

# Microcanonical Optimization Applied to Visual Processing: An Assessment

JOSÉ R. A. TORREÃO  
EDWARD ROE

Departamento de Informática, Universidade Federal de Pernambuco  
50.732-970 Recife, PE, Brasil  
{jrat,er}@di.ufpe.br

**Abstract.** The microcanonical optimization algorithm ( $\mu O$ ) is an algorithm based on the microcanonical simulation techniques of statistical physics, which has been recently proposed as a suitable alternative to the simulated annealing approaches. The  $\mu O$  algorithm retains all of the positive features of the microcanonical annealing introduced by Barnard (which follows the pioneering work by Creutz), but avoids the necessity of an annealing schedule, thus resulting in improved computational efficiency. Here we present applications of the  $\mu O$  algorithm to some prototypical visual processing problems (binary-image smoothing, halftoning, and random-dot stereogram matching), and show that it performs better than the traditional, or canonical, simulated annealing (SA) and than Barnard's microcanonical annealing (MA).

## 1 Introduction

Since the 1980's, techniques originating in the statistical mechanics branch of physics have found their way into image processing and image analysis applications [3,5]. Such techniques are usually based on, or modifications of, the simulated annealing algorithm [3], which is a stochastic heuristic appropriate for global optimization problems where a non-convex functional has to be minimized. The simulated annealing procedure is most commonly run on top of a Metropolis-type Monte Carlo process [3], and is therefore based on the so-called *canonical ensemble*, a parallel being drawn between the image-related problem at hand and the evolution of a physical system in equilibrium at a given temperature. Lately, there have also appeared simulated annealing applications based on the *microcanonical ensemble*, where the system's evolution is controlled not by its temperature, but by its internal energy [2,7]. Microcanonical annealing (MA) has some definite advantages over the standard (canonical) annealing (SA), since it does not require the generation of random numbers, and can be implemented with low-precision integer arithmetic, but it seems that the potentiality of this approach hasn't been fully exploited, so far.

Recently, we have presented some further contributions to the use of microcanonical techniques for image-related applications [8]. We have developed a microcanonical optimization algorithm ( $\mu O$ ) which does not involve annealing, but which retains the positive features of the MA algorithm. In [8], we have

shown that, for typical image processing and computational vision applications,  $\mu O$  yields results comparable to those obtained with the canonical annealing approach, but in considerably shorter times. Here, we continue our assessment of the potential of  $\mu O$  for visual processing applications, showing that it outperforms the microcanonical annealing algorithm. The efficiency of  $\mu O$  has been tested on some prototypical image applications (binary-image restoration, halftoning, random-dot stereogram matching) and the quality of the obtained results has been remarkable.

In the following section, we present a review of the annealing procedure, both in its canonical and microcanonical versions. Next, we introduce our microcanonical optimization algorithm and illustrate its application to the three processes mentioned above, analysing its efficiency when compared to SA and MA. The paper ends with our concluding remarks and the bibliography.

## 2 Simulated Annealing

Simulated annealing is a technique developed in statistical physics, which can be applied to the resolution of combinatorial optimization problems based on the minimization of non-convex functionals (cost functions or objective functions). In annealing, the cost function associated with any such problem is identified as the energy of an imaginary physical system with many degrees of freedom, whose ground

state (lowest energy state) is sought. The annealing rationale is then to stochastically generate samples of the possible states of this system, following a prescription which guarantees the approach to the minimum energy states.

#### Canonical Annealing (SA)

Basically, there have been two prescriptions for implementing simulated annealing. The traditional one, which we treat now, is based on the so-called *canonical ensemble*, where the system is assumed to be in equilibrium with a large heat bath at a definite temperature,  $T$ . The probability distribution of its possible states is then known to be of Gibbs (or Boltzmann) form, i.e.,

$$P(E_i) = \frac{\exp(-E_i/T)}{Z(T)}, \quad (1)$$

where  $P(E_i)$  is the probability of finding the  $i$ -th state, whose energy is  $E_i$ . The normalization quantity in the denominator is called the partition function, and is given by  $Z(T) = \sum_j \exp(-E_j/T)$ . ( $T$  is here considered as a control parameter in the same units as the energy  $E$ ).

The prescription for generating the possible states of the system, in this case, is thus to obtain samples of the Gibbs distribution in (1). The Metropolis algorithm [3], which is a Monte Carlo process known to the physicists since the early 1950's, can be employed for this purpose. The Metropolis algorithm is implemented by starting with the system in an arbitrary state, and proposing random state transitions, which are accepted with probability equal to one, if the system energy decreases, and with probability given by  $\exp(-\Delta E/T)$ , if the energy increases by  $\Delta E$ . It has been proven that the sequence of states generated in this fashion converges to a sample of the distribution in (1).

In the Metropolis-based, or canonical, annealing, the convergence to the lowest-energy states of the system is guaranteed by the introduction of an appropriate schedule for decreasing the heat-bath temperature, in the sampling process above, while still maintaining thermal equilibrium. The idea is to allow the system to undergo many energy-increasing transitions in the beginning of the process (high  $T$ ), so that it will be able to avoid getting trapped at local-minima states, and then to progressively restrict the positive energy jumps, as  $T$  decreases, to guarantee that it will settle in a global minimum configuration.

#### Microcanonical Annealing (MA)

An alternative method for the simulation of large statistical systems has been introduced by Creutz in [4]. Instead of considering the system's

evolution at a fixed temperature, as with Metropolis, Creutz has proposed a procedure based on the *microcanonical ensemble*, which assumes the system to be in isolation – in contrast with the heat bath assumption of the *canonical ensemble* –, and thus to have constant total energy,  $E_S$  (in practice, the system will be found in a narrow range of energies,  $E_S - \delta E < E < E_S + \delta E$ ).

Creutz's algorithm postulates an extra degree of freedom, called a *demon*, which travels around the system carrying a variable amount of (always positive) energy,  $E_D$ , in its bag ( $E_D \ll E_S$ ). The purpose of the demon is to exchange energy with the system, in such a way that the total energy,  $E_{Total} = E_S + E_D$ , is preserved. As with the Metropolis algorithm, in Creutz's procedure the system is considered initially in an arbitrary state, and random state transitions are proposed. All transitions to lower energy states are accepted, and the difference  $-\Delta E_S > 0$  is added to the demon's bag, where  $\Delta E_S$  is the decrease in the system's energy. On the other hand, those transitions which bring about an increase in the system's energy are accepted only if the energy difference  $\Delta E_S > 0$  satisfies  $\Delta E_S < E_D$ , so that it can be provided by the demon (recall that  $E_D$  is constrained to be always positive). In this way, the total energy  $E_S + E_D$  is guaranteed to remain constant.

Since the demon is assumed to be very small, it acts as a thermometer for the system, which now functions as a heat bath. In equilibrium, we have

$$P(E_D = E) \propto \exp(-E/T). \quad (2)$$

Thus, in Creutz's microcanonical Monte Carlo simulation, the system's energy becomes the control parameter, while the temperature emerges as a statistical feature from the equilibrium distribution of the demon's states. From standard physics arguments, we obtain

$$T = \langle E_D \rangle, \quad (3)$$

where the brackets denote an average taken over all possible demon states.

An implementation of annealing based on the microcanonical ensemble is straightforward, as suggested by Barnard in [2] (see also the work by Hérault and Horaud in [7]). The approach to the low energy states, in this case, is guaranteed by a gradual decrease in the initial value of the demon energy,  $E_D$ . While presenting the advantages of not requiring the generation of random numbers or the evaluation of the transcendental function  $\exp(x)$ , Barnard's algorithm is still computationally expensive, since the simulation of the system's evolution must be repeated for the sequence of decreasing values of the

demon's energy, according to an (energy-based) annealing schedule. Trying to preserve the advantages of the microcanonical approach, but also aiming at further reductions of the computational burden, we have proposed a different algorithm for non-convex optimization, which is still based on Creutz's ideas, but which avoids the simulation of annealing. Let us refer to it here as the microcanonical optimization ( $\mu O$ ) algorithm.

### 3 The Microcanonical Optimization Algorithm

The technique which we have proposed as an alternative to the annealing approach is based on the microcanonical simulation algorithms which have been employed in statistical physics for the study of Ising systems (see [4] and [6], for examples of such applications). It consists of two procedures which are alternately applied, starting from an arbitrary initial configuration, until the system has stabilized in a low-energy state: an initialization procedure and a sampling procedure.

In the initialization phase, our goal is to place the system in a state with a lower energy than its starting configuration. This is accomplished via an iterative improvement process by which we propose changes in the system's dynamical variables, accepting only those that lead to lower energies. The initialization procedure stops when a given amount of energy (established in advance) has been drawn from the system.

From the state obtained in the initialization, the sampling phase proceeds, as in Creutz's algorithm, to generate microcanonical samples of the system's states at fixed energy (see preceding section). In contrast to Creutz's (and Barnard's) approach, in the  $\mu O$  algorithm, we impose an upper bound on the energy that the demon can hold. Thus, assuming that the demon's initial energy is  $E_I$ , and that it carries a bag of capacity  $E_F$ , the sampling phase in our algorithm corresponds to a microcanonical simulation of the system in the energy range  $E_S - E_F + E_I < E < E_S + E_I$ , where  $E_S$  is the final energy obtained in the initialization step.

The sampling phase is made to stop when thermodynamical equilibrium has been reached, which is relatively simple to establish for a microcanonical simulation (see [2] or [7]). The final state resulting from this step will then be used as the initial configuration for another initialization run, and thus consecutively, until no further reduction in the system's energy can be obtained.

When compared to the annealing-based approaches, we see that the  $\mu O$  algorithm, besides pre-

serving the advantages of the microcanonical simulations, brings the extra benefit of not requiring a time-consuming annealing schedule, with the associated trial-and-error process of determination of free parameters, such as initial temperature, cooling rate, and number of steps at each temperature. In our algorithm, only the values for  $E_I$  and  $E_F$ , respectively the demon's initial energy and capacity, have to be chosen a priori, and the prescriptions for halting the two procedures of the algorithm are unequivocal.

### 4 Experiments

Here we show examples of the application of the  $\mu O$  algorithm to some image-based processes, and compare its performance to those of the canonical and the microcanonical annealing algorithms.

The application problems which we discuss are all easily solved by stochastic optimization methods. The first two are image processing applications (binary-image smoothing and halftoning), and were first tackled via simulated annealing by Carnevali et al., in [3]. The third one is a random-dot stereogram matching, formulated as a stochastic optimization problem and solved through annealing in [1], by Barnard. In our experiments, we have followed the formulations in the above references, which involve constructing the appropriate energy functions, and looking for the configurations which give minimum energy. We have arrived at solutions via SA, MA, and the  $\mu O$  algorithm, which we compare.

(Remark: In the figures, we only show results obtained via  $\mu O$  and MA. The results obtained via SA have been reported in [8])

#### Binary-Image Smoothing

The problem, here, is to obtain a smoothed version of a given binary image, which has been corrupted by random (also binary) noise. The solution is obtained as the image  $\beta = \{\beta_{ij}\}$  which minimizes the energy function (cost function)

$$E(\beta) = \sum_{\langle ij,kl \rangle} (\beta_{ij} - \beta_{kl})^2 + \lambda \sum_{ij} (\alpha_{ij} - \beta_{ij})^2, \quad (4)$$

where  $\alpha = \{\alpha_{ij}\}$  is the degraded image ( $\alpha_{ij}$  and  $\beta_{ij}$  can assume the values 0 or 1), and where the notation  $\langle ij,kl \rangle$  indicates that the summation is over nearest-neighbor pixels, only. The first term in (4) encodes a smoothness condition for the reconstructed image,  $\beta$ , while the second term enforces its fidelity to the given image,  $\alpha$ .  $\lambda$  is a factor which controls the balance between the two terms.

Figure 1a shows the original image from which the degraded version  $\alpha$ , in Figure 1b, was obtained. Figures 1c and 1d show the smooth image,  $\beta$ , ob-

tained from the minimization of (4) via the  $\mu O$  algorithm and microcanonical annealing, respectively.

#### Halftoning

The problem of halftoning is to construct a binary image whose average intensities emulate a given continuous-tone image. Calling  $\mu = \{\mu_{ij}\}$  the binary image (intensities +1 or -1) and  $\gamma = \{\gamma_{ij}\}$  the gray-level image (intensities between +1 and -1), and defining the average intensities of  $\mu$  through the introduction of a filter (blurring function)  $V$ , as

$$\rho_{ij} = \sum_{kl} V_{ij,kl} \mu_{kl}, \quad (5)$$

we can formulate the problem of halftoning as that of minimizing the cost function

$$E(\mu) = \sum_{ij} (\gamma_{ij} - \rho_{ij})^2. \quad (6)$$

The images  $\mu$  obtained through the minimization of (6), via the  $\mu O$  algorithm and via MA, are presented in Figures 2b and 2c, respectively, with the input image  $\gamma$  being shown in Figure 2a.

#### Random-Dot Stereogram Matching

The matching problem in stereoscopy refers to the establishment of a correspondence between points in a pair of images obtained from different viewing positions, in order to determine a disparity field from which depth estimates can be obtained. There have been different methods proposed for the extraction of the disparity fields from stereo images. The stochastic optimization approach introduced in [1] involves the minimization of the cost function

$$E(D) = \sum_{ij} \|I_l(i, j) - I_r(i, j + D_{ij})\| + \lambda \sum_{ij} \|\nabla D_{ij}\|, \quad (7)$$

where  $D = \{D_{ij}\}$  is the disparity field,  $I_l$  and  $I_r$  are the left and right input images, respectively, and  $\nabla$  is a gradient operator. The first term in (7) embodies a photometric constraint, by which one tries to guarantee that points in the two images are matched which have approximately the same intensities (the match is along horizontal lines); the second one is a smoothness term, favouring disparity fields which do not vary much in a small neighborhood of sites.

We have compared the minimization of (7) through the  $\mu O$  and MA algorithms for the random-dot stereogram shown in Figures 3a and 3b. The results are presented in Figures 3c and 3d.

#### 5 Relative Efficiency of $\mu O$ , SA, and MA

Table 1 summarizes the performance of the  $\mu O$ , SA and MA algorithms, for the three experiments discussed above (The results obtained with SA have

been reported in [8]). The CPU times were obtained with a Sun Sparc-II station. The algorithms were run sequentially, with SA (respectively, MA) following a linear schedule for the decrease of temperature (respectively, energy), and both MA and  $\mu O$  being implemented with a single demon per lattice, in contrast with the multiple-demon implementations of [2] and [7]. When scanning the image lattices for updating the system's states, a maximum of 100 sweeps were performed at each temperature (respectively, energy) for the SA (respectively, MA) algorithm, and at each sampling step of the  $\mu O$ . The results presented confirm the higher efficiency of the  $\mu O$  approach.

		Exp 1	Exp 2	Exp 3
$\mu O$	Energy	13255.91	1.74	23440.00
	Time (s)	73.8	217.16	1176.48
MA	Energy	13446.50	2.04	23410.00
	Time (s)	455.88	434.10	4600.00
SA	Energy	13558.70	2.07	23467.00
	Time (s)	1917.06	729.26	9634.79

Table 1

#### 6 Concluding Remarks

The microcanonical optimization algorithm ( $\mu O$ ) has been proposed by us, in [8], as an alternative to the simulated annealing approaches to non-convex optimization problems. Such algorithm follows the ideas introduced by Creutz [4], and first exploited by Barnard [2] in the context of stochastic optimization but, unlike the latter, it avoids the recourse to annealing. Here we have made an assessment of the performance of  $\mu O$  when compared to the standard (canonical), and to the microcanonical annealing algorithms, and we have shown the higher computational efficiency of our approach.

#### 7 References

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Figure 1a

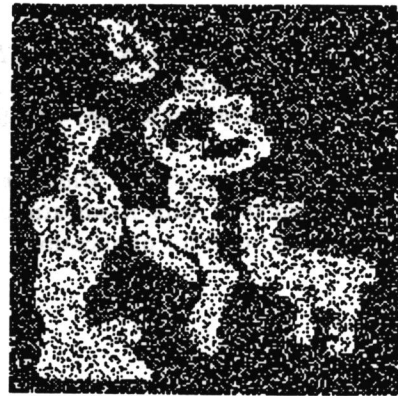


Figure 1b



Figure 1c



Figure 1d

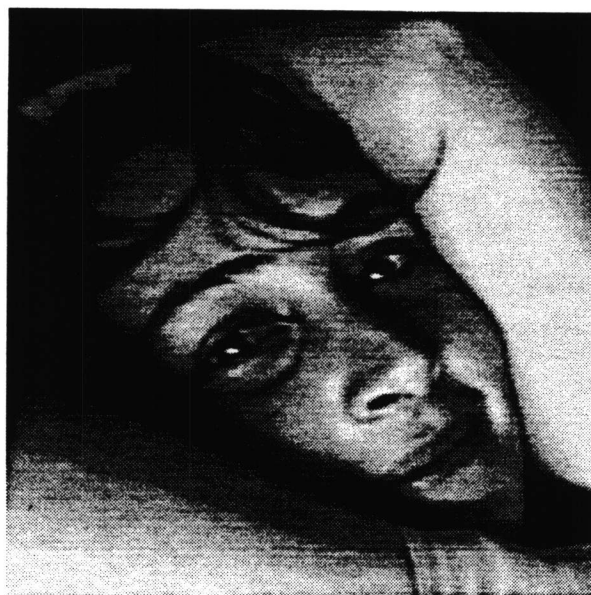


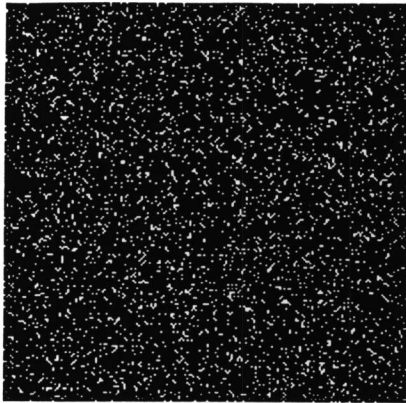
Figure 2a



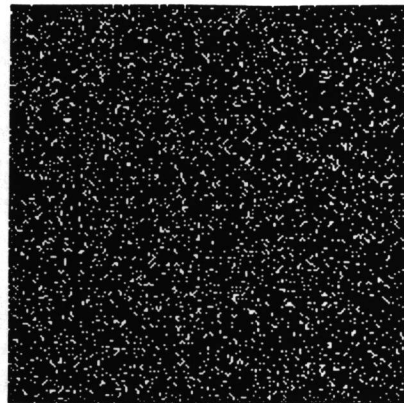
Figure 2b



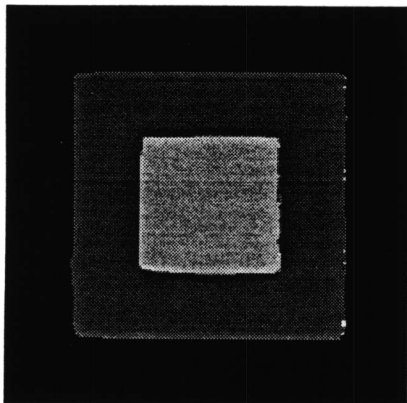
Figure 2c



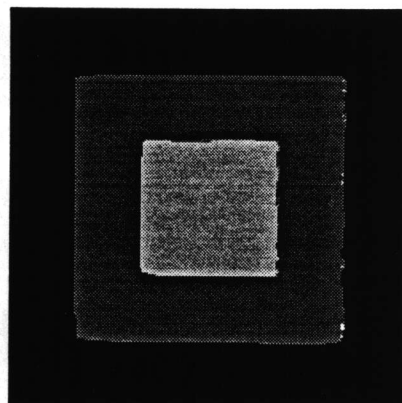
**Figure 3a**



**Figure 3b**



**Figure 3c**



**Figure 3d**