Linear Disparity-Based Photometric Stereo

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Abstract. Disparity-Based Photometric Stereo (DBPS) is a computer vision process based on the correspondence between pairs of photometric stereo images, from which a photometric disparity map can be obtained which encodes surface curvature information. Here we formulate a linear approximation to DBPS through the linearization of the reflectance map function, which is the function relating image intensities to surface orientation. We show that linear DBPS allows the recovery of relative depth from the photometric disparity map, and illustrate our approach in simple experiments with synthetic imagery.

Introduction

The problem of the estimation of shape is a central one in computer vision, and several techniques have been devised to deal with it. In the processes known as Shape-from-Shading (SFS) [2] and Photometric Stereo (PS) [7], the shading information (smooth spatial variation of the recorded pixel intensities) in monocular images of opaque non-textured objects can be used for the inference of surface orientation.

Shape-from-Shading and Photometric Stereo are formulated in terms of a reflectance map function, which encodes the image-formation process and relates the brightness in each image point to the local orientation at the corresponding surface patch in the scene. Such relation is usually highly nonlinear, but linear approximations to the reflectance map have been proposed which yield good shape estimates in SFS [3].

On the other hand, a new approach to shape reconstruction has been introduced by Torreão et al. [5,6], based on the matching of pairs of photometric stereo images (monocular images obtained under different illumination conditions), which can yield shape attributes such as relative depth [5] and surface curvature information [6]. This new kind of reflectance-map based process has been called Disparity-Based Photometric Stereo (DBPS).

Here we formulate a linear approximation to the DBPS process, also based on the linearization of the reflectance map function, as in [3,4]. In this approximation, the disparity field obtained in DBPS can be related to the relative depths of the observed surface points, in a much more general and direct way than has been done in [5], where the depth reconstruction necessitated the previous estimation of the orientation map of the surface.

In what follows, we review the concept of the reflectance map, introducing its linear approximation, and then employ it in the linear formulation of the Disparity-Based Photometric Stereo. Next, we illustrate the application of linear DBPS to simple imagery, in some preliminary experiments. Finally, we present our concluding remarks, commenting on possible extensions to the present work.

Linear Reflectance Functions

Let us consider a surface whose shape is given by $z = z(x, y)$, and whose gradient is denoted by (p, q), where

$$p = \frac{\partial z}{\partial x}, \quad q = \frac{\partial z}{\partial y}. \tag{1}$$

Assuming that such surface is being distantly illuminated by a collimated light source in the direction $\hat{s}$, and that it is being imaged by a distant camera, such that an orthogonal projection in the direction $\hat{v}$ can be considered, we have that the brightness at each point (x, y) in the image plane can be expressed as

$$I(x, y) = R(p, q, \hat{s}, \hat{v}), \tag{2}$$

where $R(p, q, \hat{s}, \hat{v})$ is the reflectance map function [2]. Assuming, henceforth, that the direction of observation is fixed at $\hat{v} = (0, 0, 1)$, we will omit the dependence of R on $\hat{v}$.

Given an image region with mean surface orientation (p_0, q_0), a linear approximation to R can be obtained through a Taylor series expansion around (p_0, q_0). Equation (2) thus becomes

$$I(x,y) \approx R(p_0,q_0,\hat{s}) + (p-p_0)\left(\frac{\partial R(p,q,\hat{s})}{\partial p}\right)_{(p_0,q_0)} + $$

$$+(q-q_0)\left(\frac{\partial R(p,q,\hat{s})}{\partial q}\right)_{(p_0,q_0)}. \quad (3)$$

For a smooth surface, this kind of expansion will give an accurate approximation to the observed intensities around a given point in the image, provided that the neighborhood is chosen small enough so that it contains only a restricted range of (p,q) values.

As an example of the above, we have that, for a Lambertian surface, i.e., for a reflectance map of the form

$$R(p,q,\hat{s}) = \frac{1 + pp_s + qq_s}{[(1+p^2+q^2)(1+p_s^2+q_s^2)]^{1/2}}, \quad (4)$$

the linear expansion around $(p_0, q_0) = (0,0)$ gives

$$I(x,y) \approx \cos\sigma + p\sin\sigma\cos\tau + q\sin\sigma\sin\tau, \quad (5)$$

where $\hat{s} = (1, -p_s, -q_s)/(1+p_s^2+q_s^2)^{1/2}$ (the illumination vector) can be written in terms of the slant (σ) and the tilt (τ), as $\hat{s} = (\cos\sigma, \sin\sigma\cos\tau, \sin\sigma\sin\tau)$. It has been observed that equation (5) provides a perceptually indistinguishable approximation to the Lambertian reflectance map over the range $-2 < p, q < 2$, and that the accuracy of this approximation increases with the slant σ (the angle between the illuminant $\hat{s}$ and the z-axis) [4]. This is a quite general result: in practice, any reflectance map can be accurately modeled by the linear approximation when a sufficiently oblique illumination is employed, and the range of (p,q) values over which the approximation is good increases with the slant of the illumination vector [3].

Linear Disparity-Based Photometric Stereo

Disparity-Based Photometric Stereo (DBPS) [5] is a computer vision process which incorporates aspects of Photometric Stereo and Stereoscopy [1]. It is based on the fact that, for a smooth surface, a change in the illumination direction entails a displacement of intensities in the image lattice, from which a photometric disparity field can be obtained. Such disparity field encodes information about the surface curvature [6], and can also be employed for the estimation of relative depth, once the orientation map of the surface has been recovered [5].

DBPS can be thus formulated: Given a pair of photometric stereo images, I_1 and I_2, a photometric disparity map can be produced by making the correspondence between the two images along a given direction. The correspondence in DBPS is made in such a way that the absolute difference

$$|I_1(s) - I_2(s + D_d(s))| \quad (6)$$

is minimized at each point $s = (x,y)$, where $D_d(s)$ is the disparity along the direction d. Assuming that a horizontal match is performed, the resulting disparity map can be related to the image intensities, as follows:

The matched intensities in the second image, $I_2(x + D_X(s), y)$, can be expressed as

$$I_2(x + D_X(s), y) \approx I_2(x,y) + D_X(s)\left(\frac{\partial I_2}{\partial x}\right)_s, \quad (7)$$

where terms of higher order in $D_X(s)$ have been neglected. Now, recalling that the DBPS match minimizes the absolute difference in (6), we can rewrite (7) as

$$I_1(s) \approx I_2(s) + D_X(s)\left(\frac{\partial I_2}{\partial x}\right)_s,$$

and thus

$$\Delta I(s) = I_1(s) - I_2(s) \approx \left(\frac{\partial}{\partial x}[D_X(s) I_2]\right)_s -$$

$$-I_2(s)\left(\frac{\partial D_X(s)}{\partial x}\right)_s. \quad (8)$$

Now, let us make two further approximations in the above equation: First, we assume that the linear expansion of the reflectance map is applicable, such that the left-hand side of (8) can be rewritten as

$$\Delta I(s) = I_1(s) - I_2(s) \approx k_0 + k_1 p + k_2 q, \quad (9)$$

with

$$k_0 = k_0^{(1)} - k_0^{(2)},\ k_1 = k_1^{(1)} - k_1^{(2)},$$

$$k_2 = k_2^{(1)} - k_2^{(2)}, \quad (10)$$

where the superscripts refer to the two input images.

On the other hand, we further assume that the second term on the right-hand side of equation (8) can be neglected, when compared to first one, since the photometric disparity map is generally much smoother than the intensity field. This is usually true, not only because the disparities are related to the second derivatives of $z(x,y)$ (curvatures), while the intensities depend on the first derivatives (surface gradients), and we are assuming slowly curving surfaces, but also because an explicit smoothness restriction is imposed in our photometric matching algorithm [5,6], through which the disparities are obtained.

Taking the above approximations into account, equation (8) becomes

$$k_0 + k_1 p + k_2 q = \frac{\partial}{\partial x}[D_X I_2], \tag{11}$$

where, for simplicity, we have omitted the dependences on s.

Now, we can take advantage of the form of the coefficients on the left-hand side of the above equation (see (10)) to choose the illumination directions for the two input images in such a way that we obtain $k_0 = k_2 = 0$, and so (11) becomes

$$k_1 p = \frac{\partial}{\partial x}[D_X I_2]. \tag{12}$$

Thus, recalling that $p = \frac{\partial z}{\partial x}$, we finally get

$$z(x,y) = \frac{D_X I_2}{k_1} + F(y). \tag{13}$$

As an example of the above, let us consider a Lambertian reflectance map, and illumination directions given by $\{\sigma_1, \tau_1\}$ and $\{\sigma_2, \tau_2\}$, with $\sigma_1 = \sigma_2 = \sigma$, $\tau_1 = 0$ and $\tau_2 = \pi$. From the expression for the linear approximation to the Lambertian reflectance map (equation (5)), we find that, in this case, $k_0 = k_2 = 0$ and $k_1 = 2\sin\sigma$, and thus, from (13),

$$z(x,y) = \frac{D_X I_2}{2\sin\sigma} + F(y). \tag{14}$$

Equation (14) has a very simple geometric interpretation, since the first term on its right-hand side is exactly the expression for the relative depth estimates that we would obtain from a horizontal convergent stereoscopic system with angle 2σ between the optical axes of the two cameras, if the stereoscopic disparities were given by $D_X I_2$ [5].

We thus see that our linear approach to DBPS is consistent with our early claim (see [5]) that relative depth estimates could be obtained from the photometric disparities if an equivalent stereoscopic system could be found. In [5] we have shown that such a stereoscopic equivalent can be determined (at least in some restricted situations) if an orientation map has been reconstructed from the input images, for instance via the traditional Photometric Stereo process. In linear DBPS, no such orientation map is required, but we still have to grapple with that extra term, $F(y)$, on the right-hand side of (13), if a depth map for the observed surface is to be recovered. Before we do just that, let us remember that disparities along other directions can be considered in DBPS. In particular, the whole analysis from (7) to (11) could be repeated for a disparity $D_Y(s)$, instead of $D_X(s)$, and we would thus obtain

$$k'_0 + k'_1 p + k'_2 q = \frac{\partial}{\partial y}[D_Y I'_2]. \tag{15}$$

Again, choosing the appropriate illumination directions, such that $k'_0 = k'_1 = 0$, we can get

$$z(x,y) = \frac{D_Y I'_2}{k'_2} + G(x), \tag{16}$$

which, in the case of a Lambertian reflectance map, would become

$$z(x,y) = \frac{D_Y I'_2}{2\sin\sigma} + G(x), \tag{17}$$

for illuminations with $\sigma'_1 = \sigma'_2 = \sigma$, $\tau'_1 = \pi/2$ and $\tau'_2 = -\pi/2$.

Now, from equations (13) and (16), we find

$$F(y) - G(x) = H(x,y), \tag{18}$$

where

$$H(x,y) = \frac{D_Y I'_2}{k'_2} - \frac{D_X I_2}{k_1}, \tag{19}$$

and, from the form of (18), we can make some inferences about $F(y)$ and $G(x)$ in some simple cases, as below:

i) $H(x,y) = C$ *(constant)* :

In this case, we get $F(y) = C_F$ and $G(x) = C_G$, with $C_F - C_G = C$, and thus

$$z(x,y) = \frac{D_X I_2}{k_1} + C_F = \frac{D_Y I'_2}{k'_2} + C_G. \tag{20}$$

ii) $H(x,y) = H_x(x)$:

Now, even though $G(x)$ is no longer a constant, we still have $F(y) = C_F$, and thus $z(x,y)$ can be obtained as

$$z(x,y) = \frac{D_X I_2}{k_1} + C_F. \tag{21}$$

iii) $H(x,y) = H_y(y)$:

Similarly, in this case, although $F(y)$ is no longer a constant, we still have $G(x) = C_G$, and thus $z(x,y)$ can be obtained as

$$z(x,y) = \frac{D_Y I'_2}{k'_2} + C_G. \tag{22}$$

Other simple forms for $H(x,y)$ could be similarly treated, but we limit ourselves here to the above three, which we illustrate, in the following section, by experiments of depth estimation through linear DBPS for synthetic image pairs.

Experiments

We show examples of the application of linear DBPS to synthetic images of Lambertian ellipsoids.

In our experiments, for obtaining the horizontal disparity map employed in (13), we have considered two pairs of photometric stereo images. Each of the pairs includes a centrally illuminated image ($\sigma = 0$), with the other images corresponding to illuminations $\{\sigma = 0.35, \tau = 0\}$, and $\{\sigma = 0.35, \tau = \pi\}$. The disparity D_X employed in (13) is given by the average of the disparity maps obtained from the two image pairs. Similarly, the vertical disparity D_Y used in (16) is obtained as the average of the disparity maps resulting from two photometric image pairs, sharing a common centrally illuminated image: one pair including an image corresponding to illumination $\{\sigma = 0.35, \tau = \pi/2\}$, while the other includes an image obtained for $\{\sigma = 0.35, \tau = -\pi/2\}$. This approach to the construction of the disparity maps has proved useful for reducing the loss of information due to dark areas in the input images (see [5]).

Figures 1a, 2a and 3a show examples of the input images employed. The average photometric disparity maps D_X and D_Y are shown in the letters (b) of Figures 1 to 3, for the respective experiments, while letters (c) present the obtained $z(x, y)$ estimates (The stochastic algorithm already discussed in [5,6] has been employed for the DBPS matchings). It is easy to see that the first experiment illustrates the case $H(x, y) = C$, the second one illustrates $H(x, y) = H_x(x)$, and the third illustrates $H(x, y) = H_y(y)$, corresponding to the three situations considered in the previous section. Our estimates for $z(x, y)$ have therefore been obtained, in each case, through the appropriate formula among the ones given in equations (20) to (22).

Concluding Remarks

We have presented a linear approximation to the Disparity-Based Photometric Stereo process, whereby relative depth estimates can be obtained from photometric disparity maps. We have also applied linear DBPS to surface reconstruction from synthetic photometric stereo pairs, in simple experiments. Presently, we are experimenting with more realistic applications of our process, in order to prove its usefulness as a general tool for shape estimation in computational vision, permitting the inference of both depth and curvature information in a quite simple, straightforward framework.

References

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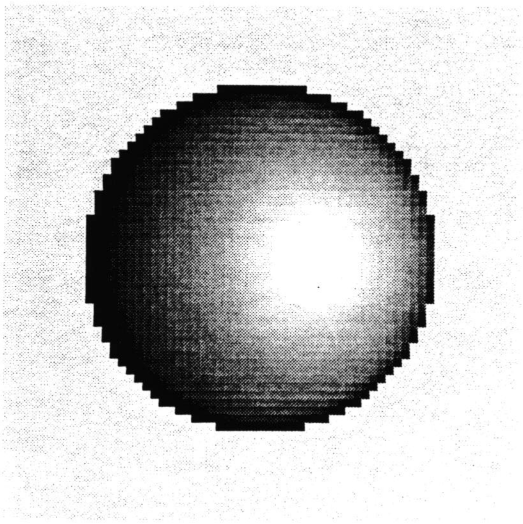

Figure 1a

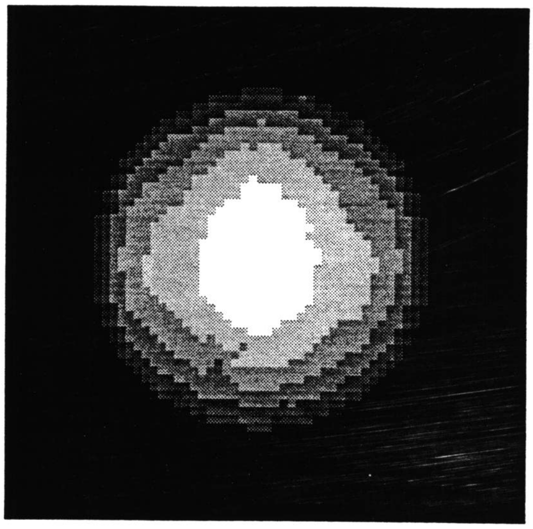

Figure 1b

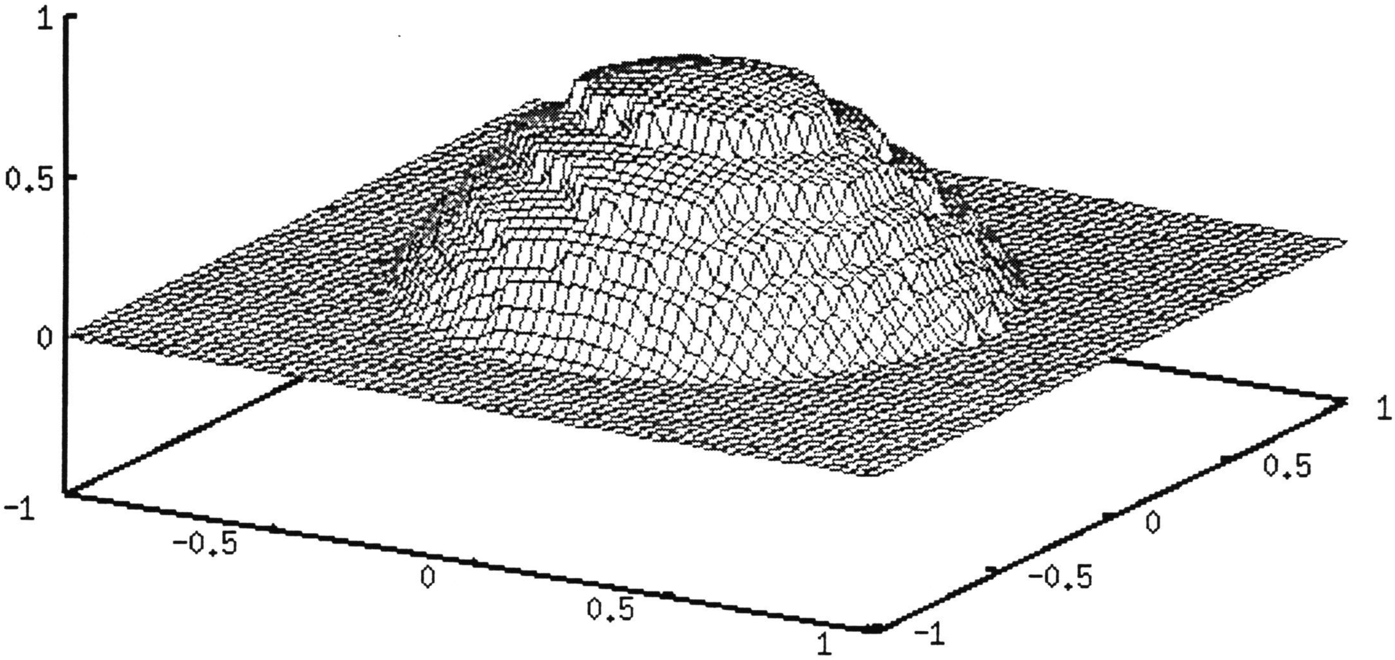

Figure 1c

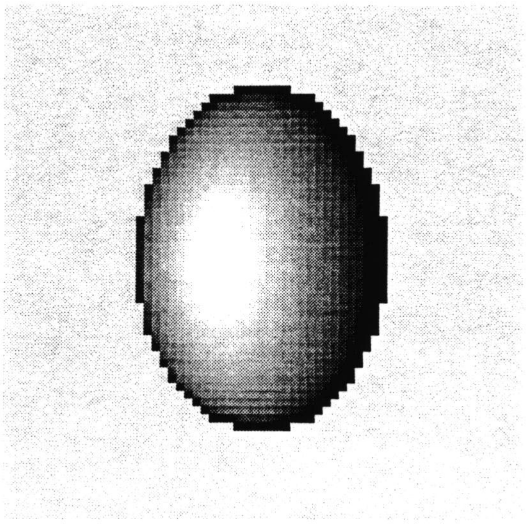

Figure 2a

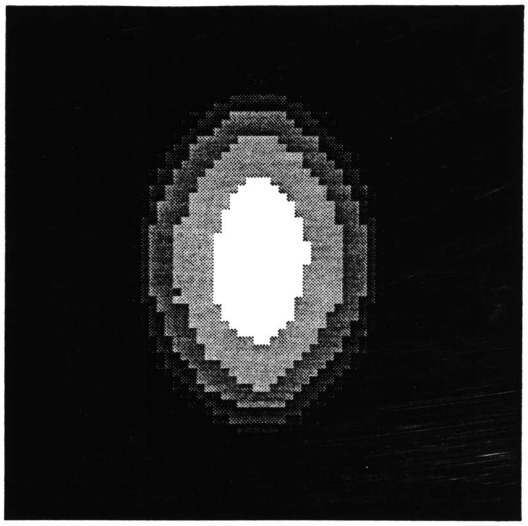

Figure 2b

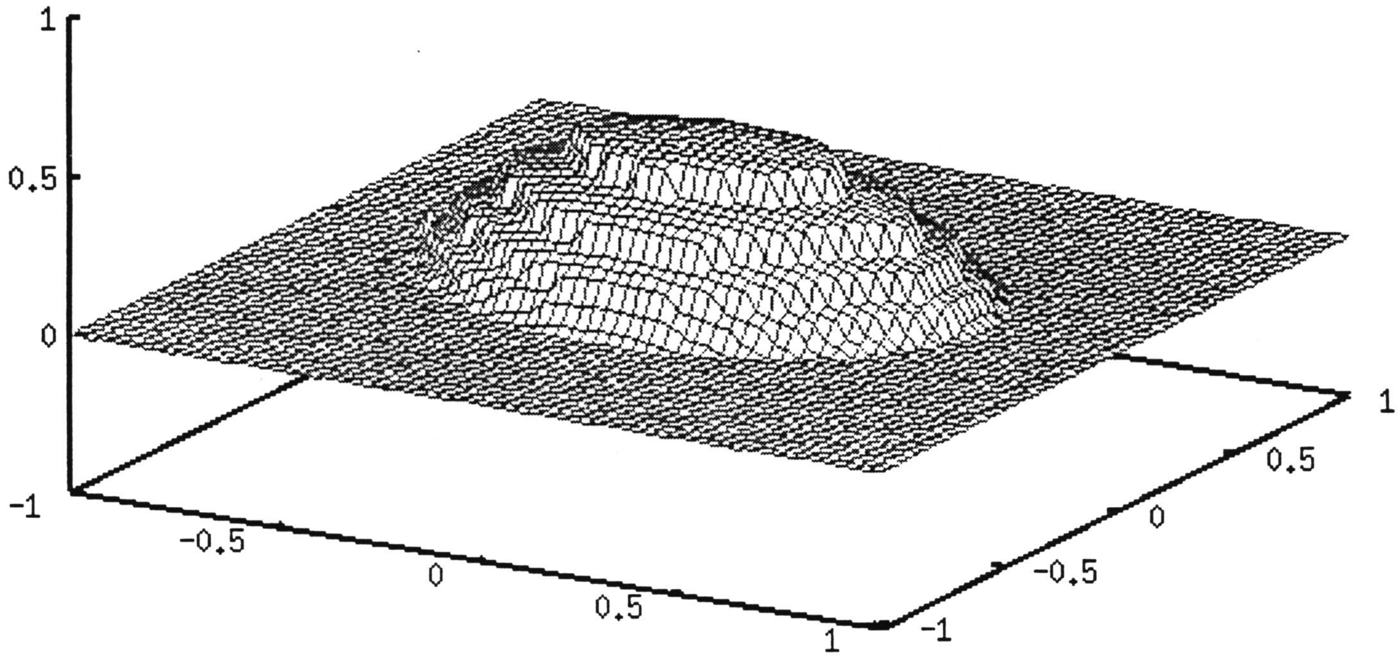

Figure 2c

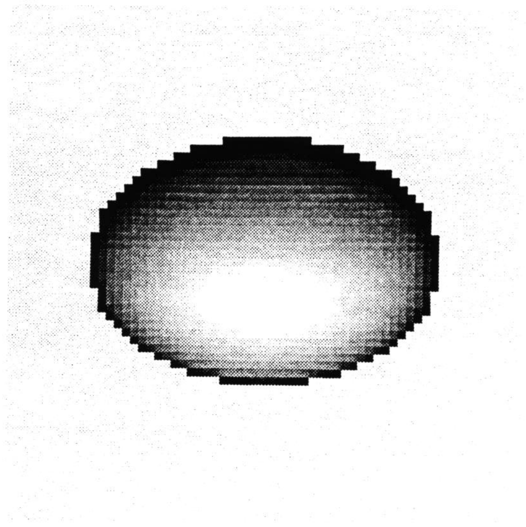

Figure 3a

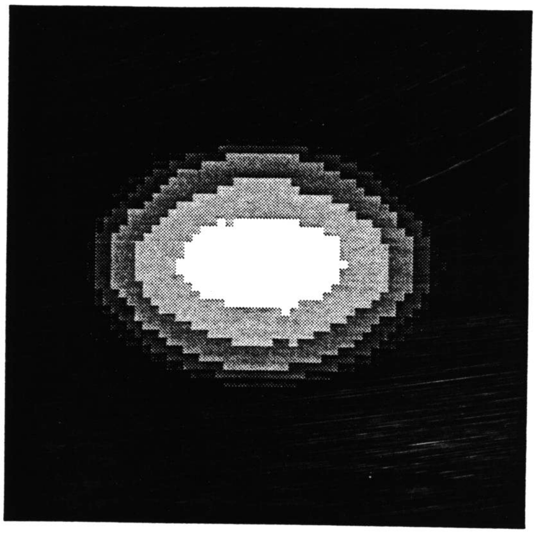

Figure 3b

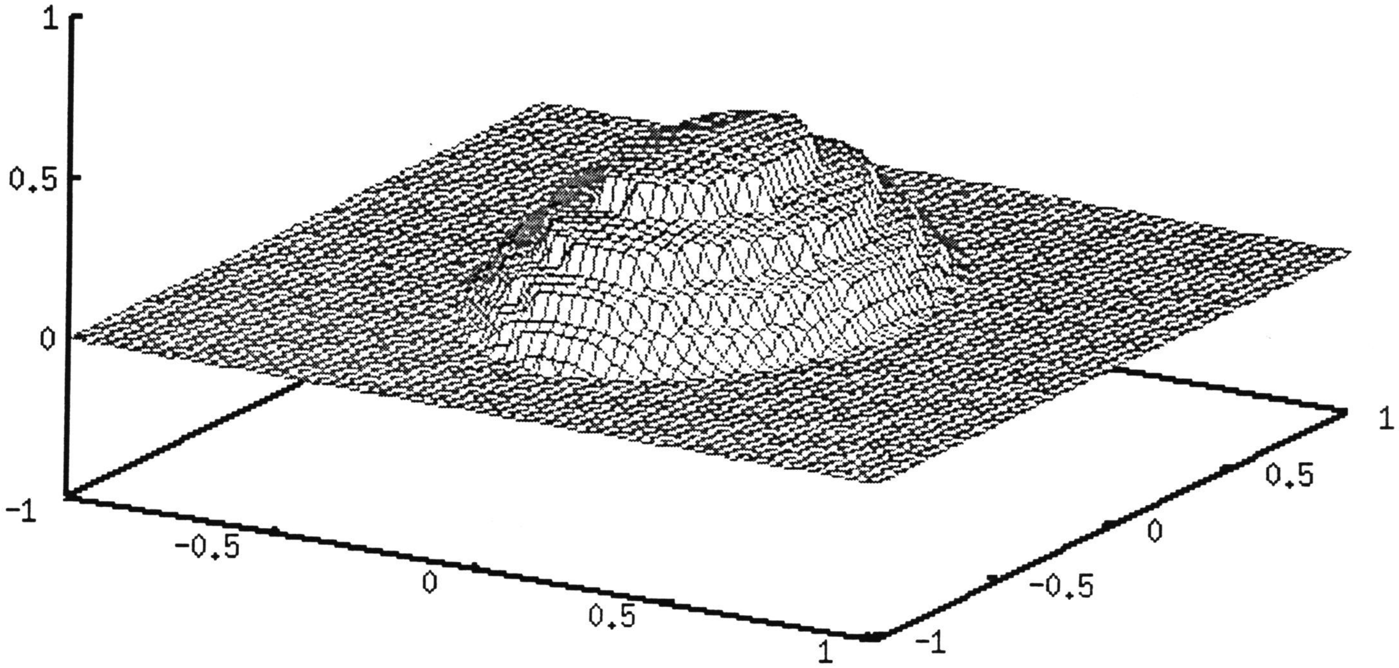

Figure 3c