Computing the q-index for Tsallis Nonextensive Image Segmentation

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***Abstract*—The concept of entropy based on Shannon Theory of Information has been applied in the field of image processing and analysis since the work of T. Pun [1]. This concept is based on the traditional Boltzaman-Gibbs entropy, proposed under the classical thermodynamic. On the other hand, it is well known that this old formalism fails to explain some physical system if they have complex behavior such as long rang interactions and long time memories. Recently, studies in mechanical statistics have proposed a new kind of entropy, called Tsallis entropy (or non-extensive entropy), which has been considered with promising results on several applications in order to explain such phenomena. The main feature of Tsallis entropy is the q-index parameter, which is close related to the degree of system nonextensivity. In 2004 was proposed [2] the first algorithm for image segmentation based on Tsallis entropy. However, the computation of the q-index was already an open problem. On the other hand, in the field of image segmentation it is not an easy task to compare the quality of segmentation results. This is mainly due to the lack of an image ground truth based on human reasoning. In this paper, we propose the first methodology in the field of image segmentation for q-index computation and compare it with other similar approaches using a human based segmentation ground truth. The results suggest that our approach is a forward step for image segmentation algorithms based on Information Theory.**

***Index Terms*—Image segmentation; q-entropy; Tsallis entropy**

I. Introduction

Image segmentation plays an important role on the basis of computational vision tasks, such as image analysis, recognition and tracking, to name a few. This is a basic but important and complex problem which has been intriguing the researches for decades. The issue behind image segmentation is to decompose the image into regions of coherent properties in an attempt to identify objects and their parts.

Gray level image segmentation techniques can be classified into the following categories: thresholding, methods based on feature space, edge detection based methods, region based methods, fuzzy logic techniques and neural networks. Besides, most of these methods can be extended to color images by representing color information in an appropriate color space. In addition, it is possible to combine more than one approach to achieve better performance such as Zhu and Yuille [3], Shi and Malik [4], Malik *et al*. [5] and Ma and Manjunath [6].

Among these methods, those based on threshold are fundamental for our work. Thresholding is a large class of segmentation techniques that are based on the assumption that the objects can be distinguished and extracted from the background by their gray levels. The output of traditional thresholding operations is a binary image whose intensity pattern distinguish the foreground (e.g. gray level 0) from the background (e.g. gray level 255). Interesting surveys underlining the thresholding segmentation can be found in references [7] and [8]. In general, threshold selection can be categorized into two classes: local methods and global methods. By the way, a threshold can be based on different criterions, such as Otsus method [9], minimum error thresholding [10], and entropic methods [1], [11], to name a few.

Applying the concept of entropy in order to segment a digital image is a common practice since PUN's work [1] shows how to find out a threshold that maximizes the information measure, the very celebrated Shannon entropy, of the resulting binary image. Other works following the same philosophy were proposed, e.g, Kapur *et al* [11] maximized un upper bound of the total a posteriori entropy in order to obtain the threshold level. Abutaleb [12] extended the method using two-dimensional entropies. Li and Lee [13] and Pal [14] used the directed divergence of Kullback-Leibler for the selection of the threshold, and Sahoo *et al.* [8] used the Reiny entropy model for image thresholding. A recent review about entropy methods for image segmentation can be read in [15].

The concept of Shannon entropy was proposed in the Theory of Information based on Boltzaman-Gibbs entropy for the context of classic thermodynamic. However, for several decades it is well known that this concept fails to explain some phenomena which have complex behaviors such as long range interactions and long-time memories [16], [17]. Such systems are called "non-extensive systems", and those following the BGS formalism are called "extensive systems".

In 1988, Tsallis proposed a new formalism for the generalization of BGS entropy, which is called q-entropy or Tsallis entropy. This new entropy has reached relative success in explanation complex phenomena for several applications. The main feature of Tsallis entropy is the introduction of a q parameter, called extensiveness parameter. It has been proved in the literature that each physical system is close related to a specific value for q, and to achieve the optimal q value for a specific physical system is a challenge and has been issue of great debates between current researchers. A complete list of non-extensive systems is vast and can be fully find in [17].

In 2004, Albuquerque *et. al* [2] applied the concept of non-extensive entropy for mamographic gray scale images. They assume a probability distribution of gray scale luminance, one for background and other for foreground class of pixels. Then, they take the threshold that maximizes the separation between these two classes. The work of Albuquerque *et al* was an advance of the method based on Shannon entropy for image segmentation.

The main drawback of Tsallis entropy, as well as the algorithm proposed by [2] is the choose of the q-index. Since it is not an intuitive idea, several applications should to randomly choose its value. Up to our knowledge, there is no automatic method or theory proposed for its automatic computation.

Another important questions remain regarding the image segmentation methods. How to measure the quality of output segmentation. This is due to the lack of a database for ground truth and due to the difficult to build a function to compare the similarity between two segmentations.

The work described in this paper present three contributions. firstly, we introduce an automatic method to the computation of q parameter; then we present a database for ground truth, which was segmented by human subjects. Finally, we present a new similarity measure which compute the quality of image segmentation in x and y euclidian dimensions and z luminance dimension.

This paper is organized as follows. In Section II we introduce the q-entropy under the context of non-extensive systems and explains the original non-recursive method. In Section III we show how to compute the q parameter. In Section IV we explain our database. Is Section V we present our proposed measurement method. In section VI we explain how we carried out our experiments and in final Section VII we discuss our main conclusions.

II. Theoretical Background

The traditional equation for entropy, over a probability density function $p(x)$, also called Boltzmann-Gibbs-Shannon entropy (BGS), is defined as:

$$S = -\sum_i p_i \ln(p_i) \tag{1}$$

Generically speaking, systems which can be described by Equation (1) are called extensive systems and have the following additive property: Let A and B be two random variables, with probability densities functions $A = (a_1, \ldots, a_n)$ and $B = (b_1, \ldots, b_n)$, respectively, and S be the entropy associated with A or B. If A and B are independent, under the context of the Probability Theory, the entropy of the composed distribution[1] verify the so called additivity rule:

$$S(A * B) = S(A) + S(B) \tag{2}$$

This rule was used by several researchers of Computational Vision Systems to achieve an optimal threshold aiming to separate foreground from background of intensity images [18], [19]. The general idea, historically presented by T. Pun [1], considers the gray level histogram with L bins a symbol source, with all the symbols statistically independent.

This traditional form of entropy is well known and for years has achieved relative success to explain several phenomenon *if* both the effective microscopic interactions *and* the effective spatial microscopic memory are *short*-ranged. Roughly speaking, when the system does not has such behavior, the standard formalism became only an approximation, and some kind of extension appears to became necessary. A complete review about this theory can be see [16], [17], [20].

Recent developments based on the concept of non-extensive entropy, also called Tsallis entropy, have generated a new interest in the study of Shannon entropy for Information Theory [21]. Tsallis entropy (or q-entropy) is a new proposal for the generalization of BGS traditional entropy applied to non-extensive physical systems.

The non-extensive characteristics of Tsallis entropy have been applied through the inclusion of a parameter q, which generates several mathematical properties which the general equation is the following:

$$S_q(p_1, \ldots p_k) = \frac{1 - \sum_{i=1}^{k} p_i^q}{q-1} \tag{3}$$

[1] we define the composed distribution, also called direct product of $A = (a_1, \ldots, a_n)$ and $B = (b_1, \ldots, b_n)$, as $A * B = \{a_i b_j\}_{i,j}$, with $1 \leq i \leq n$ and $1 \leq j \leq n$

where k is the total number of possibilities of the whole system and the real number q is the entropic index that characterizes the degree of non-extensiveness.

In the limit $q \rightarrow 1$, Equation (3) meets the traditional BGS entropy defined by Equation (1). These characteristics give to q-entropy more flexibility to explain several physical systems, which can not be properly explained by traditional BGS formalism. Then, this new kind of entropy does not fail to explain the traditional physical systems since it is a generalization.

Furthermore, a generalization of some theory may suppose the violation of one of its postulates. In the case of the generalized entropy proposed by Tsallis, the additive property described by Equation (2) is violated in the form of Equation (4), which apply if the system has a non-extensive characteristic. In this case, the Tsallis statistics is useful and the q-additivity describes better the composed system. In our case, the experimental results (Section VI) show that it is better to consider our systems as having non-extensive behavior.

$$S_q(A * B) = S_q(A) + S_q(B) + (1-q) S_q(A) S_q(B) \tag{4}$$

In this equation, the term $(1-q)$ stands for the degree of non-extensiveness. Note that, as we said before, when $q \rightarrow 1$, this equation meets the traditional Equation (2).

Recently, Albuquerque *et al.* [2] proposes an algorithm using the concept of q-entropy to segment general images. Their idea is quite the same as that proposed by T. Pun, however, under the concept of Tsallis entropy, having the followinf formalism. Suppose an image with L gray-levels. Let the probability distribution of these levels be $P = \{p_i = p_1; p_2; \ldots; p_L\}$. Then, we consider two probability distribution from P, one for the foreground (P_A) and another for the background (P_B). We can make a partition at luminance level t between the pixels from P into A and B. In order to maintain the constraints $0 \leq P_A \leq 1$ and $0 \leq P_B \leq 1$ we must re-normalize both distribution as:

$$P_A : \frac{p_1}{p_A}, \frac{p_2}{p_A}, \ldots, \frac{p_t}{p_A}$$

and

$$P_B : \frac{p_{t+1}}{p_B}, \frac{p_{t+2}}{p_B}, \ldots, \frac{p_L}{p_B}$$

where $p_A = \sum_{i=1}^{t} p_i$ and $p_B = \sum_{i=t+1}^{L} p_i$.

Now, following the Equation (3), we calculate the a priori Tsallis entropy for each distribution as $S_A = \frac{1 - \sum_{i=1}^{t} (\frac{p_i}{p_A})^q}{q-1}$ and $S_B = \frac{1 - \sum_{i=t+1}^{k} (\frac{p_i}{p_B})^q}{q-1}$. Allowing the pseudo-additive property given by Equation (4), for two statistically independent systems, we can compute the pseudo-additive property of systems A and B as:

$$\begin{aligned} S_{A*B}(t) = & \frac{1 - \sum_{i=1}^{t} (\frac{p_i}{p_A})^q}{q-1} + \frac{1 - \sum_{i=t+1}^{L} (\frac{p_i}{p_B})^q}{q-1} \\ & + (1-q) \frac{1 - \sum_{i=1}^{t} (\frac{p_i}{p_A})^q}{q-1} \frac{1 - \sum_{i=t+1}^{L} (\frac{p_i}{p_B})^q}{q-1} \end{aligned} \tag{5}$$

To accomplish the segmentation task, in the work of M. Albuquerque *et al.* [2] the information measure between the two classes (foreground and background) is maximized. In this case, the luminance level t is considered to be the optimum threshold value (t_{opt}), which can be achieved with a cheap computational effort of

$$t_{opt} = argmax[S_A(t) + S_B(t) + (1-q) S_A(t) S_B(t)] \tag{6}$$

Note that the value t which maximizes Equation (6) depends on mainly the entropic parameter q. Up to now in the literature the value of q which generates t_{opt} is not explicitly calculated and must

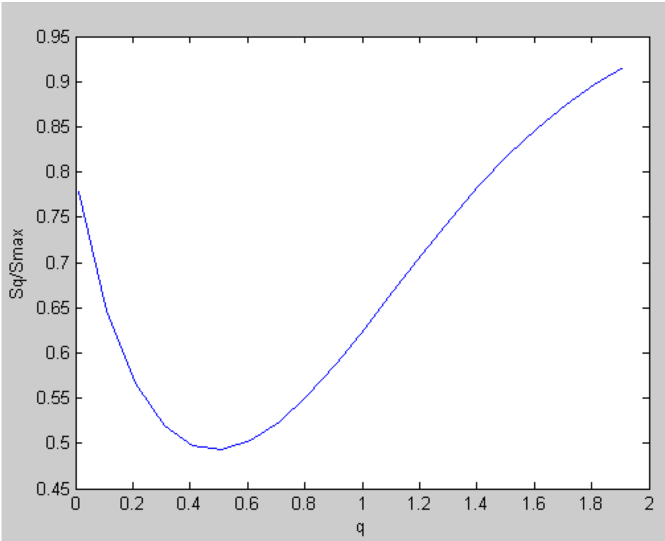

Fig. 1. S_q/S_{max} as a function of q range. The lower value, corresponding to $q = 0.46$, is the optimal q used for initial segmentation (Fig. 2-middle).

be defined empirically. In this paper, we propose an algorithm to compute the optimal q-value, which is justified and described in the next section.

III. Computation of the q index

Considering the background and foreground of an image as independent physical (sub)systems, the very celebrated strategy proposed by T. Pun [1] for image segmentation was to use the additive property (Equation (2)) of the extensive systems to achieve the optimal threshold between both (sub)systems. This idea comes from the fact that the maximum possible information is transferred when the maximal global entropy is achieved through the sum of the both systems. The same argument works for nonextensive systems. However, the formalism used in this case turns according to Equation (5), where t is the optimal threshold which maximizes the self-information.

As posed in Section II, the Tsallis formalism are a generalization of the Shannon entropy, meeting the traditional system when $q \rightarrow 1.0$ only. Thus, we can conclude that the q-entropy (as also has been called this formalism) can capture both the nonextensivity and extensivity behaviors. So, it is reasonable to investigate the segmentation entropic approaches under both contexts. Later we will show that for our image database we achieve better segmentation performance (regarding the human reasoning) under nonextensive formalism.

Of course, the usage of a new parameter has an extra computational price to pay, and despite of its class, each image or region may demand for a different q value (including $q = 1.0$) in order to achieve information maximization. Then, it is interesting to evaluate the value of the computed entropy for each image in several ranges for q; e.g, regarding sub-extensive systems ($q < 1.0$), extensive ones ($q = 1.0$) and super-extensive ones ($q > 1.0$).

From the point of view of Theory of Information, as smaller the maximum entropy S_q produced by a q value related to the theoretical maximum entropy S_{max} of a physical system (in this case, an image), larger is the self-information contained in this system. This is a well known principle of Theory of Information and yields to the idea of that the optimal q-value can be reached by minimizing the S_q/S_{max} ratio. Then, before applying the proposed formalism stated by Equation (8), we compute the optimal q value underlining the image. This is accomplished as the following. For each q value in the range $[0.01, 0.02, ..., 2.0]$ we get the optimal q as that which minimizes the S_q/S_{max} ratio. In this paper we work the hypothesis that not only each natural image may behave as a singular non-extensive system – and as such demanding for a different q value for segmentation – but also its internal regions also may be singular non-extensive ones – also demanding for different q values as well. Later, experiments will show that this is a promise hypothesis.

In order to apply different q values to segment different image regions, and to achieve most of the image's main regions, we carried out two levels of segmentation. Initially, we compute the q value minimizing S_q/S_{max} and apply the Equation (6) to get a first optimal t_{opt} threshold, obtaining a first segmentation, separating background (R_B) from foreground (R_F). Then, for each achieved region (R_B and R_F) we compute new q values, treating R_B and R_F as different physical systems, and apply the algorithm again, obtaining two new t_{opt}s as well. Thus, we can achieve at most four intensity separations and several regions in the image. Fig. 2 shows an example. Fig. 2-left is the original image, and the Fig. 2-middle is its first segmentation in two regions (R_B and R_F), achieved with the optimal $q = 0.46$, which corresponds to the minimal value of the curve of Fig. 1 (S_q/S_{max}). Following the same idea for R_B and R_F regions, we compute new q values by minimizing new S_q/S_{max} curves and achieve two new optimal thresholds t_{opt}. The result can be seen in the Fig. 2-right. In this case we found $q = 0.15$ for R_B and $q = 0.73$ for R_F, suggesting sub-extensive system behavior for all regions.

Fig. 2. (left image) a natural image; (middle image) the first segmentation with $q = 0.46$ achieving R_B and R_F; (right image) the final segmentation with $q = 0.15$ and $q = 0.73$ for the previous R_A and R_F, respectively.

IV. Database

As discussed in the Section I, the task of automatic image segmentation into its individual cognitive regions is already an open problem. We can state at least two main reasons to not consider this as an easy task: (i) a good segmentation does depend on the human subjectiveness as well as its point of view and cognitive visual target; and (ii) it is rare in the Computer Science and correlated research areas finding a database for formal result comparisons. Typically, researchers show their results on a few images and point out why the results ‘ look good’. It is not clear from these results if the technique will work for other images from the same class. At the and of the papers, the same question remains: “ What is a correct segmentation”. An alternative is to carried out a segmentation *only* in the context of a system task, such as object recognition, as did Borra and Sarkar [22].

Clearly, under the lack of a precise response to this question, we need at least a “ lighthouse” to follow as a relative point in order to compare several techniques under the same database and or parametrization. By the way, the Berkeley database, presented by D. Martin and colleagues [23], can be considered a tentative in the way to stand a point from which we can carried out measures.

The Berkeley database consists of a public available ' ground truth' segmentation produced by humans for images of a variety of natural scenes. This database has been continuously updated, and, at the moment we were writing this paper, it had 1000 images with 481x321 RGB images from the Corel image database, which is also a large usage database with 40,000 images widely used in Computer Vision (e.g. [24], [25])

In our work we use a subset of 100 images from Berkeley database with 5 segmentation by each image. Fig. 3 shows some examples of images from this database and the 5 different segmentations superimposed, where we can see the high degree of consistency between different human subjects. Additional details of database construction may be found in [23].

Fig. 3. 10 pairs of image-segmentation from our 100 images used for experiments. Each edge-map corresponds to five segmentations superimposed in order to observe the consistency between human subjects.

In the five edge-maps superimposed for the same image in Fig. 3 not all edges from each human subject meet each other. The effect is that as more subjects choose the same line more this line is highlighted. The contrary is also true. Then, in our work we use 100 edge-maps images as a base for comparison inter algorithms (comparing their output segmentations under a same parametrization) as well as intra algorithms (comparing their output segmentations under different parameterizations).

Obviously, the Berkeley database may not be considered an absolute ground truth. But, since it was generated by several independent human subjects (having high degree of cognitive consistency) it is reasonable to use this database as a relative point for segmentation comparison. However, the divergence (in absolute value) of information between some machine segmentation and the ground truth (human segmentation) will not be taken as a segmentation quality measurement. This database is only a base for relative comparison between input algorithms or algorithm's parameters. In the case of the non-extensive algorithm proposed in this paper, it is reasonable to try to response the question about *what q-value most approaches machine segmentation to human segmentation?*; or, *what we should to use?: a random constant q-value or the proposed automatic calculation for each image region?* Besides, this is an open door for a posed further question: *which class of image is better segmented with a non-extensive parameter $q \neq 1.0$?*; or *which images may be better segmented with the traditional Shannon entropy?*

V. Segmentation Measurement

In order to measure the similarity between two segmentations (in this case, between a human and a machine segmentation), we need to define a similarity function. However, this is also a difficult task and an open problem. Sezgin and Sankur [19], in their image segmentation survey, proposed a set of five quantitative criteria in order to measure the region luminance and shape uniformity of 20 classical methods for image segmentation. Since their criteria are not based on ground truth data, it is an intrinsic quality judgment of the segmented areas: e.g, an output segmentation with uniformity shape regions may not approach to expected human segmentation.

On the other hand, measuring techniques based on ground truths are also difficult to propose when the system demands for detecting several image's regions together, a common task in several computational vision applications. Also, the problem of match corresponding boundaries carries the problem to detect their corresponding whole regions, as well as their spatial localization. But in several Computer Vision applications, this will be important to infer inter-regions relationships.

Some algorithms can be useful as they tolerate any localization error, approaching slightly to mislocalized boundaries. Then, simply detecting coincident boundary pixels and consider all unmatched pixels either false positives or misses would yields to severe low performance. Clearly, as we can see from Fig. 3, the machine boundary pixels assigned to ground truth boundaries must tolerate localizations errors since even the ground truth data contains boundary localization divergencies. Then, some slope correspondence in order to permit small localization divergencies may be useful, as did the approach in [23].

On the other hand, on a 2D edge-map, such as the Berkeley one, we can find two kind of information: geometrical and luminance scattering. The geometrical scattering measures the size and localization of the region boundaries and the luminance scattering measures the boundary intensities, which, for a human segmentation, it captures the boundary cognitive consistency between all subjects.

The geometrical scattering between two edge-maps can be measured quantifying the divergence of information between both edge-maps, for x and y dimensions, and the luminance scattering for z dimension as well. The divergence of information on the x dimension between two edge-maps can be computed as the Euclidian distance between the edge-map's (e.g. M_x histogram for machine segmentation and H_x for the corresponding human segmentation) of a $M \times N$ image. Then, in this paper, we propose to use the following matching function between both edge-map's histograms, M_x and H_x, of the x-dimension in order to measure how far a machine segmentation will be from a human segmentation in this specific direction:

$$Sim_x(M_x|H_x) = \sqrt{\sum_M (M_x(i) - H_x(i))^2}. \tag{7}$$

where M_x and H_x are the probability mass functions (luminance histograms) of the boundary distribution along the x direction, and M is the size of x distribution (image resolution in x direction).

Similarly, we propose the following matching functions for y and z directions, respectively:

$$Sim_y(M_y|H_y) = \sqrt{\sum_N (M_y(i) - H_y(i))^2}. \tag{8}$$

$$Sim_z(M_z|H_z) = \sqrt{\sum_L (M_z(i) - H_z(i))^2}. \tag{9}$$

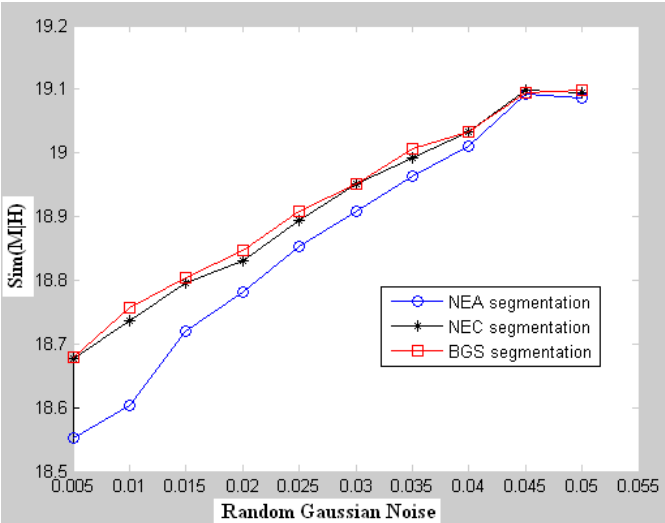

Fig. 4. Simulation Results under increasing gaussian noise.

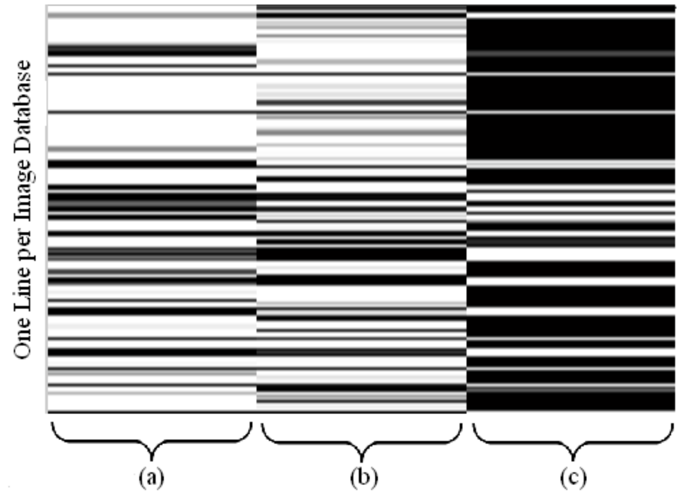

Fig. 5. Simulation of general performance. (a) is the NEA performance; (b) is the NEC performance; (c) is the BGS performance.

where N and L are the size of y and z distribution, respectively. Note that N is the image resolution in y dimension and L is the total luminance levels (e.g. 256).

Thus, we propose the following matching function to measure information between two edge-maps of a machine and a human segmentation:

$$Sim(M|H) = Sim_x + Sim_y + Sim_z \tag{10}$$

VI. Experimental Results

We have three types of segmentation to analyze: (BGS) that proposed by T. Pun [1], which is based on traditional BGS entropy; (NEC) that proposed by M. Albuquerque and colleagues [2], which uses the generalized non-extensive entropy, but with a constant manually chosen q value; and (NEA) our proposed method, which is also based on generalized non-extensive entropy but with an automatic calculated q value, according to Section III.

Firstly, it is interesting to observe how the three algorithms behave under noise situation. Then, in the first experiment, we randomly choose an image I_m and its corresponding I_g edge-map from the Berkeley database and apply the following four steps: (i) add to I_m gaussian noise with zero mean; (ii) apply the algorithms BGS, NEC and NEA over the noisiness I_m image and achieve an edge-map with the Canny operator; (iii) measure the Sim similarity, given by Equation (10), between I_g edge-map and the output Canny edge-maps for BGS, NEC and NEA algorithms, respectivelly; (iv) Repeat the steps i-iii 50 times (taking the average) for 10 increasing standard deviations: from $\sigma^2 = 0.005$ to $\sigma = 0.5$, given a total of 500 segmentations for each algorithm. This approach is enough to curve convergence.

In the graphic of Fig. 4, we clearly see the behavior of the three algorithms under non-increasing SNR. According to Fig. 4 all three algorithms decrease their performance in approaching to human segmentation of Berkeley image. The NEC algorithm slightly overcomes the BGS algorithm for all values of gaussian noise. On the other hand, our proposed algorithm NEA clearly overcomes both as well. All three algorithms meet for $\sigma^2 \rightarrow 0.05$, since under this value the noise is to high and there is few information to get.

It is quite subjective to get some conclusion about segmentation algorithms based on visual inspection over their output regions or edge-maps, since it is not an easy task to compare their size, spatial position and amount of output regions. Also, it is not easy to compare all results together instead of inspecting an unique image individually. Thus, our experiments use the Equation (10) in order to match a machine edge-map (e.g. given by NEC, NEA or BGS algorithm) with a human edge-map (given by Berkeley database). Also, we segment all 100 images from the Berkeley database with the three algorithms. For the NEC algorithm we use a constant $q = 0.5$ value. We normalize between $[0, 1]$ the result of Equation(10) in order to measure which algorithm most approaches the Berkeley's human segmentation. A perfect match is reached when $Sim(M|H) = 0.0$ and the worst match is reached when $Sim(M|H) = 1.0$.

In the Fig. 5 we can see an overview of all three segmentations for the whole database. Each row corresponds to an image from the database. There is three main columns: the left most column corresponds to NEA segmentation; the middle one corresponds to NEC segmentation and the right most corresponds to BGS segmentation. In order to clarify the visualization, we add a gray color to row-column pair according to their similarity to human edge-map (Equation 10), where white color corresponds to ' high similarity (near to 1.0)' and black color corresponds to ' low similarity (near to 0.0)'.

According to Fig. 5 it is clear that most of all black colors were added to the right most column, indicating that most of all images are better segmented using NEA or NEC non-extensive algorithms. Besides, our proposed method has the advantage to automatically compute the important q parameter. Numerically, for all 100 images, only 22 were better segmented with BGS entropy, 42 with our NEA and 36 with NEC. However, when we use only our NEA approach against BGS, the result is 74 NEA segmentations against 26 segmentations of BGS traditional algorithm.

VII. Main Conclusions

In this paper we proposed an automatic methodology for computing the q-index of Tsallis non-extensive gray scale image segmentation. The experimental results suggest that this is a promise technique in working with natural images even under noise influence. Besides, the results indicate better performance against the traditional similar algorithms when we compute q at running time.

The previous results have two folds. Firstly, it is possible to claim that the automatic q computing methodology is that which most approximate the obtained regions to Berkeley human segmentation. Even it is always possible to find manually a value to fit the optimal q-index, this choice is not an intuitive task as this value is very related to physical non-extensive image features. On the other hand, the automatic method proposed here can became, in the future, a start point of automatic methods for classifying images as non-extensive versus extensive systems, whose implications are now broadly investigated. The fact that most of our image database (about 74%) was better segmented (in terms of similarity to Berkeley human segmentation) is a strong indication of the non-extensive feature of such natural images.

The use of non-extensive methods for image segmentation is an advance of the celebrated BGS entropic methods, and the alternative for automatic q computing is also an advance for non-extensive methods as it opens new possibilities, not only for image analysis area, but several others with non-extensive physical behavior.

The next step is to test our proposed method to other characteristics such as color, texture and spatial information such as gradient or region shapes.

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