Wavelet and Entropy Analysis Combination to Evaluate Diffusion and Correlation Behaviors

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Abstract. Diffusion and correlation effects are two principal phenomena which have been studied for years and several visualization techniques have been proposed to help scientists to understand them. The analysis of these phenomena will help to extract important information from data sets. To understand these problems we combine wavelet and entropy analysis to evaluate the evolution of these behaviors through scale and time. We present image case studies to show several different kinds of behaviors of these effects. Some of them are fallible cases and not reliable, as the images do not show the desired information. We calculate entropy of smooth and detail coefficient sets, generated by wavelet transform of these sample images in each scale, to obtain measures that allow us to evaluate these behaviors according to the organization complexity. These measures can provide an indication about the quality of the rendered images.

1 Introduction

One of the drawbacks of visualization is the failure of it to warrant that the rendered images show the behavior of the data set. This problem is more severe in volume visualization of dynamic systems, mainly in medical applications where misdiagnosis can be caused from these errors. The density, color and flow mapping that reflect the match between desired information and visual effects is not easy. Thus, information analysis is becoming important to solve these problems. In this work, we apply wavelet transform in sample images to obtain several cases of diffusion and correlation interactions. This operation allows us to exploit multiscale analysis to study the evolution of these behaviors through scales. We also employ entropy analysis to evaluate complexity structures to facilitate rendering process.

The idea of combining several interdisciplinary concepts used by wavelet and entropy analysis was initially introduced by Chiou et al. [1]. Its benefit was to obtain local and global analysis facilities to study the features in different scale view points. They emphasized the Brooks and Wiley [2] hypothesis "Biological evolution is an entropy process" as essential to the study of evolution laws. One can notice the diffusion and correlation interaction produced by wavelet transform as one kind of evolution through scales. In this way, we can evaluate entropy production through scales to analyze informational complexity of the data.

In this paper, we introduce four measures – topological entropy, metric entropy, topological complexity and statistical complexity – defined by Crutchfield and Young [3] in Computational Mechanics where they utilize information theoretic ideas in dynamics. These measures are used to fill up the lacuna of the incomplete Shannon entropy measure to capture coherent structures (topological information) required in the hierarchical coherence information concept to classify complexity of the self-organization systems. We utilize the chaining process of the Multiscale Singularity Analysis (MSA) to obtain coherent structures which are essential for these measures.

We also evaluate diffusion and correlation behavior through scales and entropy evolution in time. In addition, we present results of these measures for this analysis.

2 EIAS - Entropy Information Analysis System

The Entropy Information Analysis System (EIAS) is composed of two subsystems: Multiscale singularity Analysis (MSA) and Hierarchical Evolutionary Information Analysis (HEIA). Each one undertakes specific tasks.

MSA MSA was described in detail in Mallat's work [4, 5]. Its importance is that it uses wavelet transform to decompose original signals into smooth and detail sig-

Figure 2: Multiscale singularity analysis and hierarchical information analysis integration.

and detail coefficients because wavelet transform decomposes a signal in both coefficients.

Feature Information Analysis System In nature, entropy or disorder increases spontaneously; hence, the information organization diffuses along time. Similarly, disorder increases through scales in wavelet multiscale representation. Thus, we can correlate time with scale to measure the information evolution through coarser scales. FIS is proposed to evaluate this purpose. We can now define the topological complexity for each scale s of the smooth space $S^t_{2^s}$, where t is a discrete time, as:

$$\begin{aligned} C^t_0(S^t_{2^s}) &= \log_2 \|Q_{S^t_{2^s}}\| && (1)\\ C^t_1(S^t_{2^s}) &= -\sum_{q_i \in Q_{S^t_{2^s}}} p^t_S(q_i) \log_2 p^t_S(q_i) && (2) \end{aligned}$$

where $p^t_S i$ is pdf (probability distribution function) of smooth coefficient sets.

Singularity Information Analysis System We know the wavelet coefficient space has singularity information. To analyze this space, we use statistical complexity measure to evaluate singularity information of data set. Let $W^t_{2^s}$ be the wavelet coefficient space. The same measures for this space are given as:

$$\begin{aligned} C^t_0(W^t_{2^s}) &= \log_2 \|Q_{W^t_{2^s}}\| && (3)\\ H^t_1(W^t_{2^s}) &= -\sum_{q_i \in Q_{W^t_{2^s}}} p^t_W(i) \log_2 p^t_W(i) && (4) \end{aligned}$$

where $p^t_W(i)$ is pdf of detail coefficient sets. Note that statistical complexity is a entropy measure.

Coherence Information Analysis System In this section we combine chaining process (CP) and hierarchical information theory (HIT). We show the hierarchical information capacity by taking advantage of the chaining process. The main idea is to join similar properties in triple parameters: space, time and scale to form structure elements (chains). In the process, we make use of spatial coherence (space similarities), spatio-temporal coherence (time-space similarities), and the local maximum propagation (scale-space similarities). We obtain chains with different sizes for all scales. In accordance with scale-space similarities, small chains do not propagate through scales. This contrasts with FIS, where order information is propagated through coarser scales as disorder information disappears with scale increases. If we combine multiscale analysis with temporal coherence, we will obtain multiscale evolution analysis. We believe it is important to analyze self-organization phenomena since we prefer to understand behavior modification through scales in time.

In the scale-space, multiscale singularities information can be grouped to form structure elements as textures or surfaces that can be linked through scales. Then, we can associate levels of the hierarchical information theory with i-sized of structures elements. As result, our expression of the topological entropy and metric entropy of chains is given respectively by:

$$H_0^{2^s,t} = \lim_{L\to\infty} \log_2 N^{2^s,t}(L), \tag{5}$$

where $N^{2^s,t}(L)$ is the number of distinct chains of length L.

$$H_1^{2^s,t} = \lim_{L\to\infty} - \sum_{\omega\in\Omega_{|\omega|=L}} p^{2^s,t}(\omega)\log_2 p^{2^s,t}(\omega), \tag{6}$$

where $p^{2^s,t}(\omega)$ is the probability of occurrence of the chain and the sum is taken over all distinct chains of lenght L. Ω is set of all chains. In the equation above, s is the scale, t is the discrete time.

3 EIAS Evaluation

3.1 Evaluation of Information Evolution through Scales

Evaluating Entropy of Smooth and Detail Coefficients We use statistical complexity for evaluation of smooth and detail coefficients. The images in Figure 6 illustrate smooth coefficients and magnitude of detail coefficients' sample images. It shows some idea of the diffusion and correlation behaviors of wavelet transform. The first line is the original image. The following three images are smooth coefficients in scales 1, 3 and 5. The last three are the magnitude of detail coefficients in the same scales.

Using Shannon's entropy definition, the maximum entropy (maximum disorder) of a 256 $\times$ 256 size image with one byte per pixel is 5.545. Zero is minimum entropy (maximum order). If we observe Noise image in the Figure 6, we see the image is randomly generated between 0 to 255. The entropy of the smooth coefficient in this image is near the maximum in scale 0 and decreases rapidly through scales because of the correlation behavior in the neighborhood. See Figure 3 for this kind of organization phenomenon. We also see this approximated decline in the detail coefficient entropy. On the other hand, the entropy is minimum in image with constant values. The resulting entropies of both smooth and detail coefficients are zero. We do not show this image because it is easy to understand.

Different from Noise image, the entropy of Rectangle (Figure 6) greatly increases in the scales. The diffusion process occurs extremely, spreading contour features around the neighborhood. This reflects the disorder behavior around border regions. We see it more clearly in Rectangle images of Figure 6. Figure 3 shows the entropy which also increases notably with scales.

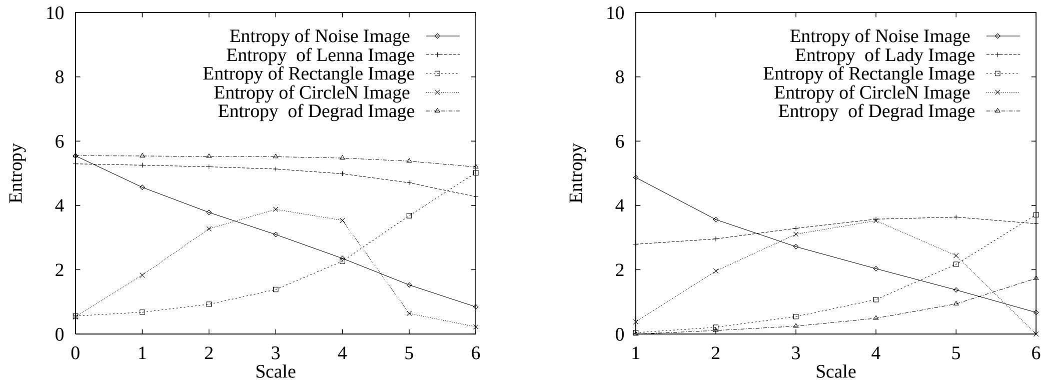
Evaluating the entropy of Lenna image in Figure 3, we note the entropies decrease in the smooth coefficient sets and rise in the magnitude of detail coefficient sets. The entropy production through scales causes increased diversity and complexity. The reason for these opposing measures is that correlation occurs with more intensity than diffusion for smooth coefficient sets in higher scales. The same does not occur in detail coefficient sets where there are few features to correlate.

We can see a more clear diffusion vs. correlation in the CircleN image case, when the entropy increases from scale 0 to 3 where there is diffusion predominant in relation with correlation. In the higher scales, correlation is predominant. This behavior occurs for both smooth and detail coefficients.

Evaluating Entropy of Chains We use metric entropy for chain evaluation. In contrast with detail coefficients, the entropy of chains is not affected by diffusion behavior around the contours. We want to evaluate the entropy of chains in accordance with their sizes – small, medium and large. We attribute weights in accordance with these sizes. After this, we calculate order and disorder capacity.

The values in Figure 4 show the entropy of these information capacities for the sample images. Comparing the values of order capacity and disorder capacity between Noise and Lenna images, we note disorder capacity is higher than order capacity in lower scales. The reverse occurs in higher scales.

The order capacity predominates in relation with disorder capacity for all scales of the Rectangle image. In the CircleN image sample, the value of disorder capac-



(a) (b)

Figure 3: Entropy of smooth (a) and detail (b) coefficients of sample images.

ity is predominant in scale 2. After this, the variation of order and disorder capacity are similar because there are small differences between diffusion and correlation.

3.2 Information Evolution Evaluation in Time-Scale

To evaluate the scale evolution we utilize the quasar animation recently scanned by the Hubble satellite telescope. We apply wavelet transform and calculate entropy for each frame. Then, we show smooth and detail (magnitude) coefficient images and graphics of the entropy vs. time in Figure 7.

One can note that light emitted by a star explosion inside of the quasar is not clearly apparent in lower scales (column 4) but it becomes evident at higher scales (column 5) because smooth features are not well detected by application of a high-pass filter in the convolution operation. Furthermore, the successive transforms can capture the global features in this case, in contrast with Degrad image where wavelet coefficients are almost zero. These are cases where compression does not work well. One observes that entropy analysis can detect and quantify these smooth change features. This is illustrated in Figure 5 where fast increase of entropy can detect the occurance of the explosion. Hence, this analysis can help to control the quality in direction to rendering of assured images.

4 Conclusion

Evolution behavior can be governed by a diffusion equation. One can perceive that wavelet transform causes diffusion and correlation effects that depend on geometric distribution of features of the objects. Entropy analysis is applicable for evaluating these outcomes to determine the complexity structure - of order and disorder - produced by evolution of features through scales.

We present several cases of images to show different behaviors of diffusion via scale and time. In general, wavelet transform has not work well in smooth variation examples where entropy analysis is required to detect and evaluate this kind of comportment.

We hope this information and complexity analysis can be applied for quality control in precise rendering where lost information is not acceptable. This analysis can also be adopted for visualization of behavior of a complex system in which inference information can not be visualized by traditional techniques. Under this circumstance, a new illumination model will need to be reformulated.

Acknowledgments

Rui C. H. Chiou was supported in part by a grant from FAPESP under process 92/4716-2. Cláudio T. Silva is partially supported by Sandia National Labs and the Dept. of Mathematics, Information, and Computer Science Office, and by the National Science Foundation (NSF), grant CDA-9626370. Arie E. Kaufman is supported in part by the NSF under grant CCR-9205047.

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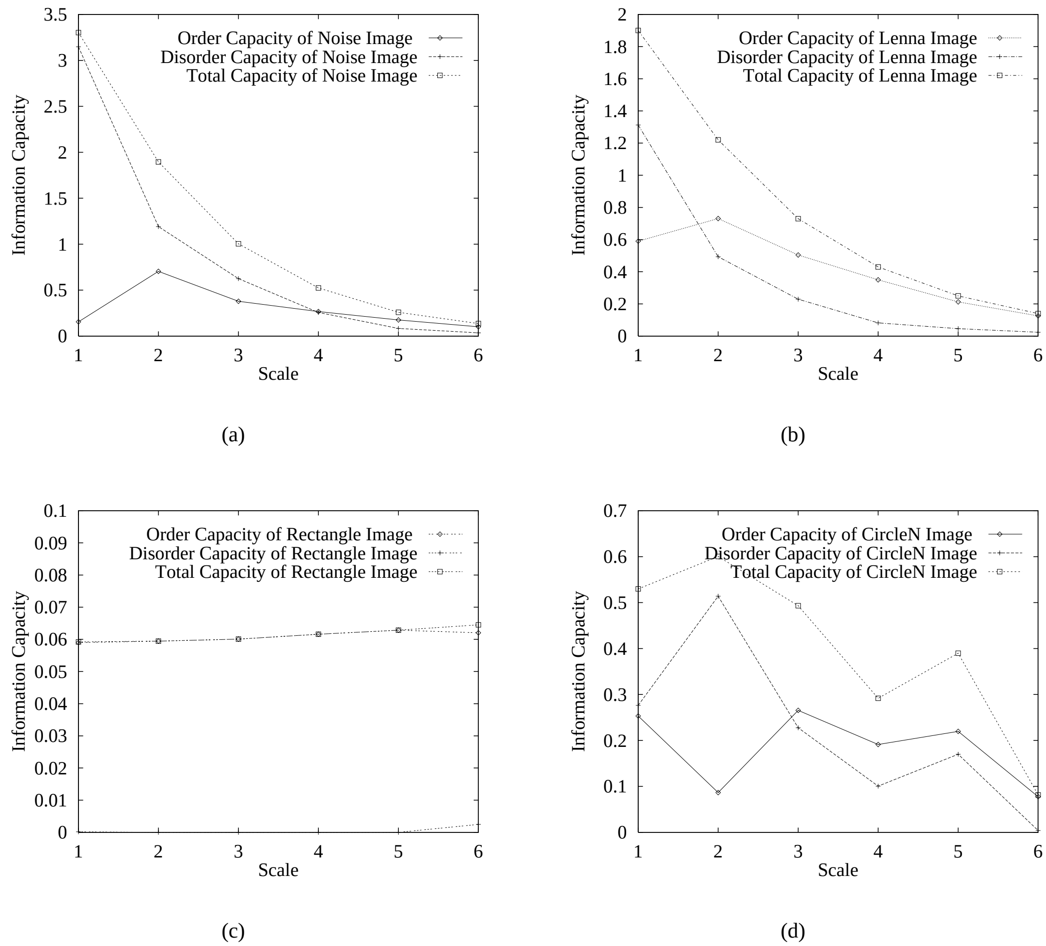

Figure 4: Information capacity of sample images

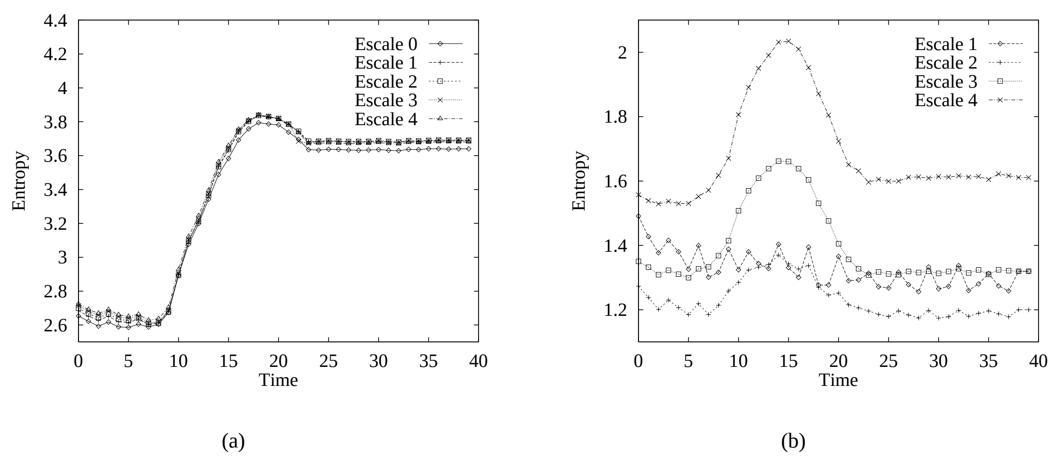

Figure 5: Entropy measure in time of Quasar animation

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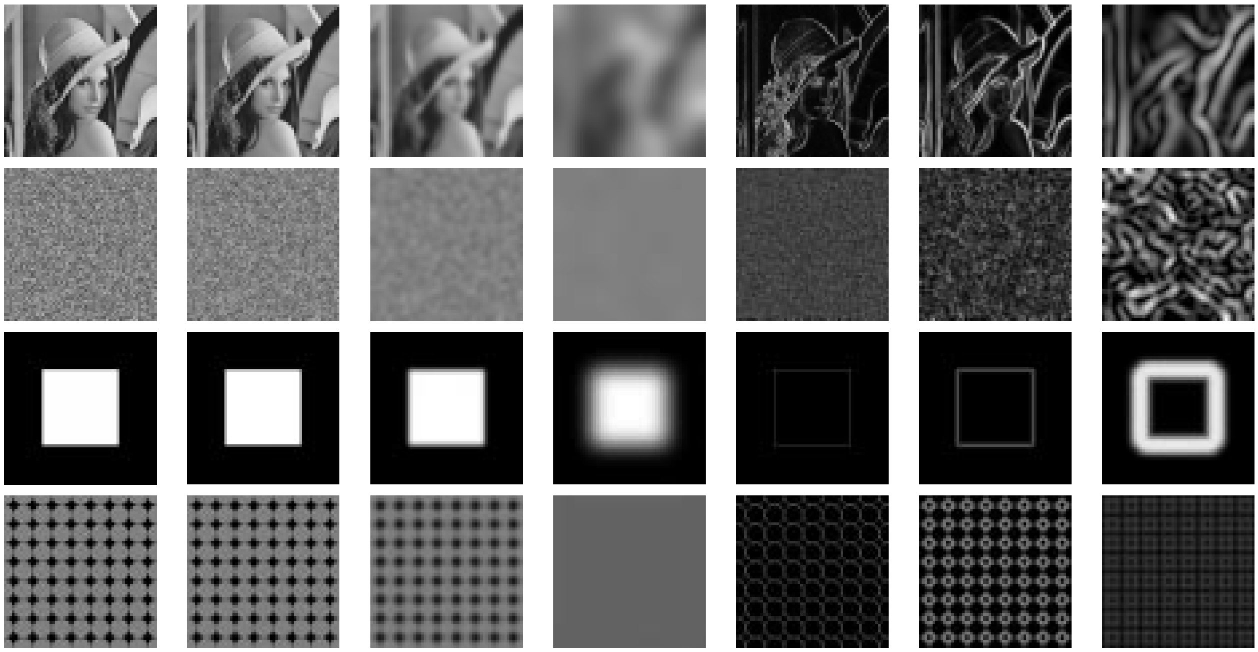

Figure 6: Original (column 1) of the image Lenna, Noise, Rectangle, CircleN (up-down) and their smooth in scales 1 (column 2), 3 (column 3) and 5 (column 5) and detail coefficients in scales 1(column 6), 3 (column 7)and 5(column 8), left to right.

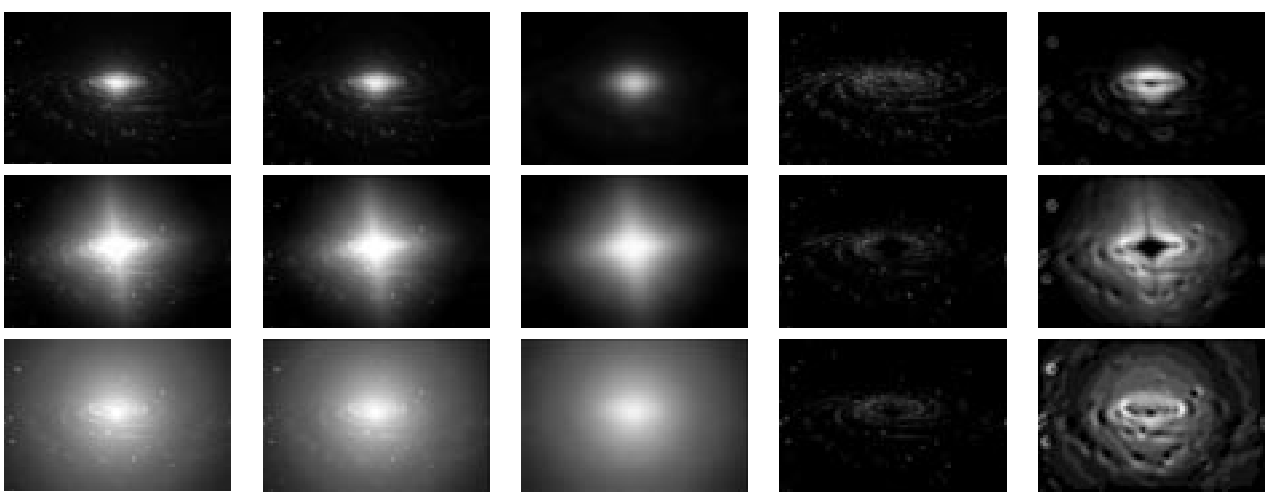

Figure 7: Original Quasar(column 1) image and its smooth in scales 1(column 2) and 4(column 3) and detail coefficients in scales 1(column 4) and 4(column 5),left-right. The frames, from up to down, are 0, 150 and 310, respectively. The original color images were scanned by the Hubble satellite telescope from NASA.