in the thinnest part of the largest binary object of Θ_d^B (see Fig. 6.a)

The central idea of this technique is determine in Θ_d^B the possible reference point, considering as final reference point the most intern of them. A variation in gray scale of this proposal is defined as:

$$\Theta_d^C(i,j) = \sum_{r=0}^{Q-1}\sum_{k=0}^{Q-1} w_d(k,r)\Theta_{\text{ mod }(k-d,Q)}(i,j), \tag{4}$$

$$w_d(k,r) = \exp\left(-\frac{1}{2\sigma^2}(k - \text{ mod }(r-d,Q))^2\right), \tag{5}$$

where w_d weights the mixture of the different Θ_k, such

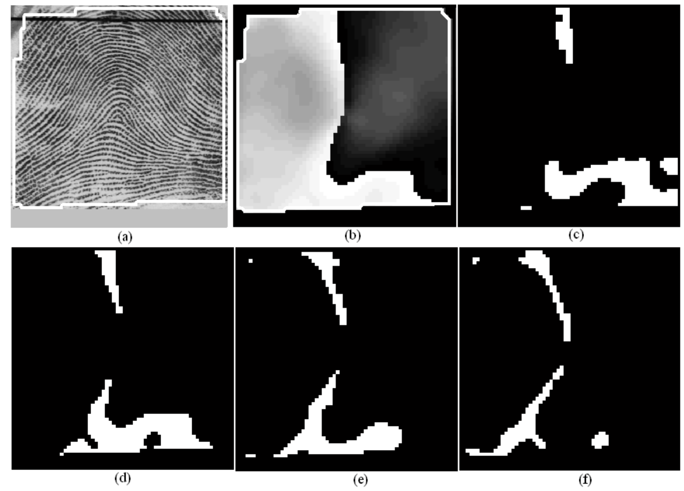

Figure 5. (a)I; (b)Θ quantized in 32 levels; (c)Θ_0^B; (d)Θ_1^B; (e)Θ_2^B; (f)Θ_3^B.

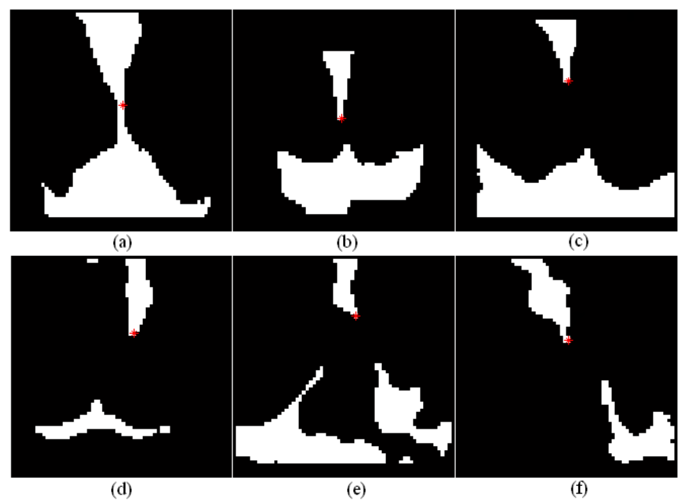

Figure 6. Images Θ_d^B associated the fingerprints type (a)A; (b)TA; (c)RL; (d)LL; (e)W; (f)TL, where $d = 1$ and $Q = 16$.

that the blocks with orientations $m\pi/Q$ and $n\pi/Q$ have the minimum values (see the appendix for demonstration). This representation in gray scale allows the use of the morphological opening operator to delete small disturbances in Θ_d^C, preserving the characteristics of importance, which are the corners belonging to the contour of the connected component constituted by the blocks with orientations $m\pi/Q$ e $n\pi/Q$ (see Fig. 7).

In function of the representation in multiple levels Θ_d^B and Θ_d^C, is proposed an original technique that allows locate the reference point of a fingerprint without considering its type. The steps are:

1) Θ is quantized in Q discrete angular values $\{0, \pi/Q, \cdots, (Q-1)\pi/Q\}$.
2) A fingerprint quality index is determined through the coherence matrix Coh and the region image Π.

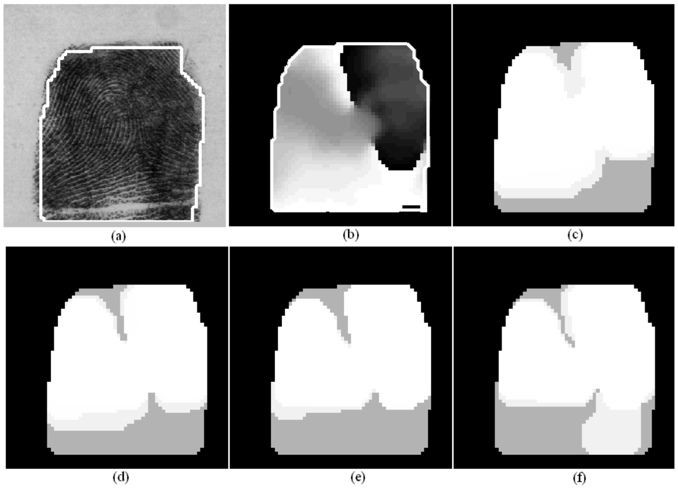

Figure 7. (a)Image I; (b)Θ quantized; images Θ_d^C for (c)$d=0$; (d)$d=1$; (e)$d=2$; (f)$d=3$;

First, it is obtained the number of elements of the foreground of Π, this number is called N_{Pri}. Therefore, is determined the number of elements of Coh that are inferiors to a threshold t_Q and belonging to the foreground, this number is called N_{Coh}. We defined as an indicator of Quality $Q_c(t_Q)=N_{Coh}/N_{Pri}$. Therefore, a fingerprint will have good quality if $Q_c(t_Q)$ is close to 1 (the threshold t_Q defines the degree of sensitivity of $Q_c(t_Q)$).

3) For $d=0,1,\cdots,Q/8-1$, to do:
   - If $Q_c(t_Q)$ is smaller than 0.8, then calculate Θ_d^B, otherwise calculate Θ_d^C. In case that Θ_d^C is calculated, the morphological opening operator is applied to Θ_d^C and the resulting image is thresholded to obtain the connected components of the blocks with orientations $m\pi/Q$ and $n\pi/Q$. Because of the generated image is binary it is also called Θ_d^B.
   - An element by element multiplication between Θ_d^B and Π is performed, eliminating the elements of Θ belonging to the background.
   - It is determined the thinned binary image B_m. For each line of Θ_d^B is determined the midpoint of each set of 1's belonging to this line. $B_m(i,j)=1$ if (i,j) is the position of midpoint already detected.
   - It is determined the vector of finalization points P_e of the image B_m. The Rutovitz definition of the number of crossings is applied to the image B_m [10]. This definition determines the positions of the finalization points of a thinned binary image. In this case, the finalization points of B_m are the corners of the present binary objects in Θ_d^B. These positions are stored in the vector of finalization points $P_e=\{(i_{p1},j_{p1}),(i_{p2},j_{p2}),\cdots\}$.
   - The vector P_e is filtered to delete all the positions that are not a valid corner of Θ_d^B.
4) Define the binary image B_T, such that is the union of all binary images Θ_d^B ($B_T=\Theta_0^B\cup\Theta_1^B\cup\cdots\cup\Theta_{Q/8-1}^B$). Therefore, are deleted the isolated components connected in B_T.
5) For each valid point of P_e, it is extracted the correspondent line in B_T, and is calculated the length of the set 1 to which it belongs. If this length is greater than a threshold, then the point is eliminated from P_e. Also, the point is deleted if a set of 1 doesn't exist (since the point in question may have belonged to an island of B_T already deleted).
6) If the number of valid points in P_e is greater than 0 and the number of connected components of B_T is greater or equal to 1, then the valid points of P_e belonging to the same connected component of B_T are grouped and is defined as a representative point of that connected component to the lowest of them. Therefore, the reference point is the largest representative point of connected components (the reference point cannot be in the inferior part of the fingerprint).
7) If the number of valid points in P_e is equal to 0 and the number of connected components of B_T is greater than 1, it is defined the centroid of Π as the reference point.
8) If the number of valid points in P_e is equal to 0 and the number of connected components of B_T is equal to 1, can be assumed that the fingerprint is of type A. Therefore, the reference point is the centroid of the single connected component of B_T.
9) If the reference point determined in (6) and (8) is near to the background, then the reference point is the centroid of Π. Otherwise, only the reference point determined in (6) is recalculated in a neighborhood of the coherence matrix, Coh, to improve the accuracy of the position calculation.

The step sequence of this technique is shown in the Fig. 8. Starting with the quantization of Θ in 32 values (Fig. 8.a), next, the calculation of the images Θ_d^B is carried out, where the gray box indicates the innermost corners of superior binary object of each Θ_d^B and the routes of medium points are indicated by the dark lines (Fig. 8.b-e). Fig. 8.f presents the binary image B_T where the gray box indicates all valid corners, considering as a possible reference point to validate lower corner. In the Fig. 8.g we have the recalculation of the reference point through the coherence matrix Coh and in the Fig. 8.h we have a final reference point indicated with a white box.

A. Experimental results

For the tests, it was used the database of fingerprints number 4 (BD4) of NIST [11]. The DB4 consists of 4,000 images in gray levels of 480x512 pixels, 8 bits per pixel, with a resolution of 500 dpi. These images correspond to

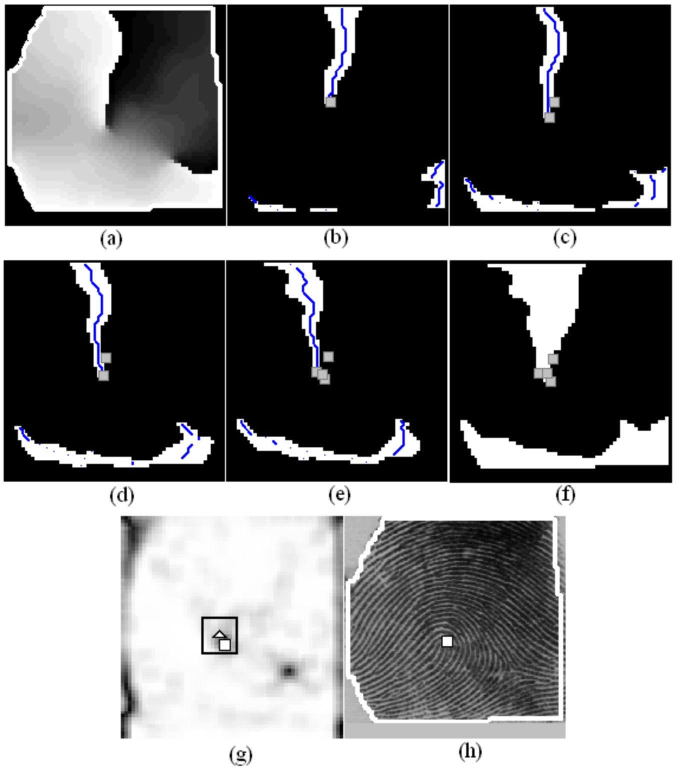

Figure 8. (a)Θ quantized in 32 values; (b)Θ_0^B; (c)Θ_1^B; (d)Θ_2^B; (e)Θ_3^B; (f)B_T; (g) recalculation of the reference point through of the *Coh*; (h)reference point final.

fingerprints scanned from cards. Each fingerprint has two different instances labeled with the letters F and S, indicating if it corresponds to the first instance or second instance of each fingerprint. In addition, each fingerprint was examined by a forensic specialist who assigned a label that sets the class to which the fingerprint belongs (A, LL, RL, TA or W). Some ambiguous fingerprints (close to 17.5%) has an additional label referred to a additional class.

Since the images of the DB4 are images of rolled fingerprints, an important part of this database is constituted by low-quality fingerprints, which is difficult to determine the reference point. To analyze the quality fingerprints impact, each one was manually labeled as poor quality, medium quality or good quality, according to their region image and orientation image. The fingerprints distribution, according to their quality, is shown in Table I, where is observed that 22.35% of the total number of images in the DB4 is of poor quality (the TA class has the largest amount of fingerprint of poor quality).

The performance of the detection technique of the reference point is validated observing its capacity of detecting the position of reference point with accuracy. Therefore, the analysis of the robustness needs to evaluate the influence that has the used technique for the calculation of the orientation image and the influence of the quality of the fingerprint.

Table I
DISTRIBUTION OF FINGERPRINTS IN THE DATABASE DB4 BY THEIR QUALITY CATEGORIES.

Quality	Class					
	LL	W	RL	TA	A	Total(%)
Poor quality	193	162	185	222	132	22,35
Medium quality	91	73	71	94	66	9,875
Good quality	516	565	544	484	602	67,775

In order to measure this influence, it was developed the following experiment:

1) For each image belonging to DB4:(a) it is determined manually the reference point of the fingerprint. For those fingerprints where the reference point is not detectable, we assumed as reference point the central point of the image. (b) The reference point of the image is calculated using the proposed technique and the technique based on the Poincaré index [5] with the improvements referred in [12]. (c) We defined a neighborhood of 61x61 pixels centered in the reference point detected manually. If the reference point detected by any of the automatic techniques is inside of the neighborhood, then this point is considered as a correct estimation of the reference point. Otherwise, we suppose that the technique doesn't determine with accuracy the reference point. The selection of the size of the neighborhood is made considering the exposed in [12], where is indicated that the most of the category information in the fingerprint lies in the lower part of the fingerprint centered in the reference point. Therefore, it is more important to detect the neighborhood where is located the reference point than its exact location.
2) The error rate is obtained by the quotient between the incorrect determination of the reference point and the total number of images in database.

The results of this experiment are summarized in the tables II, III and IV. It is observed the great influence that has the selection of the technique for the estimation of the local orientation. In this case, the technique for the estimation of the local orientation that generates the minimum error rate, for both techniques of detection of the reference point, is the technique proposal in [13]. The technique based on the Poincaré index has an error rate of 21.675% and the technique based on multiple levels has an error rate of 8.325%. The superior performance is due to the capacity of the technique based on multiple levels of detecting accurately the reference point of the fingerprints type TA and A. However, the accuracy in the detecting the reference point for the fingerprints type LL, RL and W is comparable to the technique based on the Poincaré index [6]. In the case of fingerprints type W, the technique based on the

Poincaré index is superior to the technique based on multiple levels (5.87% versus 8.125%, respectively). That is because the fingerprint type W has two *core* points: an inferior and a superior *core* point. Sometimes they are very close, making the technique based on multiple levels consider the *core* point inferior as a reference point (the main assumption of the technique based on multiple levels is that the reference point is in the proximities of the most internal element of the superior part of Θ, whose orientation quantized is 0^o).

In general, the technique based on the Poincaré index has an average error rate of 21.8% and the technique based on multiple levels have a minimum error rate of 8.32% with a maximum error rate of 23.2%. This last case happens when it is used the technique proposed in [14] for the estimation of the local orientation. Respect to the quality of the images, considering only the technique for the estimation of the local orientation proposed for [13], the technique based on multiple levels is superior for the three quality groups (poor, medium and good quality), where the maximum percentage of error rate are from images of poor quality (20.18% of the total number of images of poor quality).

Table II
THE ERROR RATES OF ACCURACY WHEN IT USES THE TECHNIQUE OF ESTIMATION OF LOCAL ORIENTATION STANDARD, WHERE THE LABEL (**MC**) INDICATES THE FINGERPRINTS OF POOR QUALITY, THE LABEL (**MD**) INDICATES THE FINGERPRINTS OF MEDIUM QUALITY AND THE LABEL (**BC**) INDICATES THE FINGERPRINTS OF GOOD QUALITY.

Technique	Error (\%)								
	Total	For class					For quality		
		LL	W	RL	TA	A	MC	MD	BC
Poincaire	22.05	8.5	7.25	6.75	25.62	62.12	28.93	18.98	19.40
Proposed	14.42	9.87	10.37	8.00	15.37	28.50	31.35	14.93	9.36

Table III
THE ERROR RATES OF ACCURACY WHEN IT USES THE TECHNIQUE OF ESTIMATION OF LOCAL ORIENTATION PROPOSED BY [14].

Technique	Error (\%)								
	Total	For class					For quality		
		LL	W	RL	TA	A	MC	MD	BC
Poincaire	21.8	8.50	7.00	10.50	21.50	61.50	32.30	19.74	19.25
Proposed	23.2	14.87	15.37	19.50	29.20	37.00	40.96	26.58	17.63

Table IV
THE ERROR RATES OF ACCURACY WHEN IT USES THE TECHNIQUE OF ESTIMATION OF LOCAL ORIENTATION PROPOSED BY [13].

Technique	Error (\%)								
	Total	For class					For quality		
		LL	W	RL	TA	A	MC	MD	BC
Poincaire	21.67	7.5	5.87	7.35	27.25	60.37	28.74	17.46	20.51
Proposed	8.32	4.75	8.12	5.00	7.0	16.50	20.18	8.86	4.72

III. Conclusions

We propose a new technique based on multiple levels of representation of the orientation image for the determination of the reference point of a fingerprint. This is a new approach that uses the morphologic characteristics of the quantized orientation image. The technique presents the following characteristics: (a) the capacity of adaptation according to the quality of the image, (b) the capacity of detection the reference point of the fingerprints A and TA obtaining acceptable error rates, (c) the use of the coherence matrix increases the accuracy in the determination of the reference point. This technique was tested in all DB4 database getting an error of estimation of the reference point of 8.32% over a total of $4,000$ images.

IV. Appendix

The following demonstration shows that the representations associated with the orientations $m\pi/Q$ and $n\pi/Q$ have a minimum amplitude in (4).

Developing the relationship (4), so that, $\Theta_d^C(i,j)$ is expressed as a linear combination of representations $\Theta_{\bmod (k-d,Q)}(i,j)$:

$$\Theta_d^C(i,j) = w_T(0)\Theta_{\bmod (-d,Q)}(i,j) + \cdots + w_T(Q-1)\Theta_{\bmod (Q-1-d,Q)}(i,j), \quad (6)$$

where:

$$w_T(k) = \sum_{r=0}^{Q-1} w_d(k,r). \quad (7)$$

Suppose that:

$$p = |k - \bmod(r-d,Q)|, \quad (8)$$

for $r = 0,1,\cdots,Q-1$, $d = 0,1,\cdots,Q/8$ is obtained:

- when $k=0$, p take the values: $0,1,\cdots,Q-1$.
- when $k=1$, p take the values: $1,0,1,\cdots,Q-2$.

  $\vdots$
- when $k=Q-1$, p take the values: $Q-1,\cdots,1,0$.

The relationship (5) can be written in function of p as:

$$w_d(k,r) = w_d(p) = \exp\left(-\frac{1}{2\sigma^2}p^2\right), \quad (9)$$

replacing (9) in (7) and considering the values what take p when k and r vary, we obtain the relationship:

$$w_T(k) = 1 + \sum_{p=1}^{Q-(k+1)} \exp\left(-\frac{p^2}{2\sigma^2}\right) + \sum_{p=1}^{k} \exp\left(-\frac{p^2}{2\sigma^2}\right). \quad (10)$$

From (10) we can prove that: $w_T(k) = w_T(Q-(k+1))$, where a recursive representation for $w_T(k+1)$ could be expressed as:

$$w_T(k+1) = w_T(k) + \exp\left(-\frac{(k+1)^2}{2\sigma^2}\right) - \exp\left(-\frac{(Q-(k+1))^2}{2\sigma^2}\right). \quad (11)$$

One should observe that when:

- $k = 0$, $w_T(1) = w_T(0) + \exp(-1/2\sigma^2) - \exp\left(-(Q-1)^2/2\sigma^2\right)$, implying that: $w_T(1) > w_T(0)$.
- $k = 1$, $w_T(2) = w_T(1) + \exp(-2/\sigma^2) - \exp\left(-(Q-2)^2/2\sigma^2\right)$, implying that: $w_T(2) > w_T(1)$.

$\vdots$

- $k = Q/2-2$, $w_T(Q/2-1) = w_T(Q/2-2) + \exp\left(-(Q/2-1)^2/2\sigma^2\right) - \exp\left(-(Q/2+1)^2/2\sigma^2\right)$, implying that: $w_T(Q/2-1) > w_T(Q/2-2)$.

So, we can see that $w_T(Q/2-1) > \cdots w_T(2) > w_T(1) > w_T(0)$ implying that $w_T(0)$ is the minimum value for $k \in [0, Q/2]$. Using the property of symmetry of w_T we conclude that $w_T(Q-1)$ is the minimum value for $k \in [Q/2, Q-1]$. From the equation (6) can see that the representations corresponding to the coefficients $w_T(0)$ and $w_T(Q-1)$ are $\Theta_{\bmod\,(-d,Q)}$ and $\Theta_{\bmod\,(Q-1-d,Q)}$ respectively. $\Theta_{\bmod\,(-d,Q)}$ and $\Theta_{\bmod\,(Q-1-d,Q)}$ are the selected representations in the mixture level d^{th}, therefore, we obtain a mixture equivalent to (3).

References

[1] C.-Y. Huang, L. min Liu, and D. D. Hung, "Fingerprint analysis and singular point detection," *Pattern Recognition Letters*, vol. 28, no. 15, pp. 1937 – 1945, 2007.

[2] V. S. Srinivasan and N. N. Murthy, "Detection of singular points in fingerprint images," *Pattern Recognition*, vol. 25, no. 2, pp. 139–153, feb 1992.

[3] B. H. Cho, J. S. Kim, J. H. Bae, L. G. Bae, and K. Y. Yoo, "Core Ű based figerprint image classification," in *International Conference on Pattern Recognition (15th)*, 2000, pp. 863Ű–866.

[4] L. Hong and A. K. Jain, "Classification of fingerprint images," in *Scandinavian Conference on Image Analysis (11th)*, 1999.

[5] Q. Zhang and H. Yan, "Fingerprint classification based on extraction and analysis of singularities and pseudo ridges," *Pattern Recognition*, vol. 37, no. 11, 2004.

[6] M. Kawagoe and A. Tojo, "Fingerprint pattern classification," *Pattern Recognition*, vol. 17, 1984.

[7] A. K. Jain, S. Prabhakar, L. Hong, and S. Pankanti, "Filterbank-based fingerprint matching," *IEEE Transactions Image Processing*, vol. 9, no. 5, pp. 846–859, 2000.

[8] C.-H. Park, S.-K. Oh, D.-M. Kwak, B.-S. Kim, Y.-C. Song, and K.-H. Park, "A new reference point detection algorithm based on orientation pattern labeling in fingerprint images," in *Pattern Recognition and Image Analysis, First Iberian Conference, IbPRIA*, ser. Lecture Notes in Computer Science, vol. 2652, Puerto de Andratx, Mallorca, Spain, June 4-6 2003, pp. 697–703.

[9] M. Liu, X. Jiang, and A. C. Kot, "Fingerprint reference point detection," *Journal of Applied Signal Processing*, vol. 4, pp. 498–509, 2005.

[10] A. K. Jain, L. Hong, and R. M. Bolle, "On-line fingerprint verification," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 19, no. 4, pp. 302–314, sep 1997.

[11] C. I. Watson and C. L. Wilson, "Nist special database 4," National Institute of Standards and Technology (NIST), Tech. Rep., 1992.

[12] A. K. Jain, S. Prabhakar, and L. Hong, "A multichannel approach to fingeprint classification," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 21, no. 4, 1999.

[13] X. Jiang, "On orientation and anisotropy estimation for online fingerprint authentication," *IEEE Transactions on Signal Processing*, vol. 53, no. 10, pp. 4038–4049, Oct 2005.

[14] Y. Wang, J. Hu, and F. Han, "Enhanced gradient-based algorithm for the estimation of fingerprint orientation fields," *Applied Mathematics and Computation*, vol. 185, no. 2, pp. 823–833, 2007.