Map Estimation Methods for Tomographic Reconstruction in Soil Science Using "A Priori" Densities Defined on the Non-Negative Real Line - Preliminary Results

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Abstract. Maximum a Posteriori estimation methods with "a priori" densities defined on the non-negative real line are used to reconstruct images with a minitomograph scanner for soil science. Preliminary results using real phantoms indicate that it is possible to obtain improved results as compared to conventional backprojection methods.

1 Introduction

Image projections that are obtained for tomographic reconstruction with low counts are corrupted by a signal dependent noise that ideally is Poisson distributed.

The statistical approach using maximum likelihood (ML) criterion and the Expectation-Maximization (EM) algorithm produces better results than traditional methods based on convolution-backprojection, but with a high computacional price.

In a previous work [MASC93], an alternative to the ML-EM algorithm was proposed, based on first estimating the average counting rate g_i at each projection point, by measuring the count y_i and using the maximum a posteriori method. For this purpose, the gaussian "a priori" density was used.

The objective of this work is to modify the previous technique, that was tested on PET images of the brain, by taking into account the fact that the average rate at each projection point is a non-negative quantity and using "a priori" densities that are defined over the non-negative real line. Futhermore, the method was tested using cylindrical phantoms on a transmission gamma-ray minitograph scanner for soil science [CRUV90].

2 Map Filtering and Parameter Estimation

By adopting the "a priori" gamma density, the MAP pointwise estimator of g_i ($\hat{g}_i$), that performs filtering of the noisy projection, is given by:

$$\hat{g}_i = \frac{(y_i + n - 1)}{(1 + \lambda)} \quad (1)$$

The local parameters n and λ of the gamma density were estimated by first perfoming a 5 - point moving average smoothing over the noisy projection, to get an initial estimate $\bar{g}_i$ of the average rate projection and using moment estimators [GIBR73] for the parameters n and λ, given by:

$$\hat{n} = \frac{m_i^2}{s_i^2} \quad (2)$$

$$\hat{\lambda} = \frac{m_i}{s_i^2} \quad (3)$$

where m_i and s^2_i are the sample mean and sample variance, respectively, over 3, 5 or 7 windows around the point $\bar{g}_i$.

3 Preliminary Experimental Results with Real Phantom Projection Data

Besides performing filtering over simulated projection data, reconstructions with real cylindrical phantoms of aluminum, iron and nylon were performed with ^{137}Ce and ^{241}Am sources.

Figure 1 displays a 31x31 image of the iron phanton with 1s exposure of the ^{137}Ce source per projection point (size 3 window), using the conventional convolution-backprojection algorithm (Ram-Lak window) [JAIN89].

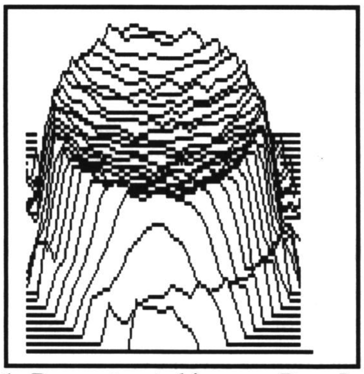

Figure 1- Reconstructed image - Ram-Lak filter

Figure 2 displays the same phantom reconstructed with the MAP filtering algorithm over the noisy projection, followed by convolution-backprojection. It can be observed that a less noisy reconstruction was obtained in latter case, with very little observable resolution loss.

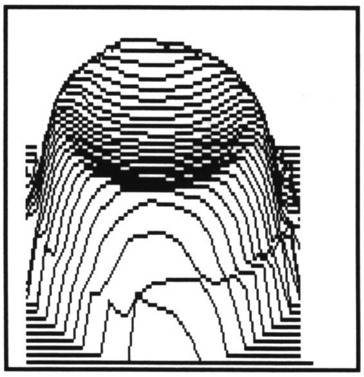

Figure 2 - Reconstructed image - MAP filter

Conclusions

The preliminary results demonstrate that, although the counting process on the minitomograph scanner does not strictly obey the Poisson distribution (histograms display, in general, greater variance than the one predicted by the Poisson model), it is possible to achieve better results with the MAP filtering than with the conventional convolution-backprojection method.

The results obtained with the "a priori" gamma density are close to those obtained with the "a priori" gaussian density. This is due to the fact that the estimated value of n ([illegible]) is high (in the order of hundreds or thousands in the simulated projections) and, by the central limit theorem, the gamma density tends to the gaussian density.

In the future, other densities defined over the no-negative real line will be tested. Examples could include Beta, Weibull, LogNormal, etc.

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