Level Lines Continuation based Digital Inpainting

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Abstract

Inpainting digital models have been since the late 1990's a powerful image reconstruction tool for missing data. After the original work of Bertalmio, Sapiro, Caselles and Ballester [1] several different approaches have been used to tackle the problem. Some are based on Partial Differential Equations to model a transport process and a diffusion process, others are based on the Euler elastica functional. This paper presents a model using the level lines continuation to perform the filling-in of the inpainting domain D. The model is proposed in a way as to satisfy the "$Connectivity\ Principle$"*. The image* $u(x,y)$ *is represented by a family of level lines and the missing part of the image is filled-in by the propagation of the available surrounding information, from outside to inside of the inpainting domain* D *along the level lines of the image. After defining the domain* D *the restoration process becomes automatic and the final result* $u(x,y,t_n)$ *is carried out by the evolutionary process starting with the initial degraded image* $u(x,y,0)$. *Examples on real and textured images show the performance of this proposed model.*

1. Introduction

The filling-in technique has been used since the Renascence period and its main goal is to reconstruct missing parts or damaged areas in an image in such way as to restore its harmony. In artwork restoration this is an ancient practice and consists of filling-in the missing or degraded areas in a way that they become non perceptive to an observer. A large number of algorithms have been proposed to tackle this problem of reconstruction of occluded or damaged areas in a digital image. The ability to perform automatic filling-in of missing information has numerous and important applications in image processing. The act of filling-in image information on a domain D is called Inpainting. This domain D can be a finite union of sub-domains, i.e., $D = \cup_{i=1}^{m} \mathrm{D}_i$ where each one is a connect region and could be, for example, a blank domain, hidden parts of objects, a superimposed text (like dates or publicity), a damaged part of an old photograph or even an entire object which one desires to remove from an image. Special effects on a scene can also be obtained by inpainting techniques. The act of inpainting in a domain D can also be understood as performing disocclusion in an occluded area D.

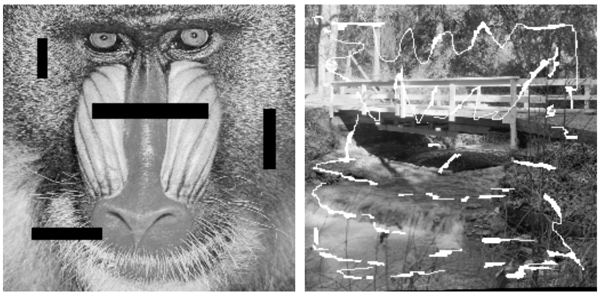


Figure 1. **Two damaged figures to be restored: The Mandrill and the bridge.**

The ability of disocclusion is a common process in human vision which has been widely studied and analyzed by psychophysicists, particularly by Kanizsa [9]. The object's boundaries continuation being smooth and straight as possible plays an important role in the process of reconstructing hidden objects.

Bertalmio, Sapiro, Caselles and Ballester, introduced the term digital inpainting into image processing in the pioneering digital inpainting work [1] based on two partial differential equations (PDE).

In this paper we present a new method for the automatic digital inpainting after a user-provided domain D specifies the region of the input and damaged image to be restored (or inpainted). The domain D will become progressively smaller until becoming an empty set. Our approach is in the deterministic methods context of the variational continuation framework to the level lines structure and can be viewed as a combination of the ideas presented in the Bertalmio et al. inpainting process [1] and Masnou variational approach using level lines to process the disocclusion [10] in a way that reaches the natural ability of humans who mostly seem to prefer the connected result, even when they are far apart ($Connectivity\ Principle$) [9].

Inspired by this work, several others techniques have been designed for inpainting small regions [5], [13].

This paper is organized as follows: Section 2 presents the geometric nature motivation for the proposed model, and the discussion of important geometric concepts used in their formulation. Connection to some related existing works in digital inpainting are made. Section 3 addresses the proposed Level Line Continuation (LLC) model. Section 4 presents some final remarks and some typical applications of the LLC model in disocclusion and scratch removal, as presented in figure 1. Finally, the conclusion is presented in the section 5.

2. Geometric inpainting models

The geometric inpainting algorithms have motivated and inspired this work mainly by the advantage of the geometric nature model that even thought the implicit contour itself can develop singularities (like cusp and corners) and can merge or split to change topology, the level lines remains well defined and thus, one can discretize the curve on a fixed cartesian grid.

2.1. The level lines

The level set method has been used to capture the interfaces and has been applied in a large number of areas, including problems that arise in geometry, fluid mechanics computer vision, etc. Numerous advances have been made since the original approach [14], including the variational level set method [10] for disocclusion. For details of level set techniques see [15].

Given an image $u : \Omega \subset R^2 \to R$, the upper level set L_λ is defined by the following subsets of R^2:

$$L_\lambda = \{x \,\epsilon\, R^2,\ u(x) \geq \lambda\}.$$

Let's suppose u a C^1 function on R^2 and λ a level such that $u^{-1}(\lambda)$ is compact and for all x belonging to $u^{-1}(\lambda)$, $Du(x) \neq 0$ then $u^{-1}(\lambda)$ is a finite union of C^1 Jordan curves [7]. The level sets of a function give a complete account of the function, then we can look at an image through its level sets.

The level lines are defined as the boundaries of the upper level sets, and the set of all level lines (or topographic map) gives a complete representation of an image u in terms of its Jordan curves [7].

The method we describe here aims at performing the disocclusion by the level lines connection, i.e., given an image u, and the inpainting domain D, the scheme aims at interpolating u inside of D by means of the continuation of its level lines.

2.2. Geodesic lines

One can understand geodesic on a surface as a straight line on the plane. In more formal terms, a geodesic on a surface S is a parameterized curve $\alpha : I \to S$ whose accelerating vector, $\alpha''(t)$, is orthogonal to S at each point. A geodesic path always maintains a constant speed once the $\alpha'(t)$ and $\alpha''(t)$ are orthogonal, and then

$$\frac{d}{dt}\|\alpha'(t)\|^2 = 2\alpha'(t) \cdot \alpha''(t) = 0.$$

These curves have various important properties which distinguish them from the others curves over the surface. These curves can be characterized as, for example, the shortest path between two points belonging to the surface and also the curves which are the straightest possible on the surface. The first characterization means that every small arc of a geodesic is the smaller curve path length, on the surface, connecting the initial and the final arc points. Analogous to the fact that there exists one and only one straight line connecting any two different points on the plane, it is true that there exists only one arc connecting any two surface points which are relatively near one to another. In this case the geodesic is called the minimal geodesic.

The second geodesic characterization means that at each point the geodesic is the curve which has the lowest curvature at that point, among all the curves of the surface passing through this point and having at this point the same tangent. If one point belonging to the curve and the tangent of this curve, at this point, are known, then the complete path is well defined. For more details on geodesics, see [8] and [4].

2.3. Energy of a curve

Let $c : [0, 1] \to\ S$ be a curve, where S is a surface. The energy of the curve is given by:

$$E(c) = \int_0^1 |c'(t)|^2 dt. \tag{1}$$

One easily notes that if $l(c)$ is the length of the curve c, then:

$$(l(c))^2 \leq E(c). \tag{2}$$

Equality will be reached if, and only if, the parameter t is proportional to the arc length s.

By considering the last inequality (2) follows that, if $g : [0, 1] \to S$ is the minimal geodesic, with $g(0) = p$, $g(1) = q$, then for each curve $c : [0, 1] \to S$, connecting p and q, we have $E(g) \leq E(c)$. Equality will be reached if, and only if c is the minimal geodesic.

2.4. Related works

Let the image u be a real value function on a spatial domain Ω, $u : \Omega \subset \mathbb{R}^p \to \mathbb{R}$, $p = 2$ or 3, called image support and I be the initial damaged image defined on $D^C \subset \Omega$. Our problem is to reconstruct u having the initial damaged image I.

In literature of this area there exist various techniques to carry out inpainting or disocclusion. The classic algorithms for disocclusion are based on the transportation of pixel information into the inpainting domain which belongs to the domain's subjacent region.

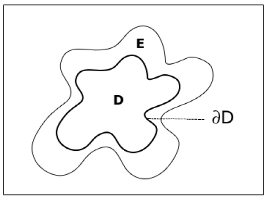


Figure 2. D **- inpainting domain,** E **- extended domain.**

The term *digital inpainting* was first introduced by Bertalmio et al. [1] and [2]. There, the authors, proposed a digital inpainting scheme taking into account the real inpainting used by the artists, to restore the images and using the PDE's as their principal tools. The authors applied the technique in text removal, restoring old photos, and creating special effects such as object disappearance from a scene. We will refer to this model as BSCB model.

The BSCB model defines D as the region of the image where the inpainting will be performed and ∂D the boundary of D. The structure of the data on E (surrounding ∂D) will be transported into the region D, see figure 2 .

Artists use different techniques to perform inpainting in theirs works, and they do this in a subjective way [3]. There is not the set method to solve the problem, but their methodology consider the following steps:

1. The global picture determines how to fill in the damaged domain (D), the purpose of inpainting being to restore the unity of the work;
2. The structure of the area surrounding D is continued into the damaged domain D, contour lines are drawn via the prolongation of those arriving at ∂D;
3. The different regions inside D, as defined by the contour lines, are filled with color, matching those of ∂D;
4. The fine details are painted, in other words,"texture" is added.

The manual concepts defined in (2) and (3) were translated to mathematical concepts, in the BSCB model, as the following transport equation:

$$u_t(\overrightarrow{x}) = \nabla(\Delta u(\overrightarrow{x})) \cdot \nabla^{\perp} u(\overrightarrow{x}), \quad \overrightarrow{x} \in E \cup D, \quad t > 0, \tag{3}$$

and the diffusion equation

$$u_t(\overrightarrow{x}) = |\nabla u(\overrightarrow{x})| div(\frac{\nabla u(\overrightarrow{x})}{|\nabla u(\overrightarrow{x})|}), \quad \overrightarrow{x} \in E \cup D, \quad t > 0. \tag{4}$$

In the numerical discretization the authors used both equations intercalated one with the other, some transport steps and some diffusion steps were used in the process.

One of the first work for the removal of occlusions was based on filtering and segmentation. The algorithm consists of connecting T-junctions at the same gray level with the elastic minimizing curves. This scheme was added to simple images, with few objects and few colors, but not being utilized for examples of natural images [12]. Inspired on the Euler elastica equation, Nitzberg, Mumford and Shiota [12] proposed, edge completion in the disocclusion process, as a solution of the following equation

$$\int_c (a + b\kappa^2) ds$$

where a are b are constants, c denote every possible path between two T-junctions and κ is the curvature of c. The T-junction are points where the edges making a "T" and c is a curve parameterized by the arc length s.

The first term of this functional has as its objective to force so that the level lines which are restored have minimal length and the second term has the objective of forcing so that the total angle variation be the minimum along these lines.

Moving to analysis of the curve $c(s)$ to the analysis of the image $u(x, y)$, Masnou and Morel [11] and [10] present an improvement to this technique which is a general variational formulation for disocclusion and a particular algorithm to implement the ideas in the theoretical formulation. They proposed minimizing the following functional:

$$J(u) = \int_D \left(a + b \left(\nabla . \frac{\nabla u}{|\nabla u|} \right)^p \right) |\nabla u| dx. \tag{5}$$

The numerical implementation of this functional considering $p = 1$ was made in the following steps:

1. the computation of the polygonal line correspondent to the boundary of the inpainting domain;

2. the computation of all T-junctions on the inpainting domain boundary;

3. the computation of the optimal set of level lines pairwise connecting the T-junctions;

4. the drawing of the geodesic paths;

5. the filling-in of the inpainting domain through the geodesic propagation [10] and [11].

As reported by the authors the inpainting domain should have simple topology with no holes. As the algorithm uses straight lines to connect two T-junctions with same gray value, the angle with which the level lines arrive at the boundary are not well preserved.

Chan, Kang and Shen give interesting results related to functional $J(u)$ considering $p = 2$. They derive the Euler-Lagrange equation for the functional:

$$\begin{aligned} J_2(u) &= \int_{D\cup E}\left(a + b\left(\nabla.\frac{\nabla u}{|\nabla u|}\right)^2\right)|\nabla u|dz \\ &- \frac{\lambda}{2}\int_E (u-I)^2 dz \end{aligned} \tag{6}$$

and give the following weighted gradient descent marching equation to minimize $J_2(u)$:

$$\frac{\partial u}{\partial t} = |\nabla u|\nabla.\vec{V} - |\nabla u|\lambda_e(u-u_0) \tag{7}$$

for all $z = (x, y) \in E \cup D$. The extended Lagrange multiplier λ_e is given by

$$\lambda_e = \begin{cases} \lambda & if \quad z \in E \\ 0 & if \quad z \in D. \end{cases}$$

The flux field $\vec{V}$ is defined by

$$\vec{V} = (\nu + \alpha k^2)\vec{n} - \frac{2\alpha}{|\nabla u|}\frac{\partial(k|\nabla u|)}{\partial \vec{t}}\vec{t} \tag{8}$$

with $\vec{n}$ as the normal field $\nabla u/|\nabla u|$ and $\vec{t}$ the tangent field. For more details see [6].

In contrast with Masnou and Morel work, curvy level lines can be well restored. However there exists two constants a and b in the functional $J(u)$ thus, good results obtained by the numerical implementation is a delicate issue once there are no results about the range of these two constants. The convergence and the stability are also two delicate issues once the equation is of the fourth order. As the equation uses the diffusion process it becomes difficult to recover sharpness, as pointed out by the authors.

Following the ideas introduced by Bertalmio et al. and Nitzberg et al., also trying to avoid the dynamical programming used in the work of Masnou and Morel, which has high computational cost, and also avoiding the fourth order differential equation proposed by Chan et al., we present an algorithm with low computational cost to restore digital images with damaged areas D_i. It is based on the continuation of the level lines arriving in each inpainting domain D_i.

3. The proposed model

The proposed model aims at recovering damaged or missing data areas of an image in such a way as to satisfy the $Connectivity\ Principle$ independent of the size or topology of the inpainting domain. Here, contrary to the technique of Masnou and Morel, [10] and [11], we can have holes in the inpainting domain.

To find the continuation of the level lines, from outside to inside of the inpainting domain, following the geodesic lines and minimizing the curve energy we propose to minimize the following functional:

$$l(c) = \int_c |c'(t)|dt \tag{9}$$

with the restrain condition

$$c'(t) \cdot c''(t) = 0, \ \ \forall t \tag{10}$$

where c is a curve.

The numerical results show the model satisfies the $Connectivity\ Principle$ (see figures 5, 6, 9 and 12).

3.1. The inpainting algorithm

Different from the algorithm presented by Masnou and Morel [11] here it does not bring about the necessity of the detection of T-junctions on the boundary ∂D of an inpainting domain. Also the solution of a fourth order differential equation is not calculated, thus avoiding serious problems such as convergence, stability or the posing constants a and b in (eq.5) and (eq.6) which should be investigated once the values set for these two constants interfere with the results.

To implement the ideas behind equations (9) and (10) to carry out the inpainting we propose an algorithm which can be described in the following steps:

1. Definition of the inpainting domain D.

2. Automatic detection of the points belonging to the boundary ∂D of D. A pixel x belongs to the border if, and only if, it belongs to the inpainting domain and any neighborhood $V(x, r)$ with its center at pixel x and radius r, with $r > 1$, contains at least one pixel not belonging to the inpainting domain.

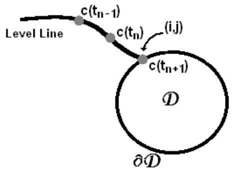


Figure 3. The level line continuation.

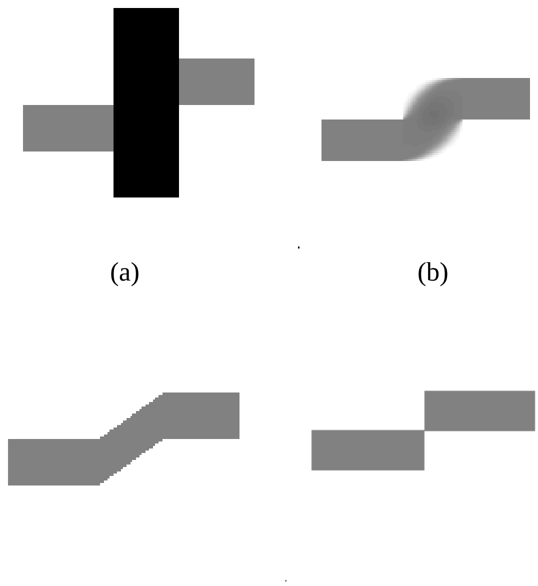
3. The filling in of information on the pixels border (belonging to ∂D) is performed in such a way as to satisfy the condition given by the equations 9 and 10. In the numerical discretization these conditions are reached by the following procedure:
   (a) For each pixel on the border, the level lines in k directions and both ways of each direction, are analyzed (for example if $k = 4$, the level lines arriving in D in the direction N-S, S-N, NE-SW, SW-NE, E-W, W-E, SE-NW and NW-SE are analyzed) and the direction of the directional derivative of least intensity is chosen (at this moment, the signal of directional derivative is not considered).
   (b) If there exist two different directions satisfying the criterion (a) we chose that one which has the greatest flow perpendicular to the level line.
   (c) If there exist yet two different level lines satisfying both criteria we chose that one which has the shortest distance between $c(t_{n+1})$ and $c(t_n)$.
   (d) The value transported to the pixel in question is

$$c(t_{n+1}) = c(t_n) + \Delta t c'(t_n)$$

   where Δt is the Euclidean distance between $c(t_{n+1})$ and $c(t_n)$, see figure 3. This means the value of the nearest pixel, in the same direction of the chosen level line and which does not belong to the inpainting domain, added to the value of the directional derivative along the chosen level line (here the signal of directional derivative is considered).
      i. This procedure is repeated until the values of the boundary points do not change anymore or until a number maximum of repetitions is reached. This procedure helps to avoid the repetition of the values obtained with the first iteration and also to consider the possibility of the existence of several different points in the boundary of D all belonging to the same level line. This procedure allows for the transportation of some kind of texture. We will refer to this procedure as the border analysis.
      ii. The elements modified by the transportation are excluded from the inpainting domain. This means, we make $D = D - \partial D$.
      iii. If elements still exist in the inpainting domain, go back to the step 2.

4. Final Remarks and Experimental Results

The method proposed in this paper is based on the same principles as the BSCB scheme (inpainting domain definitions and the transport procedure at pixels belonging to the inpainting domain), but carrying out, the transport of the information belonging to the surrounding neighborhood of the domain inpainting border and also in the form as to fill-in this domain in a different way. Also, another difference is that in the proposed model there is no diffusion process used in the BSCB technique to correct the level lines direction. The proposed model is closed to the Masnou ideas but performing the transport of the information in a com-



(a) (b)

(c) (d)

Figure 4. Inpainting of two parallel bars occluded by a black vertical rectangle. (a) Initial, (b) result obtained by the BSCB scheme (100 iterations with 10 inpaints and 100 diffusions for each iteration were performed), (c) result obtained by Masnou and Morel scheme and (d) result obtained by the proposed LLC algorithm.

pletely different way. In figure 4 we can see the main difference between the proposed model, the Masnou model and the BSCB model. The inpainting domain (a vertical box) occulting part of the two parallel bars of the same level in the gray scale, were restored in a different way. In the Masnou and BSCB schemes the two bars were connected but in a different form (figure 4-b,c respectively) while the inpainting carried out by the proposed model does not connect the two bars (figure 4-d). In the BSCB scheme the user must paint all of domain D. After that the whole domain D is modified in each iteration while in the proposed model only the boundary of the domain D will be restored in each iteration, then a new inpaint domain $D^K = D^{k-1} - \partial D^{k-1}$ is considered and the filling-in procedure is applied again and again until the domain D becomes an empty set. The transportation is carried out in such a way as to satisfy the $Connectivity\ Principle$, independent of the topology or the size of the inpainting domain (see figures 5 to 12).

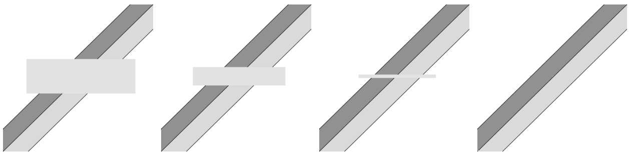


Figure 5. The transverse bar occluded by a rectangle. Initial, two partial and the final results.

Several experiments were realized using the proposed technique. We will present some of these experiments to illustrate the performance of our algorithm in several situations and how the proposed inpainting scheme works.

We present in figures 5 to 12 the performance of the proposed LLC algorithm for several different types of occlusion problems. Our first experiment shows the performance of the proposed model in the inpainting of two synthetic images, a inclined bar occluded by a rectangle and a textured synthetic image. Figure 5 shows the initial, two partial and the final results concerning the transverse bar occluded by a rectangle. Figure (6-a) shows the initial damaged textured image. Figures (6-b,c) show the final results obtained by the proposed model with and without the application of the procedure denominated border analysis, respectively.

The second experiment shows the performance of the proposed algorithm in restoration of real scene images with inpainting domains of different forms and sizes. Figures (7-b,c) show the inpainting domain and final results in the restoration of the damaged Lenna image in (7-a). Figure (8) and figure (9) show the partial and final results of the proposed model in two damaged version of Mandrill. Figure 10 shows a original bridge where the occlusion area is in white. Partial and final inpainting results obtained by the LLC algorithm are shown.

In figure (11) one can see that the discontinuities have been recovered well in the restoration of the damaged Lenna image. Finally, figure (12) illustrates the performance of the proposed LLC algorithm in a scratched landscape.

All the experiments have been executed with images of 256×256 pixels size and with 256 levels of gray intensity. The code was implemented in C language and run on a PC $2GHz$ computer with $256Mb$ of RAM memory. The time spent was less than one second in all experiments.

5. Conclusion

In this paper we have introduced a new level line based algorithm for image inpainting to fill-in a damaged domain keeping the harmony of the image through the use of a method built in a way as to satisfy the $Connectivity\ Principle$. The scheme consists of extending the level lines, in the direction of the isophotes, arriving on the boundary of the damaged domain into this same domain. The user needs only to define the area to be inpainted. After that the algorithm is automatic. The proposed technique is fast and robust to restore the level lines and to fill-in the inpainting domain. Numerical experiments show the good

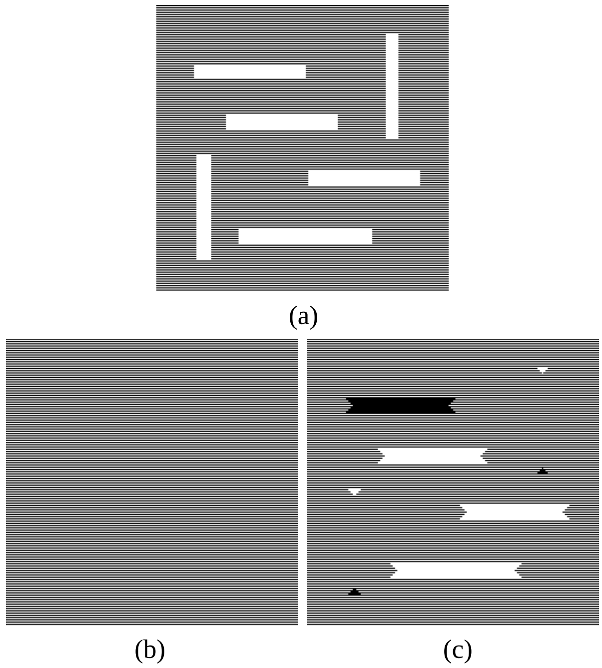


Figure 6. (a) Initial image damaged by the six white bars, (b) final result, (c) final result without the border analysis procedure.

performance of the proposed model even when the image has a natural texture, as can be seen in figure 9.

One advantage of the geometric nature model is that even though the implicit contour itself can develop singularities (like cusp and corners) and can merge or split to change topology, the level lines remain well defined and thus, one can discretize the evolutionary curve on a fixed cartesian grid.

The results produced by this geometric model are, in many cases, comparable to previous inpainting models presented in literature, but in some cases the performance is better and faster. The main reason is that the proposed algorithm does not use the diffusion process so we can restore texture better.

The proposed model can obtain good results even when the region D is not a small area as we can see in figure 9 and 11.

Acknowledgements

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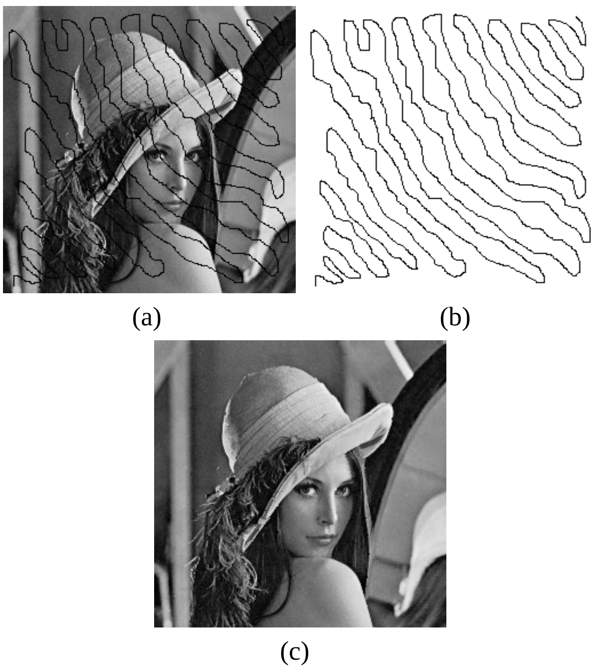

Figure 7. Lenna. (a) Initial, (b) inpainting domain and (c) final result of the inpainting in the initial image.

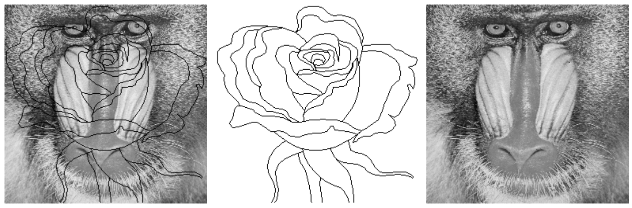

Figure 8. The Mandrill figure with an imposed flower contour. The initial image, the inpainting domain and the final result, respectively.

References

[1] M. Bertalmio, G. Sapiro, V. Caselles and C. Ballester, *Image Inpainting*, Computer Graphics, *SIGGRAPH 2000*, pp.417-424, 2000.

[2] M. Bertalmio, *Processing of flat and non-flat image information on arbitrary manifolds using Partial Differential Equations*, University of Minnesota, Thesis, 2001.

[3] C. Braverman, *Photoshop retouching handbook*, IDG Books Wordwide, 1998.

[4] M.P. Carmo, *Differential Geometry of Curves and Surfaces*, Prentice Hall, 1976.

[5] T. Chan and J. Shen, *Mathematical models for local non-texture inpainting*, SIAM J. Appl. Math., 62(3), 1019-1043, 2001.

[6] T. Chan, S. H. Kang and J. Shen, *Euler's elastica and curvature based inpainting*, SIAM J. Appl. Math., 63(2), 564-592, 2002.

[7] F. Guichard and J.M. Morel, *Partial differential equations and image iterative filtering*, TR 9535, CERAMADE, Université Paris-Dauphine, France, 1995.

[8] H. Hilbert and S. Cohn-Vossen, *Geometry and Imagination*, Chelsea, 1962.

[9] G. Kanizsa, *Organization in Vision*, Praeger, New York, 1979.

[10] S. Masnou *Disocclusion: A Variational Approach using Level Lines*, IEEE Transactions on Image Processing, vol.11, 2, 68-76, 2002.

[11] S. Masnou and J.M. Morel, *Level-lines based disocclusion*, 5th IEEE Int'1 Conf. on Image Processing, Chicago, IL. Oct 4-7, 1998.

[12] M. Nitzberg, D. Mumford T. Shiota, *Filtering, segmentation and depth*, Springer-Verlang, Berlin, 1993.

[13] M.M.Oliveira, B.Bowen, R.Mckenna, Y.S. Chang, *Fast Digital Image Inpainting*, Proc. Int. Conf. Vis. Imag. and Image Processing, Marbella, Spain, 2001.

[14] S. Osher and J.A. Sethian, *Fronts propagation with curvature dependent speed: Algorithms based on Hamilton-Jacobi formulations*, J. Comput. Phys., 79, 12-49, 1988.

[15] J.A. Sethian, *Level Set Methods*, Cambridge University Press, 1996.

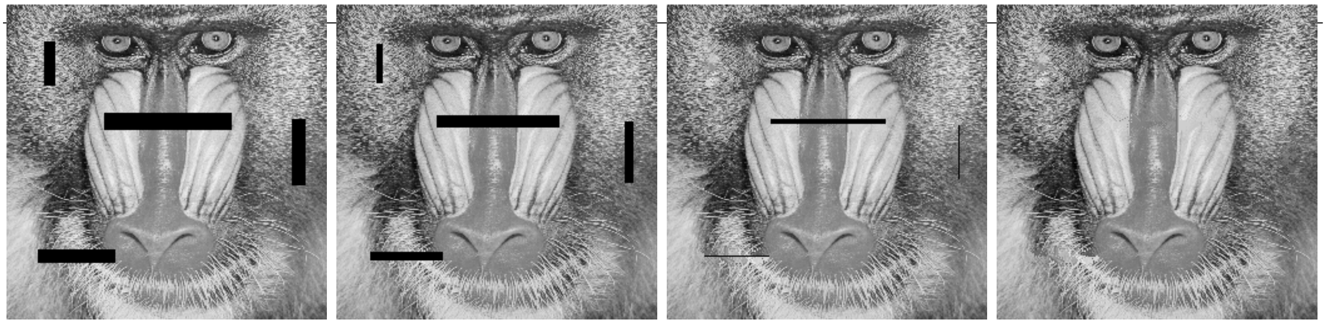

Figure 9. Mandrill damaged initial image, partial and final results obtained by the proposed model.

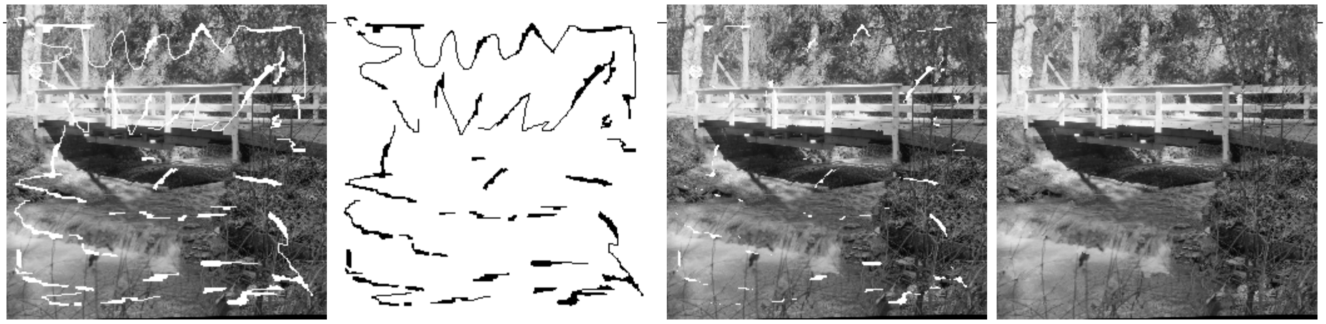

Figure 10. Initial damaged bridge, inpainting domain, partial and final results, respectively.

Figure 11. Restoration of a damaged Lenna image. Initial, partial and final results.

Figure 12. Restoration of a scratched landscape. Initial, inpainting domain, partial and final results, respectively.