Finding Patterns and Exploiting Pseudo-randomness using Complex Systems

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***Abstract*—In this work[1], we present patterns and pseudo-randomness in an approach that relates both concepts, which traditionally are seen as opposites. This approach uses the mathematical basis of complex systems for two purposes: to exploit the spectrum of pseudo-randomness of chaotic systems in a quest to achieve true randomness and, the development of pattern recognition methods based on artificial life in complex networks that finally intertwined the search for patterns in pseudo-random sequences. In the first part, we developed a method to explore the depth properties of chaotic systems, specifically in the logistic map and tent map, as sources of pseudo-randomness. We observe that the patterns disappear and the pseudo-randomness is increased by removing k-digits to the right of the decimal separator of the chaotic orbits. Thus, a rapid transition from "weak to strong" randomness was evidenced as k tends to infinity, which allows a parametrically pseudo-randomness. In the second part, it was proposed the combination of cellular automata in the network topology (also called network-automata), to characterize networks in a pattern recognition context. Four problems were explored: identifying online social networks; identify organisms from different domains of life through their metabolic networks; the problem of authorship identification; and classifying stomatal distribution patterns varying according to different environmental conditions. Finally, this same approach was used to analyze the sequences of pseudo-random numbers generated by the gold standard k-logistic map PRNG in a context of pattern recognition. The proposed approach allowed to explore patterns and pseudo-randomness extracted from a myriad of systems with successful results in terms of accuracy and good pseudo-randomness. This work has brought significant advances in real-world pattern recognition tasks across a wide range of fields such as cryptography, cryptoanalysis, biology, and data science.**

I. Exploiting Pseudo-randomness of Chaotic Maps

Pseudo-random number generators (PRNGs) are the backbone of various fields of application, starting from statistics and applied math, numerical calculus, decision theory, systems modeling, simulation, programming languages, to cryptography. Classically, the development of PRNGs is based on deterministic algorithms using linear recurrences, algebraic concepts, and bitwise operations, among other artifacts, usually without mathematical foundations, such as the linear congruence generator (LCG) or the Mersenne Twister [1]. In contrast to classical algorithms, chaos-based PRNGs (CB-PRNG) rely on chaotic systems to produces values with random-like properties. In fact, much progress has been reported, for instances, with CB-PRNGs based on differential equations and recurrence maps [2]–[5], and chaotic cellular automata [6], [7].

[1]This work relates to a Ph.D. thesis.

Sensitivity to the initial conditions is one of the most important properties of the chaos theory since the smallest variation at the initial conditions can completely disturb the system over time. Furthermore, other properties from the chaos theory include its random-like behavior, and its unpredictability, this is, the difficulty to predict over a long time what the systems' behavior might be in the future. These features are of great applicability in different branches including cryptography and PRNGs, where chaotic systems with a high Lyapunov exponent are used as pseudo-randomness sources. Consequently, the close relation between chaos and pseudo-randomness has aroused great interest inside the academic community, commercially and militarily exploited, as established via the state-of-the-art over the last 32 years.

Recently, some researchers have expressed certain doubts regarding the chaotic properties of well-known chaotic maps, such as the logistic map (given by the simple recurrence equation $x_{t+1} = \mu x_t(1 - x_t)$, with time t, control parameter $\mu \in [0, 4]$ and $x_t \in [0, 1]$). In the context of modern cryptography, this chaotic map presents non-uniform probability distribution (pattern U), where a plateau distribution is expected; dependence of the control parameter which might also lead to periodic windows; large enough ciphertext samples that may estimate the parameter [8]; short cycle period orbits in which lengthy periodicity is expected depending on machine limitations [9]; degradation of digital chaotic systems problems [10].

Notwithstanding, we have found that the authentic advantageous of the chaotic systems are based on the infinitesimal depth of the precision digits of their orbit points. As an example, considering the well-known Mandelbrot set, when displayed on a computer screen, it exhibits curious but constrained patterns. But, while this pattern is again and again prolonged, a huge number of complex patterns may be distinguished and, surely, is in those magnifications in which legitimate chaos occurs. Therefore, higher computational precision is required to exploit the deep-zoom of a chaotic system and hence to research the pseudo-random properties of such chaotic systems.

In this thesis, we proposed a generalized approach to generate a new orbit from an underlying orbit of a one-dimensional discrete dynamical system (such as the logistic map, tent map, etc) where each point is composed by removing k-digits to the right of the decimal separator given by Equation (1). In such manner that the resulting orbit $\mathcal{O}^k(\mu, x_0)$ maintains both parameters: $\mu \in [0, 4]$ and its phase space inside the unit interval $[0, 1]$ of the corresponding chaotic map, e.g. k-logistic map.

$$x_t^k = \lfloor x_t 10^k \rfloor - \lfloor x_t 10^k \rfloor, \tag{1}$$

where x_t^k, e $\lfloor \ \rfloor$ represents the floor function.

Henceforth, we summarized the main outcomes concerning the first topic of this thesis: to exploit the pseudo-randomness properties of a chaotic map. All subsequent experiments were focused on the most chaotic regions of the k-logistic map provided by the parameter $\mu = 4$ which relates to the largest Lyapunov exponent. In Fig. 1a-b, we can observe the Poincaré plot, which relates the arrangements x_t^k, x_{t+1}^k and x_{t+2}^k, respectively. The original orbit k_0 shows the typically inverted parabola of the logistic map, whose similar pattern is retained in the 3D version. At that point, in a top-down vision, we introduce the phase diagrams from k_1 to k_4, we can observe how the parabola curve changes in a zig-zag sort, which is consistently vanished until it turns out to be outwardly arbitrary, as it can be seen from k_2 onwards. In fact, it is seen that the phase space is being filled as k increases, that is, that the k-logistic map generated almost all attainable values within $[0, 1]$, as it is expected of a good PRNG and furthermore turning it into a non-invertible map, i.e. past and future values are getting uncorrelated. In Fig. 1c is shown the 2D Fourier power spectrum. From top to bottom, this spectral assessment is demonstrated with the parameter k, for k_0, k_1, k_2, k_3 and k_4. For the original logistic map (k_0), the parallel lines already suggest the presence of patterns, however, as the parameter k is increased, the spectral density shows a regular constant at the center, which is a patternless indicator.

One of the main goals to construct a good PRNG is to obtain a uniform distribution as much as possible [11], [12]. The logistic map distribution shows a U pattern, which is clearly observed on the first chart of Fig. 1d, which represents frequency curve of the original logistic map k_0 using the parameter $\mu = 4$. This U pattern is outstanding in dynamic system theory since the logistic map follows an invariant probability density function. Nonetheless, from top to down, it can be seen that this distribution turns out to be more uniform as k increases, i.e. the pseudo-randomness properties of the k-logistic map changes, in spite of redundancy, more random as $k \gg 1$. Moreover, this uniformization procedure is illustrated in Fig. 1e, which compares the former curves [13].

Besides the former plots, we also explored the pseudo-randomness DIEHARD tests results, which are reported in Table I. Each column corresponds to the number of files that passed the sub-tests. The failed tests (at least 50 files) were highlighted in gray, which can be observed in the case of k_0, k_1, k_2 and k_3-logistic map. We observed that $k = 0$ fails to the DIEHARD tests, however, the panorama changes as the parameter k increases, since the k-logistic map passes on all of the tests when $k \geq 4$.

TABLE I

AVERAGE NUMBER OF FILES THAT PASSED DIEHARD TESTS USING THE k-LOGISTIC MAP PRNG FROM 100 FILE SAMPLES. MAJOR FAILED TESTS ARE SHOWN IN GRAY. ALL TESTS PASSED USING THE INTERVAL $0.0001 <$ P-VALUE < 0.9999. SOURCE: [13].

Diehard tests	k_0	k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9
BirthdaySpacings [KS]	100	100	100	100	100	100	100	100	100	100
OverlappingPermutations	99	97	98	95	98	96	98	98	99	100
Ranks31x31 matrices	100	100	100	100	100	100	100	100	100	100
Ranks32x32 matrices	100	100	100	100	100	100	100	100	100	100
Ranks6x8 matrices [KS]	0	0	25	99	100	100	100	100	100	100
Monkey20bitsWords [KS]	0	99	100	100	100	100	100	100	100	100
OPSO [KS]	98	99	100	100	100	100	100	100	100	100
OQSO [KS]	98	100	100	100	100	100	100	100	100	100
DNA [KS]	100	100	100	100	100	100	100	100	100	100
Count1sStream	0	0	0	98	100	100	100	100	100	100
Count1sSpecific [KS]	0	0	0	0	94	100	100	100	100	100
ParkingLot [KS]	100	100	100	100	100	100	100	100	100	100
MinimumDistance [KS]	96	100	100	100	100	100	100	100	99	100
RandomSpheres [KS]	100	100	100	100	100	100	100	100	100	100
Squeeze [KS]	100	100	100	100	100	100	100	100	100	100
OverlappingSums [KS]	100	100	100	100	100	100	100	100	100	100
Runs (up)	100	100	100	100	100	100	100	100	100	100
Runs (down)	100	100	100	100	100	100	100	100	100	100
Craps (wins)	100	100	100	100	100	100	100	100	100	100
Craps (throws/game)	100	100	100	100	100	100	100	100	100	100

In these experiments, we have found that the pseudo-randomness properties of the logistic map can be notably enhanced when k is increased. In fact, patterns turn out to be progressively widespread until they turn out visually indistinguishable ($k \geq 4$). Regarding the pseudo-randomness tests, we also have verified that the sequences produced by the map $k \geq 4$-logistic map passed effectively both randomness tests of DIEHARD and NIST (data found in Ref. [13]). Therefore, we formulated the following conjecture: as the parameter k increases, the pseudo-randomness is improved, going from regular (k_0) to the most random (k_∞). Clearly, in computational terms, k_∞ would be impossible to demonstrate, yet we did not mean to explore this side of the conjecture, however, to exploit the fact that k increases the pseudo-randomness, which thereby allows us to create a helpful tool: a gold standard PRNG. With this gold standard, it is conceivable to generate datasets with different parameters k thus providing distinct classes of pseudo-randomness, which would permit the study and development of methods aimed for pattern recognition purposes, which is the second part of this thesis.

II. FINDING PATTERNS IN COMPLEX NETWORKS

Networks have been used effectively in many fields. The increasing demand involving networks is because it incorporates an alternate point of view from the conventional data analysis. Typical examples of data modeled as networks include metabolic networks, protein-protein interaction networks and social networks, etc. In the most recent decades, unstructured models, such as time series, plots of recurrence, contours [14], textures [15], have managed to model their connections as networks. The organization of all these systems can be represented by graphs, i.e vertices associated by edges.

The joint of pattern recognition and networks emerge as an essential approach to deal with the high demand for methods

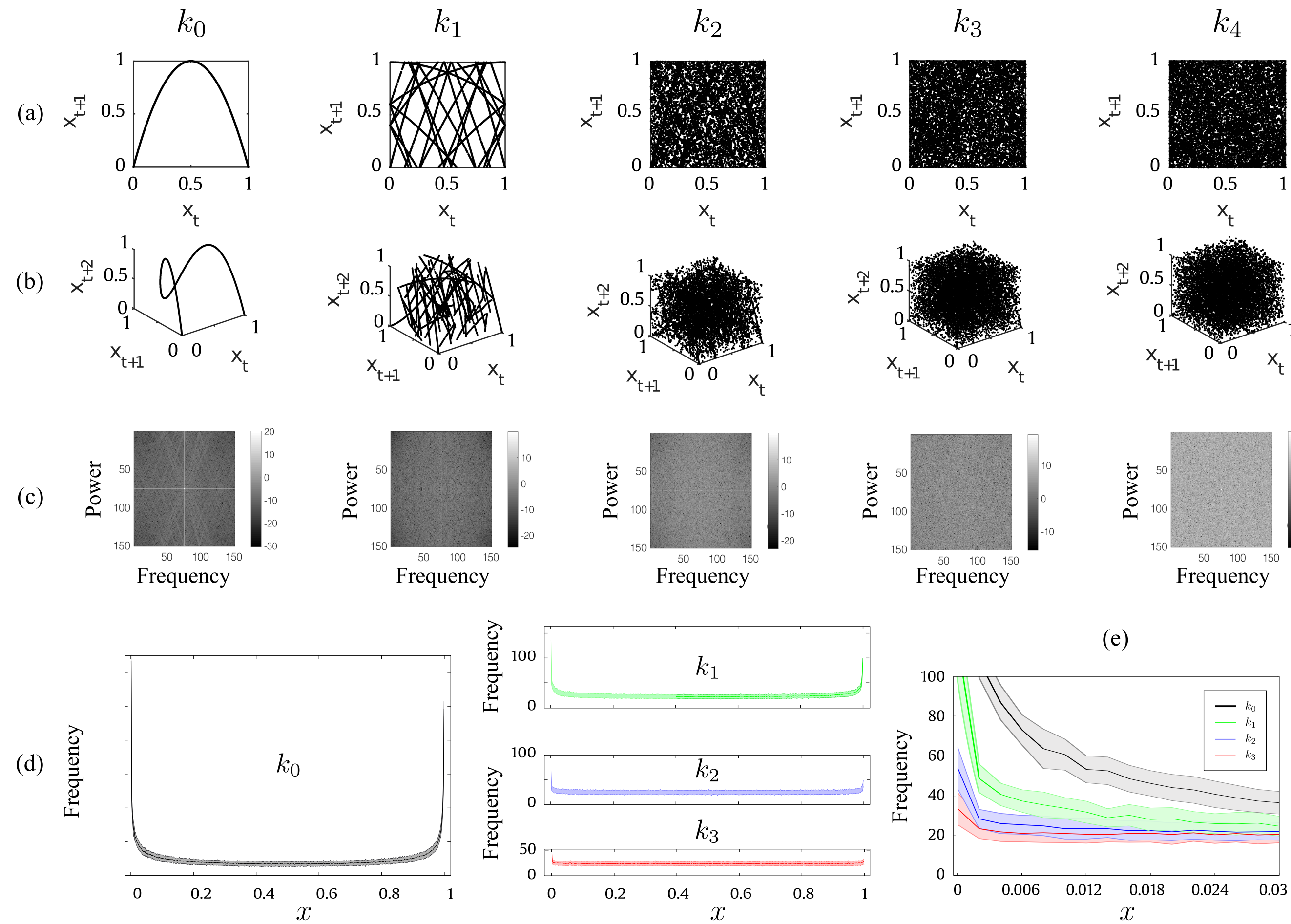

Fig. 1. Various visual results for the k-logistic map for k_0, k_1, k_2, k_3 and k_4 (top to down) utilizing $\mu = 4$. (a-b) Two-and three-dimensional plots are shown on the left and right column, respectively. The horizonal and vertical axes demonstrate the phase space of x_t^k against x_{t+1}^k. Each orbit contains 10^4 points started from arbitrary initial conditions, where the initial 200 iterations were dismissed of (transient time). (c) 2D Fourier power spectrum for 150^3 numbers produced by the PRNG k-logistic map. (d) Frequency distribution curves. Horizontal axis shows the $x \in [0, 1]$ (500 bins) and vertical axis shows the frequency of the 10^4 values removing of firsts 10^3 transient values. The curves represent the mean and standard deviation (shaded error bar) for arrangements generated over 100 random initial conditions. (e) The inset plot portrays a zoom on the windows $x \in [0, 0.03]$. Adapted from [13].

that handle in a big data scenario. Since information extracted from networks can prompt a comprehension of network patterns that are intrinsically related to the network model. In this way, pattern recognition in networks aims to characterize networks by extracting information from the correlation among vertices and their association with the network topology.

Three phases for pattern recognition in networks are well established: the modeling of data as networks, the extraction of characteristics and the classification and/or analysis of patterns. Thus, the choice of adequate network descriptors is crucial for applications in pattern recognition in general, and consequently for pattern recognition in networks. The feature extraction is typically based on well-established network structural measures, such as connectivity attributes such as the average degree and distributions and correlations of degrees, distances and paths, and so on, which are able to identify global properties shared by a large majority of empirical and synthetic networks such as random, small-world, scale-free networks, and geographic networks models [16]–[18]. However, there are still several limitations that must be considered. For example, the degree distribution is used to discriminate among the various theoretical models of networks, but when it comes to real-world networks, this same measure is no longer adequate, because there are networks with different topologies that may have a similar degree distribution, thus failing in their correct classification [17], [19].

Thus, keeping in mind the main goal to address these challenge issues, we proposed a method based on the embedding of cellular automata (CA) over the topology of a network aiming to characterize networks from the spatio-temporal dynamics of these networks. We use a family of CAs motivated by the rules of Life-Like, in this way we present the Life-Like Network Automata (LLNA) [20], as illustrated in Fig. 2. First, given a networks dataset aiming for an automatic classification is split into a training and test set. Each network is thought to be initiated with ones and zeros (alive and dead) conditions (Fig. 2a). Second, considering a set of best rules that maximizes the problem are chosen. One rule is used to evolve a network, from which is acquired a spatio-temporal pattern (Fig. 2). Third, considering the later formation, various features can be extracted, for example, the Shannon entropy, the word length and the Lempel-Ziv complexity (Fig 2). Fourth, once a feature vector is obtained for each of the networks, then various classifiers methods can be used, in this thesis we used the Support Vector Machine (SVM) and k-fold validation [21], in order to guarantee a fair examination of the performance of the method.

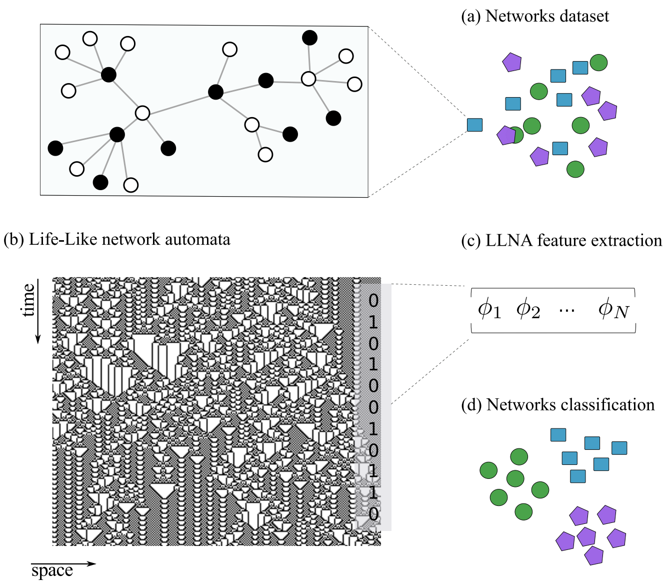

Fig. 2. Scheme based on the LLNA method. The following steps are applied: (a) given a network dataset; (b) a network is setup with alive/dead initial conditions, following the Life-like network automata method; (c) a selected LLNA rule evolves over the textual network topology; (d) LLNA spatio-temporal patterns are extracted and then used for the networks' classification task.

All the experiments regarding the LLNA classification performance were analyzed first on a synthetic network dataset, i.e. well-known theoretical models used here as a benchmark, as follows:

- Synthetic dataset: produced according to the network models: random, small-world, scale-free, and geographical with number of nodes $N = \{500, 1000, 1500, 2000\}$ and mean degree $\langle k \rangle = \{4, 6, 8, ..., 16\}$. This dataset encloses 11200 networks (2800 per model) (Fig. 3a);
- Synthetic scale-free: dataset composed only of scale-free networks generated using the modeld proposed by Barabasi & Albert and Dorogovtsev & Mendes. The dataset consists of 100 networks per each of the five classes with $N = 1000$ nodes and $\langle k \rangle = 8$ (Fig. 3b).

This same figure shows an examination with the outcomes obtained from the structural network measurements, which correspond to the concatenation of the average degree, average hierarchical degree, average clustering coefficient, average path length and degree Pearson correlation.

Furthermore, the LLNA performance was analyzed on five real-world network datasets, which are summarized as:

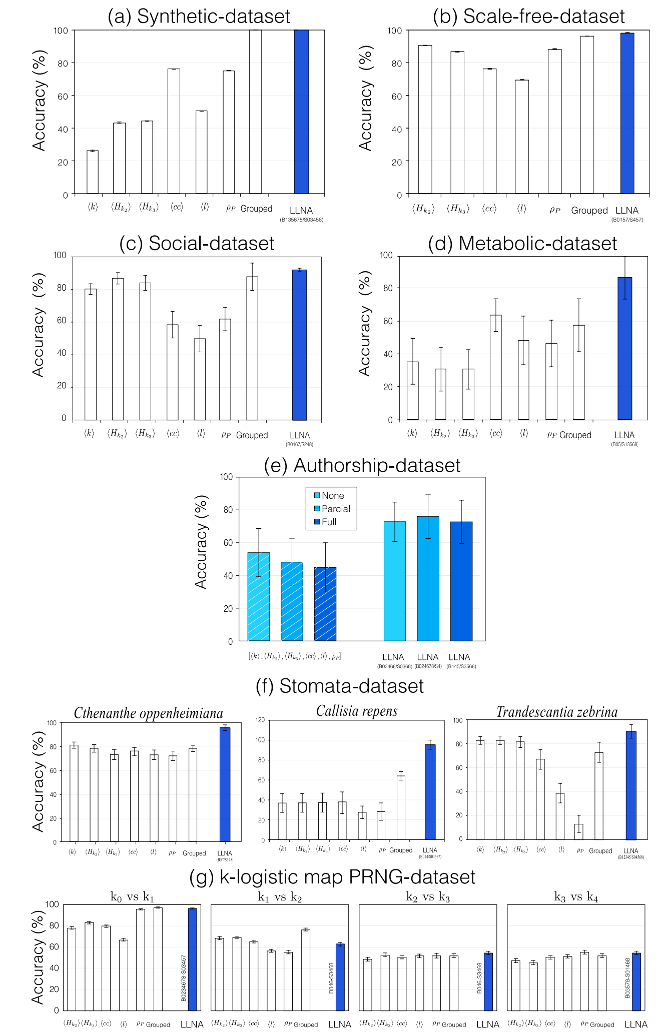

Fig. 3. Comparison summary reporting the classification performance between the proposed method and the structural measures of networks for different networks dataset. Bar graphs exhibit the mean accuracy (%) and standard deviation (error bar) using the best combinations of LLNA feature vectors and structural measures, (isolated and combined). The performance of the LLNA is contrasted with the structural measurements of networks: average degree ($\langle k \rangle$), average hierarchical degree of level 1 ($\langle H_{k_1} \rangle$) and level 2 ($\langle H_{k_2} \rangle$), average clustering coefficient ($\langle cc \rangle$), average path length (l) and degree Pearson correlation (ρ_P). Adapted from [20].

- Identification the structural patterns in *online* social networks, in which Twitter and Google+ users networks are examined (Fig. 3c);
- Identification of organisms from specific domains of life (*archaea*, *bacteria* and *eukaryotes*) through their metabolic networks (Fig. 3d);
- Authorship identification of literary manuscripts, which consists of three networks datasets with respect to the lemmatization treatment of the texts used to produce the networks (partial, complete and none (Fig. 3e);
- Regarding plant phenotypic plasticity analysis, the classification of distribution patterns in microscopic photos of stomata, by fluctuating various lighting conditions, using a distance-based stomata network (Fig. 3f);

- Finally, a method to find patterns, if possible, within the pseudo-random number sequences produced by the gold-standard k-logitisc map. For this situation the task is to separate PRNG classes k_0 vs k_1, k_1 vs k_2, $k_2$vs k_3 and k_3 vs k_4 containing sequences of numbers transformed as networks by using a distance-based threshold (Fig. 3g).

From these outcomes, we should notice that the LLNA accomplished the best performance regarding the identification problem of living organisms. Besides that, similar results were obtained for the analysis of phenotypic plasticity of plants through stomatal networks, in all species that were submitted to the conditions of environmental stress. Moreover, regarding the authorship identification using literary networks, which is considered a challenging task in the area of Natural Language Processing, our results also outperforms compared to the classical networks measurements.

III. Conclusions

In this manuscript, we presented, in a brief manner, how patterns and pseudo-randomness can be interrelated for two purposes: to exploit the pseudo-randomness properties of chaotic maps, and to explore a combination of cellular automata and networks aiming for pattern recognition purposes.

On the first part of this thesis, we developed two chaos-based PRNGs based on the k-logistic map and the k-tent map. In that regard, we observed that the pseudo-random properties of a chaotic map can be improved as k increases. In fact, by means of all the visualization tools (bifurcation diagram, Poincar diagram, frequency histogram), Lyapunov exponent analysis, randomness tests, spectral analysis and pseudo-randomness DIEHARD and NIST test suites (Fig. 1 and Table I); suggest that the quality performance of the proposed PRNG with $k \geq 4$-logistic map overpass the pseudo-randomness properties of classical PRNG such as LCG and Mersenne Twistter. Therefore, one main contribution of this thesis is a parameterized gold-standard PRNG, which is the first of its kind into the literature. The gold standard represents an important research tool that can aid the development of three different areas: pattern recognition, cryptography, and cryptanalysis. Since it can generate virtually infinite sets of random numbers with known theoretical basis.

In the second part of this thesis, we exhibited the great multidisciplinary of the LLNA as a tool for pattern recognition in networks. The LLNA surpassed the accuracy rates when compared to the topological descriptors of networks in almost all of the problems presented here, as shown in Fig. 3.

Finally, the relevance of the LLNA method was additionally extended to sequences generated by the proposed PRNG. In this way, the main motivation is the analysis of sequences of pseudo-random numbers aiming to discover patterns and/or to find approaches to recognize among PRNGs classes. The deliberation of such data sequences, first represented as time series and then modeled as mind complex networks, opens up the possibility of using pattern recognition strategies in networks in order to "measure pseudo-randomness" in such systems. Hence, throughout all of these points, the proposed approaches have conveyed huge advances to an extensive variety of fields including cryptography, cryptoanalysis, science, and information science.

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IV. Scientific production

This thesis has yielded the following list of manuscripts.

A. Publications

- MACHICAO, J.; BRUNO, O. M.. Patterns and pseudo-randomness using complex systems. Anais Estendidos do Simpósio Brasileiro em Segurança da Informação e de Sistemas Computacionais (SBSeg), p. 49 - 56. 2018. [22]
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- MACHICAO, J.; BAETENS, J. M.; MARCO, A. G.; DE BAETS, B.; BRUNO, O. M. A Dynamical Systems Approach to the Discrimination of Cryptographic Modes of Operation. In: 8th International Congress on Industrial and Applied Mathematics, 2015, Proceedings... Beijing, China 2015, p. 61.

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